Percentages

Pupil Name

Handbook Designed by Dr Buxton
## Timetable

### Timetable – Tutorials

<table>
<thead>
<tr>
<th>Tutorial</th>
<th>Date</th>
<th>Time</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Baseline assessment)</td>
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<td>2</td>
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<td>3</td>
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<td>4</td>
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<td>5</td>
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<tr>
<td>6 (Final assessment)</td>
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<tr>
<td>7 (Feedback)</td>
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</tbody>
</table>

### Timetable – Homework Assignments

<table>
<thead>
<tr>
<th>Homework Assignment</th>
<th>Description</th>
<th>Due Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tutorial 1</td>
<td></td>
<td>/ /</td>
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<tr>
<td>Tutorial 2</td>
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<td>Tutorial 3</td>
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<td>Tutorial 4</td>
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<tr>
<td>Tutorial 5</td>
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Course Rationale

Have you considered going to university when you finish school? The reason you have been selected for this course is because we believe you have the potential to thrive in higher education – you just need to be given some more tools and opportunities to succeed! This is what Uni Pathways will provide for you – an opportunity for you to improve your skills and think about how you can unlock your potential.

This year, the Uni Pathways course has a special focus on Percentages. This is a topic that features heavily in the Mathematics specification, therefore your mastery of it will go a long way to help you achieve the highest possible grade that you can by the end of Year 11.

Uni Pathways will not only help you with this topic, but it will also provide a glimpse into the world of metacognition. This effectively means “thinking about thinking”, which is a strategy that will help you succeed in all of your GCSE subjects, not just Maths! Examples of the application of metacognition include revision strategies, self-evaluation of progress and deliberate practice.

But what does this mean? What is deliberate practice? Well here is an example of it in action: a group of eight-year olds practiced tossing bean bags into buckets in a PE lesson. Half of the kids tossed into a bucket from three feet away. The other half mixed it up by tossing into buckets two feet and four feet away. After 12 weeks, they were all tested on tossing beans bags into a three-foot bucket. Which group do you think performed best? It may surprise you to learn that the second group fared much better in this test, because they were exposed to variation and had chance to adjust their skills to different scenarios.

This strategy of deliberate practice is something you will see in the Uni Pathways course, and hopefully by the time you complete a final assessment, you will be able to decide for yourself which methods of revision/practice work best for you and why.
<table>
<thead>
<tr>
<th>Word</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>A specified amount in every hundred (out of one hundred).</td>
</tr>
<tr>
<td>Multiplier</td>
<td>A number used to multiply by an amount to find a new percentage.</td>
</tr>
<tr>
<td>Growth</td>
<td>Increase (the amount gets larger).</td>
</tr>
<tr>
<td>Decay</td>
<td>Decrease (the amount gets smaller).</td>
</tr>
<tr>
<td>Compound</td>
<td>More than one/once.</td>
</tr>
<tr>
<td>Decimal</td>
<td>A whole number followed by a decimal point and further digits (e.g. tenths, hundredths, ...).</td>
</tr>
<tr>
<td>Iteration</td>
<td>The repetition of a process for a certain number of times.</td>
</tr>
</tbody>
</table>

Add more words to this list as you come across them! If you are unsure about a word, look it up online (look at a few sources for definitions because some definitions can be hard to understand).
Lesson 1 – Multipliers and Percentages of Amounts

Objectives in this tutorial:

1. Convert between percentages and multipliers.
2. Calculate percentages of amounts using multipliers.
3. Set up and solve worded percentage problems (including with fractions).

Objective 1. Convert between percentages and multipliers.

The word ‘percent’ translates to ‘out of one hundred’. Hence, typically we see percentages as numbers between 0 and 100, although it is possible for percentages to go beyond this upper limit. For example, 200% represents twice the original amount.

In order to convert from a percentage to a multiplier, you need to divide by 100.

Percentage \( \div 100 \) Multiplier

In order to convert from a multiplier to a percentage, you need to multiply by 100.

Multiplier \( \times 100 \) Percentage

Worked Example 1. Convert 32% to a multiplier.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

When dividing by 100, move the place value entries twice to the right.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The above place value table describes the number 32.
To obtain the correct multiplier, move all place value entries twice to the right:

The above place value table describes the number 0.32, which is the multiplier.

Hence, \( 32 \div 100 = 0.32 \).
The above place value table describes the number 0.2.
To obtain the correct multiplier, move all place value entries twice to the right:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The above place value table describes the number 0.002, which is the multiplier.

Hence, \(0.2 \div 100 = 0.002\).

**Worked Example 3.** Convert the multiplier 2.3 to a percentage.

When multiplying by 100, move the place value entries twice to the left:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The above place value table describes the number 2.3.
To obtain the correct percentage, move all place value entries twice to the left:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The above place value table describes the number 230, which is the percentage.

Hence, \(2.3 \times 100 = 230\), therefore our answer is 230%.

**Practice 1. Convert between percentages and multipliers.**

1. Convert the following percentages to multipliers:
   (a) 53%   (b) 19%   (c) 25%   (d) 74%
   (e) 65%   (f) 50%   (g) 70%   (h) 10%
   (i) 90%   (j) 3%    (k) 8%    (l) 5%

2. Convert the following percentages to multipliers:
   (a) 15.2%  (b) 23.5%  (c) 90.3%  (d) 62.81%
   (e) 1.7%   (f) 6.8%   (g) 8.15%  (h) 0.5%
   (i) 102.5% (j) 352.8% (k) 1047% (l) 2938%
3. Convert the following multipliers to percentages:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>1.63</td>
<td>(b)</td>
<td>1.25</td>
</tr>
<tr>
<td>(c)</td>
<td>1.8</td>
<td>(d)</td>
<td>1.01</td>
</tr>
<tr>
<td>(e)</td>
<td>2.5</td>
<td>(f)</td>
<td>2.97</td>
</tr>
<tr>
<td>(g)</td>
<td>3.15</td>
<td>(h)</td>
<td>3.82</td>
</tr>
<tr>
<td>(i)</td>
<td>4.7</td>
<td>(j)</td>
<td>10.62</td>
</tr>
<tr>
<td>(k)</td>
<td>15.8</td>
<td>(l)</td>
<td>10.08</td>
</tr>
</tbody>
</table>

Assessment 1. Convert between percentages and multipliers.

1. What is 400% as a multiplier?
   - A 0.400
   - B 40,000
   - C 4
   - D 400

2. When all are converted to a percentage, which is the largest?
   - A 0.471
   - B 40.7%
   - C 4.007
   - D 4.7%

Objective 2. Calculate percentages of amounts using multipliers.

You can use the following formula to find the percentage of a particular amount:

\[
\text{Amount} \times \text{Multiplier} = \text{Percentage of amount}
\]

This formula works for any type of amount, such as ordinary numbers, currency and measurements.

**Worked Example 1.** Find 60% of 235.

The amount is 235.
The percentage is 60%.
Therefore, the multiplier is \(60 \div 100 = 0.6\).

Using the formula:
235 \(\times\) 0.6 = 141.

60% of 235 is 141.

**Worked Example 2.** Find 35.5% of £6,816.

The amount is £6,816.
The percentage is 35.5%.
Therefore, the multiplier is \(35.5 \div 100 = 0.355\).

Using the formula:
£6,816 \(\times\) 0.355 = £2,149.68.
1. Calculate the following:
   (a) 10% of 60  (b) 20% of 60  (c) 20% of 30  (d) 10% of 30
   (e) 5% of 30  (f) 5% of 3  (g) 1% of 3  (h) 1% of £270
   (i) 10% of £270  (j) 11% of £270  (k) 21% of £270  (l) 121% of £270
   (m) 221% of £270  (n) 221% of 20g  (o) 79% of 20g  (p) 158% of 20g
   (q) 15.8% of 20g  (r) 1.58% of 20g  (s) 1.58% of 40g  (t) 1.58% of 160g

2. Describe any patterns/links that you noticed between questions for part 1.

Assessment 2. Calculate percentages of amounts using multipliers.

1. Which of these is largest?
   A 150% of 60  B 40% of 200  C 60% of 100  D 99% of 80

2. In Year 9, there are 160 students. 7.5% of the students are left-handed. How many students are left-handed?
   A 120  B 12  C 21  D 1,200

Objective 3. Set up and solve worded percentage problems (including with fractions).

A question in your GCSE exam is likely to involve a real-life situation where you are expected to recognise and set up key parts such as the multiplier and amount(s). The situations can vary, but the key information will always remain.

Sometimes, there may be more than one percentage to calculate, or you may be asked to choose the better option from a list. What is important is that you can identify the amount and the percentage multiplier in every case.

**Worked Example 1**. Maxwell is paid £460.
   He spends 38% on his rent, 13% on his food and 20% on his bills.
   How much money does he have left over?

   [Rent] 38 ÷ 100 = 0.38
           £460 x 0.38 = £174.80

   [Food] 13 ÷ 100 = 0.13
           £460 x 0.13 = £59.80

   [Bills] 20 ÷ 100 = 0.2
           £460 x 0.2 = £92
Therefore, the remaining amount that Maxwell has left over is £460 - £326.60 = £133.40.

**Worked Example 2.**
There are 80 teachers in a school.
The headteacher says that exactly 89% of the teachers drive to work.
Explain why the headteacher is wrong.
\[
\begin{align*}
89 & \div 100 = 0.89 \\
80 & \times 0.89 = 71.2
\end{align*}
\]
The headteacher is incorrect because the number of teachers that drive must be an integer (whole number), whereas the answer is not (71.2).

**Practice 3. Set up and solve worded percentage problems (including with fractions).**

1. Solve the following percentage problems:

   (a) Oliver’s salary is £18,000 and he is due to get an increase of 4%.
       How much will this increase be?

   (b) Barry earns £1,300 a month. He spends 30% of his money on rent and 12% on bills.
       How much of the original £1,300 does he have left?

   (c) There are 52,800 fans at a football match between Rovers and City.
       37% of the fans support Rovers.
       How many fans at the match support City?

   (d) Here is a list of political parties and the number of votes they received at an election:

       Gold Party  12,598 votes  
       Pink Party  9,112 votes    
       Brown Party 20,059 votes  
       Blue Party  4,466 votes

       There are 52,852 people who can vote.
       The target was that 88% of people would vote.
       Was the target met?

   (e) A ball is dropped from a height of 3m and is allowed to bounce repeatedly. Each time it rises to a height which is 80% of the height it fell from. What height does the ball rise to after the second bounce?

   (f) An adult ticket for the cinema costs £12.80.
       A child ticket is half the price of an adult ticket.
       Mr and Mrs Henderson and their six children go to see a movie.
       Mrs Henderson has a voucher that means she pays 78% of the total normal price.
       How much money does she spend?
1. Mrs Smith sold 72% of the 50 cakes she made at the charity event. How many cakes did she have left?

A 36  B 14  C 22  D 70

2. The total marks for a test are 120. 80 marks are required for the grade that Sally wants. She gains 65% of the total marks in the test. Does she get the grade she wants? State the number of marks she scored.

A No (52)  B No (78)  C Yes (96)  D Yes (108)
Lesson 2 – Percentage Increase and Decrease

Objectives in this tutorial:
1. Calculate the appropriate percentage multiplier when an amount increases or decreases.
2. Increase or decrease a quantity by a given percentage.
3. Set up and solve worded percentage change problems (including with fractions).

Objective 1. Calculate the appropriate percentage multiplier when an amount increases or decreases.

Exam questions can commonly ask for percentage increases/decreases by using different words. Here is a list of words that mean 'increase': growth; earn; appreciate; rise. Here is a list of words that mean 'decrease': fall; lose; depreciate; decay.

Worked Example 1. Write an increase of 68% as a multiplier.

When increasing by a percentage, we add the increase to 100%.

\[
100\% + 68\% = 168\%
\]

The resulting percentage is then converted to a multiplier by dividing by 100.

\[
\frac{168\%}{100} = 1.68
\]

Worked Example 2. Write a fall of 11.2% as a multiplier.

When decreasing by a percentage, we subtract the decrease from 100%.

\[
100\% - 11.2\% = 88.8\%
\]

The resulting percentage is then converted to a multiplier by dividing by 100.

\[
\frac{88.8\%}{100} = 0.888
\]

Practice 1. Calculate the appropriate percentage multiplier when an amount increases or decreases.

What is the multiplier?
1. Rise of 27%
2. Decrease of 13%
3. Growth of a half
4. Fall of one fifth
5. Earn an extra 2.5%
6. The value depreciates by 0.3%
7. The mass decays by one eighth
1. Simon’s car has depreciated in value by 2.5%. What multiplier should he use to calculate the new value of his car?

   A 1.25  B 0.975  C 0.75  D 2.5

2. Lichfield's population has grown by 4.1% last year. Which multiplier would be used to calculate the new population?

   A 1.41  B 1.041  C 4.1  D 0.41

Objective 2. Increase or decrease a quantity by a given percentage.

Once the correct multiplier is found, use it to multiply by the original amount to find your answer.

**Worked Example 1.** Increase 54 by 68%.

   When increasing by a percentage, we add the increase to 100%.

   100% + 68% = 168%

   The resulting percentage is then converted to a multiplier by dividing by 100.

   168% / 100 = 1.68

   Finally, multiply your result by the original amount to find the final answer.

   1.68 \times 54 = 90.72

**Worked Example 2.** Decrease 300 kg by 11.2%.

   When decreasing by a percentage, we subtract the decrease from 100%.

   100% - 11.2% = 88.8%

   The resulting percentage is then converted to a multiplier by dividing by 100.

   88.8% / 100 = 0.888

   Finally, multiply your result by the original amount to find the final answer.

   0.888 \times 300 kg = 266.4 kg
Practice 2. Increase or decrease a quantity by a given percentage.

Question 1
(a) Increase 20 by 50%  
(b) Increase 60p by 10%  
(c) Increase 12g by 25%
(d) Increase 400 litres by 20%  
(e) Increase 32ml by 75%  
(f) Increase 70m by 40%
(g) Increase 9000 by 5%  
(h) Increase £7 by 20%  
(i) Increase 9kg by 100%

Question 2
(a) Decrease 40 by 10%  
(b) Decrease 30 hours by 50%  
(c) Decrease 8kg by 25%
(d) Decrease 55cm by 40%  
(e) Decrease 64 by 75%  
(f) Decrease £3 by 10%
(g) Decrease 1400 by 30%  
(h) Decrease 500g by 3%  
(i) Decrease 6kg by 5%

Question 3
(a) Increase 80ml by 9%  
(b) Increase 420g by 70%  
(c) Decrease 8 by 12%
(d) Decrease £1250 by 38%  
(e) Increase 6000km by 23%  
(f) Decrease 48GB by 6%
(g) Increase 204 by 98%  
(h) Decrease 149mm by 91%  
(i) Increase 88 by 185%

Question 4
(a) Decrease 90ml by 7.5%  
(b) Increase £670 by 1.2%  
(c) Increase 3 by 67.4%
(d) Increase 750cm by 0.8%

Assessment 2. Increase or decrease a quantity by a given percentage.

1. Increase 67.5 by 32%.
   A 21.6   B 89.1   C 216   D 69.66

2. Decrease 5700 by 17.5%.
   A 4702.5   B 6697.5   C 997.5   D 6909.09
Objective 3. Set up and solve worded percentage change problems.

Exam-style questions will often ask you a question in context, rather than simply finding the percentage increase/decrease of an amount. The key skills you have picked up so far are what you will use for these questions — all you have to do is identify three pieces of information from the text:

1. The original amount.
2. Whether it is a percentage increase or decrease.
3. The increase/decrease as a percentage.

Once you have all this information, you can proceed, just like you did in the previous objective.

Worked Example 1. The average bus ticket for customers last year costed £2.60. Politicians this year have announced that they plan to cut this price down by 15%. What will the new average bus ticket price be?

Remember to identify your three pieces of information:
1. The original amount is £2.60.
2. “Cutting the price down” means it is a percentage decrease.
3. The decrease is by 15%.

\[
\begin{align*}
100\% & \quad 15\% & \quad 85\% \\
85\% & \div 100 & \equiv 0.85 \\
0.85 & \times £2.60 & \equiv £2.21
\end{align*}
\]

Practice 3. Set up and solve worded percentage change problems. You can add a text box next to the question to have your workings out and answer, or write / type it somewhere else.

1. Last year, there was 20 students in a class. This year, there are 30% more students. How many students are in the class this year?

2. A TV normally costs £520. In a sale, all prices are reduced by 10%. Calculate the sale price of the TV.

3. Over the past 10 years, the population of a town has increased by 25%. The population of the town 10 years ago was 18000. What is the population of the town now?

4. A standard bag of flour contains 600g of flour. A special edition bag contains 35% more flour. How much flour is in the special edition bag?

5. Richard owns a coffee shop. In February, 4500 hot chocolates were sold. The number of hot chocolates sold in March was 3% less. How many hot chocolates are sold in March?
6. **Gabriel’s salary is £24500.**
Next year he is due to get a 9% increase.
What will his new salary be?

7. **Iris spends £40 a month on water.**
By changing company, Iris can save 16%.
How much would Iris pay each month?

8. **An empty flowerpot has a mass of 800g.**
The mass of the flowerpot increases to 4kg when filled with soil.
A different flowerpot is 25% lighter but holds 40% more soil.
Calculate the mass of this flowerpot when it is full of soil.

---

**Assessment 3. Set up and solve worded percentage change problems.**

Train ticket prices will rise next year by 3%. If a single ticket currently costs £6, what will the new price be?

- A £7.80
- B £6.18
- C £18
- D £5.83

There was a 37% fall in newspaper sales this week, from an initial 61,000 sales in the previous week.
How many were sold this week?

- A 22570
- B 44526
- C 38430
- D 1649
Lesson 3 – Reverse Percentages

Objectives in this tutorial:
1. Calculate initial amounts when given the final amount and the percentage change.
2. Set up and solve worded reverse percentage problems.

Objective 1. Calculate initial amounts when given the final amount and the percentage change.

As seen before, you can use the following formula to find the percentage of a particular amount:

\[ \text{Final amount} = \text{Initial amount} \times \text{Multiplier} \]

This formula works for any type of amount, such as ordinary numbers, currency and measurements.

But what if we already knew the final amount and wanted to calculate what the initial amount was? The solution is to rearrange this formula to make the initial amount the subject:

\[ \text{Initial amount} = \frac{\text{Final amount}}{\text{Multiplier}} \]

Worked Example 1. 45% of a number is 90. What is the number?

- The final amount is 90
- The percentage multiplier is 0.45

Substituting this information into the formula gives:

\[ \frac{90}{0.45} = 200 \]

Therefore, the initial amount was 200.

Worked Example 2. After finding 31% of a value, my answer was 207.7. What is the missing value?

- The final amount is 207.7
- The percentage multiplier is 0.31

Substituting this information into the formula gives:

\[ \frac{207.7}{0.31} = 670 \]

Therefore, the initial amount was 670.
Practice 1. Calculate initial amounts when given the final amount and the percentage change.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% of (x) is 7. What is (x)?</td>
<td>30% of (x) is 12. What is (x)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% of (x) is 6. What is (x)?</td>
<td>30% of (x) is 48. What is (x)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20% of (x) is 6. What is (x)?</td>
<td>16% of (x) is 48. What is (x)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20% of (x) is 12. What is (x)?</td>
<td>16% of (x) is 4.8. What is (x)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25% of (x) is 12. What is (x)?</td>
<td>1.6% of (x) is 4.8. What is (x)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5% of (x) is 12. What is (x)?</td>
<td>1.6% of (x) is 1. What is (x)?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assessment 1. Calculate initial amounts when given the final amount and the percentage change.

1. The final amount after taking 67% of a number is 562.8. What was the original number?
   - A 377.076
   - B 840
   - C 185.724
   - D 1705.45

2. A number was increased by 23% to make 418.2. What was the original number?
   - A 96.186
   - B 1818.26
   - C 543.12
   - D 340
For exam-style questions, be sure to identify key pieces of information:

- Final amount
- Percentage multiplier

Then, substitute the pieces of information into the formula to find the initial amount.

\[
\text{Final Amount} \div \text{Multiplier} = \text{Initial amount}
\]

Practice 2. Set up (in a separate document / on paper) and solve worded reverse percentage problems.

1. Apple is having a 20% off sale. I bought my new iPad for £40, how much was it originally?

2. Christmas is on its way, so the price of a box of Thornton’s chocolates has been increased by 15%!! A box of Thornton’s Continental now costs £17.25. How much did it cost originally?

3. The price of all oyster card season tickets increased by 4%. The price of a season ticket from zone 1 to zone 4 increased to £98.80. What was the price before this increase?

4. A special bottle of coke contains 10% more than a normal bottle. The special bottle contains 660 ml. How much does the normal bottle contain?

5. Katie gets a 20% pay rise. Her new wage is £264 per week. What was her wage before the pay rise?

6. Ivan Ukhov the 2012 Olympics high jump gold medallist jumps 2.4 metres. This is 4% lower than the best height he can jump. What is the best height he can jump?

7. A packet of Hobnobs claims to be 24% bigger!! It now contains 31 biscuits. How many did it have before?

8. Delboy sells his goods at Peckham market for £66.70 making a 15% profit. How much did he pay for the goods when he bought them?

9. Since being on Strictly Come Dancing Vanessa Feltz has lost 20% of her body weight. She now weighs 90 kg. How much did she weigh before the show?
10. During a mid-season sale a shop keeper sold 60% of his Kindles. He then found that he was left with 50 Kindle’s. How many did he have in stock to begin with?

11. Jacob answered 80% of the questions in a test correctly. He answered 32 of the questions correctly. Work out the total number of questions in the test.

12. In a class there are 9 people absent with the flu. This is 20% of the class missing out on vital mathematics! How many are there in the class when no one in the class is ill? How many are in class today?

13. Rosie sells her laptop to John for an unknown price. John then sells the laptop to Michael for £391. John makes a 15% loss. How much did John pay for the laptop?

**Assessment 2. Set up and solve worded reverse percentage problems.**

1. A bowl of Cheerios provides 16.2 mg of vitamin C. This is 24% of the recommended daily intake. What daily intake of vitamin C is recommended?
   - A 67.5 mg
   - B 20.088 mg
   - C 21.32 mg
   - D 3.88 mg

2. A new Ford Fiesta drops in value by 30% in a year. After a year, it is worth £8,400. Find the price of the car when it was new.
   - A £5,880
   - B £10,920
   - C £12,000
   - D £28,000
Lesson 4 – Compound Growth and Decay

Objectives in this tutorial:
1. Calculate sequential percentage increases/decreases.
2. Calculate compound growth and decay using the correct formula.
3. Set up and solve worded compound percentage problems.

Objective 1. Calculate sequential percentage increases/decreases.

In the real world, percentages can change. For example, a house may go up in value and be worth 110% of the original price, that same house could also drop in value and be worth 95% of the original cost. To model these changes from an initial amount, we can simply calculate using a series of multipliers, rather than just one.

Worked Example 1. Increase £1,000 by 6%, and then decrease by 8%.

From the initial amounts given, calculate the final amounts by following the various percentage increases and/or decreases:

<table>
<thead>
<tr>
<th>Amount</th>
<th>Multiplier 1</th>
<th>Multiplier 2</th>
<th>Final amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>£1,000</td>
<td>1.06</td>
<td>0.92</td>
<td>£975.20</td>
</tr>
</tbody>
</table>

Practice 1. Calculate sequential percentage increases/decreases.

1. £5,000 Increase by 5%, Increase by 12%, Decrease by 37%
2. 672 kg Increase by 11%, Decrease by 2%, Increase by 23%
3. $450 Increase by 8%, Decrease by 10%, Decrease by 6%
4. 330 ml Decrease by 33%, Increase by 6.5%, Decrease by 3.2%
5. 39.5 cm Increase by 80%, Increase by 1%, Decrease by 5.9%
6. 4,000 miles Decrease by 36%, Decrease by 9%, Decrease by 31%
7. 668 mm Decrease by 2%, Increase by 67%, Decrease by 67%
8. £6,750 Increase by 8.2%, Increase by 12.7%, Increase by 20.3%

Assessment 1. Calculate sequential percentage increases/decreases.

1. Increase 314 by 60%, then by 15.5%.

   A 29.202  B 580.272  C 778.72  D 217.602

2. Decrease £600 by 17.5%, then increase by 4%.

   A £514.80  B £733.20  C £693  D £676.80
Objective 2. Calculate compound growth and decay using the correct formula.

Fairly often, instead of amounts changing by a different percentage each time, they can change by exactly the same amount. For example, a bank might promise to pay you interest on your savings at a fixed rate of 4% every year. In other words, the bank will pay you the same amount of money every year for holding your savings. If you chose this deal at the bank for three years, the formula to calculate your final amount would be as follows:

\[
\text{Amount} \times 1.04 \times 1.04 \times 1.04 = \text{Final amount}
\]

However, if the multiplier is the same each time, I can simplify my formula, because multiplying a number by itself is the same as writing that number with a power/index:

\[
\text{Amount} \times 1.04 \times 1.04 \times 1.04 = \text{Final amount}
\]

\[
\text{Amount} \times (1.04)^3 = \text{Final amount}
\]

Here, the 3 represents the number of iterations (repetitions). In this example, there are 3 years that interest is earned for, so the number of iterations is 3. This method is known as compound interest (if there is a consistent percentage increase), or compound decay (if there is a consistent percentage decrease). The general formula is as follows:

\[
\text{Amount} \times (\text{Multiplier})^{\text{iterations}} = \text{Final amount}
\]

**Worked Example 1.** Increase 5,000 by 12% for 4 iterations.

- The amount is 5,000
- The multiplier is 1.12
- The number of iterations is 4

Substituting these numbers into the formula gives:

\[
5,000 \times (1.12) = 7,867.5968
\]

**Worked Example 2.** Decrease 250 by 3% each week for 5 weeks.

- The amount is 250
- The multiplier is 0.97
- The number of iterations is 5

Substituting these numbers into the formula gives:

\[
250 \times (0.97)^5 = 214.6835...
\]

For another example, see here
Use the compound interest/decay formula to calculate the following compound decreases:

1. **£4500** depreciates by 4% every year for 5 years.
2. **£8000** depreciates by 5.5% every month for 14 months.
3. **£250** depreciates by 16% every year for 4 years.
4. **£128** depreciates by 7.8% every week for 10 weeks.
5. **£750** depreciates by 10.21% every decade for 9 decades.

Use the compound interest/decay formula to calculate the following compound decreases:

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4. **£128** depreciates by 7.8% every week for 10 weeks.
5. **£750** depreciates by 10.21% every decade for 9 decades.

**Practice 2. Calculate compound growth and decay using the correct formula.**

1) Find the compound interest when £400 is invested at 7% for 3 years.  
2) Find the compound interest when £200 is invested at 9% for 5 years.  
3) Find the compound interest when £200 is invested at 9% for 4 years.  
4) Find the compound interest when £600 is invested at 5% for 2 years.  
5) Find the final amount when £180 is invested at 8% compound interest for 4 years.  
6) Find the final amount when £470 is invested at 10% compound interest for 3 years.  
7) Find the final amount when £210 is invested at 9% compound interest for 3 years.  
8) Find the final amount when £330 is invested at 9% compound interest for 5 years.  
9) Find the final amount when £28000 is invested at 1.9% compound interest per month for 2 years.  
10) Find the final amount when £26500 is invested at 3% compound interest per month for 3 years.

**Assessment 2. Calculate compound growth and decay using the correct formula.**

1. A mass of 56 kg increases by 2% every month. What is the mass after 2 years?   
   - A 90.07 kg  
   - B 4451.82 kg  
   - C 58.26 kg  
   - D 80.64 kg

2. A distance of 500 miles decreases by 5.5% every week. What is the distance after 5 weeks?  
   - A 25.16 miles  
   - B 653.48 miles  
   - C 376.82 miles  
   - D 397.18 miles
Objective 3. Set up and solve worded compound percentage problems.

In worded (exam-style) questions, you just need to identify the same pieces of information as before:

- Initial amount
- Percentage multiplier
- Number of iterations

Be warned: sometimes the number of iterations may not be immediately obvious. For example, a question may ask you to calculate compound interest for 2 years on a monthly basis. In this case, your number of iterations would be 24 (12 months in a year), rather than simply 2.

Practice 3. Set up and solve worded compound percentage problems.

1. The number of people living on a remote island decreases by 9% every 10 years. In 1950 there were 18000 living on the island. Calculate how many less people will be living on the island in 2020.

2. A car was bought for £20,000. Its value depreciates by 31% each year for the first four years. What is its values at the end of the four years?

3. A tree is 80cm when planted. Each year the height of the tree increases by 22%. After how many complete years will the height of tree be at least 3m?

4. The number of polar bears in a region is decreasing by 5% per year. There are 3000 polar bears in the region in 2017. What year will be the first year with less than 1000 polar bears in the region?

5. Michael has started working for a company on a salary of £15000. Each year he will be given a 6% pay rise. How many years will it take for Michael’s salary to exceed £30000?

6. The value of a car decreases by 7.2% each year. When bought the car cost £6200. How many years will it take the car to have a value less than £1000?

7. A full water tank has sprung a leak. 4% of the water is lost every minute. What percentage of water is left in the tank after twenty minutes?
1. The value of a television was £600 on 1st March 2013. Every four months, the value of the television decreased by 8% of its value at the start of that four months.

What was the value of the television on 1st March 2014?

- A £1510.90
- B £467.21
- C £220.60
- D £307.20

2. A radioactive substance decays over time. Every year its mass decreases by 14%.

How many years will it take for 500kg of the substance to decay to a mass less than 200kg?

- A 1 year
- B 6 years
- C 7 years
- D Never
Reflecting on this booklet

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<tr>
<th>What did you most enjoy about Uni Pathways?</th>
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<table>
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<th>What did you find challenging about the programme?</th>
<th>How did you overcome these challenges?</th>
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## Answers

### Lesson 1

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<thead>
<tr>
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<tr>
<td>1a) 0.53 b) 0.19 c) 0.25 d) 0.74 e) 0.65 f) 0.5 g) 0.7 h) 0.1 i) 0.9 j) 0.03 k) 0.08 l) 0.05</td>
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<td>2a) 0.152 b) 0.235 c) 0.903 d) 0.6281 e) 0.017 f) 0.0675 g) 0.815 h) 0.05 i) 1.025 j) 3.528 k) 10.47 l) 29.38</td>
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<td>3a) 163% b) 125% c) 180% d) 101% e) 250% f) 297% g) 315% h) 382% i) 470% j) 1062% k) 1580% l) 10008%</td>
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| Assessment 2 | D |

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<td>1a) 720 b) 754 c) 33264 d) 87.5% e) 1.92 f) 49.92</td>
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| Assessment 3 | B |

### Lesson 2

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| Assessment 1 | B |

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\[ 5A = \frac{Q}{300} \]

\[ = 1500C \]

| Assessment 2 | B |

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| Assessment 3 | B |
### Lesson 3

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**Assessment 1**

- B
- D

**Practice 2**

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**Assessment 2**

- A
- C

### Lesson 4

**Practice 1**

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**Assessment 1**

- B
- A

**Practice 2**

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**Assessment 2**

- C
- C

**Practice 3**

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<td>e) 12 years</td>
<td>f) 25 years</td>
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<td>g) 44.2%</td>
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**Assessment 3**

- B
- C
Notes