Researchers in Schools

Algebraic Expressions and Equations

Key Stage 4 Revision Programme

Pupil Name

Handbook Designed by Dr. Olayinka Oladiran

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1 (Baseline assessment)			
2			
3			
4			
5			
6 (Final assessment)			
7 (Feedback)			

Timetable – Homework Assignments

Tutorial 1		
Tutorial 2		
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Course Rationale

The main objective of this course is to acquire a deep understanding of algebraic expressions and equations, especially highlighting the difference between both and whether to simply or solve. In addition, a deeper knowledge and understanding of the different components contained in either the algebraic expressions or equations is essential to be able to provide subsequent solutions. In order to fulfil this goal, this workbook is composed of 4 tutorials, aiming to cover the 5 major aspects of algebraic expressions and equations. The work has been created to guide you through this content by increasing the complexity of the tasks involved.

However, in order to access the desired skills, these tutorials have been developed using the principle of deliberate practice, an approach to learning that aims at practising specific skills at a time by tackling problems with increasing difficulty but similar structure. In a way, the individual variations of each problem should show you, the learner, what the key components of each of exercise consist of in basic terms.

Algebra is a core area of the mathematics curriculum. Studying algebra helps your mind to think logically, break down and solve problems. There is also the fact that modern technology relies on mathematics and algebra – Google, the internet, mobiles phones; satellites and digital televisions wouldn't exist without algebra. We are really relying on other people having studied algebra when we use a phone or play a computer game and as technology is everywhere, more and more people are needed to work behind the scenes with the knowledge of mathematics and algebra. If you get to understand the basic principle of algebra, you will get to like it and then give yourself a chance of getting involved in the rapidly expanding technology sector.

We hope that throughout this course, you will discover the pleasure of simplifying or solving algebraic expressions and equations respectively. Then subsequently learn the importance of sound thinking and expressing solutions in a specific order of operation.

Subject Vocabulary

Word	Definition	In a sentence
Variable	**Variable** – A variable is a letter or symbol that represents a number (unknown quantity). A variable can use any letter of the alphabet.	
Substitute	**Substitution** - A strategy for solving systems of equations that include solving for one variable and using that solution to find the other variable. Substitute numbers into formulae and expressions.	
Expression	**Algebraic expression** – a group of numbers, symbols, and variables that express an operation or a series of operations	
Like-terms	"**Like terms**" are **terms** whose variables (and their exponents such as the 2 in x^2) are the same. In other **words**, **terms** that are "**like**" each other. Note: the coefficients (the numbers you multiply by, such as "5" in 5x) can be different.	
Coefficient	A **coefficient** is the number multiplied by the variable in an algebraic expression.	
Simplify	**Simplify** – Combine like terms and complete all operations	
Term	A **term** is the name given to a number, a variable, or a number and a variable combined by multiplication or division.	
Solve	To **solve** something is to find a solution, like figuring out the answer to a complex riddle. The verb **solve** is often used in **mathematics**, and it means to answer a **math** problem.	
Expanding brackets	To **expand** a **bracket** means to multiply each term in the **bracket** by the expression outside the **bracket**	

Tutorial 1 – Introducing Algebraic expressions and equations

Objective: Writing Expressions and Equations; simply and solve (the basics)

Keywords: *expressions, equations, variables*

Objective 1: Write out Algebraic expressions for word phrases

Objective 2: Explain / describe the differences between Algebraic expressions and equations and find solutions to worded problems.

Example A

Write Algebraic Expressions for These Word Phrases

- Ten more than a number: **n + 10**
- A number decrease by 5: **w – 5**
- 6 less than a number: **x – 6**
- A number increased by 8: **n + 8**
- The sum of a number & 9: **n +9**
- 4 more than a number: **y + 4**

What is the difference between an Algebraic expression and an equation? Discuss in pairs.

Model Answer

An **algebraic expression** is a **mathematical phrase** that can contain ordinary numbers, variables (like **x or y**) and operators (like **add, subtract, multiply, and divide**). Examples: ***a* + 3, 4ab – 9**

An **algebraic equation** is the **combination of two equivalent expressions** that are **separated** by an **equals** sign. Examples: **3x + 2 = 14**, **5y + 7** = **4y + 10**

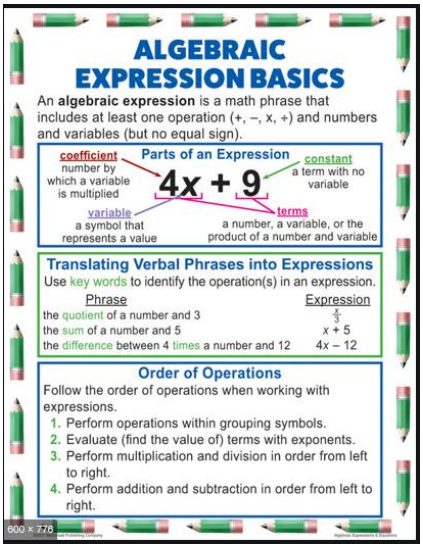
Quiz

1. Mark which is an expression or an equation:

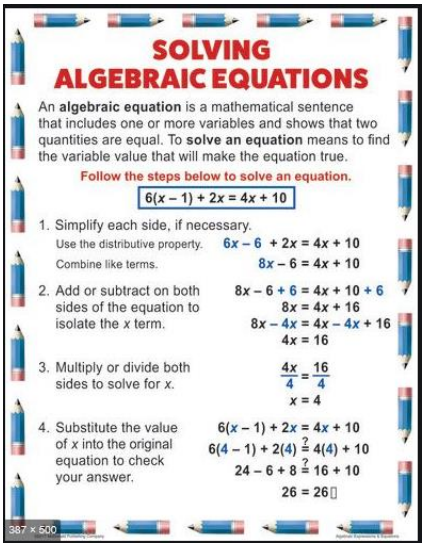
a). 5 + y = 8 b). 7a + 6 c). a = y + 3 d) $\frac{x}{5}$

e) 3b + 2c f). 7ab – 2c = 20

Algebraic Expressions Basics



Algebraic Equations Basics



Example 2 (One-Step Equation)

1. A boy has m mangoes. He sells three of them.
    a. Write an expression to represent how many mangoes he now has.
    b. If he now has 10 mangos, write an equation to represent the situation.
    c. Solve the equation from part (b).

Why is this called a 1-step equation example?

Model Answer:

Tip: **m** is a **variable** and can be any number: Check the definition of variable in the glossary.

a). **m** – 3

b). **m** – 3 = 10

c) **m** – 3 (+3) = 10 + 3

m = 13

In this case our **m** has the value of **13**

Practice A

1. A boy is 12 years old. How old will he be in *y* years' time?
    a. Write an expression to represent the boys age in *y* years' time.
    b. In *y* years' time the boy will be 27. Write an equation to represent the situation.
    c. Solve the equation from part (b).

2. A man walks *h* km towards a village which is 5 km away.
    a. Write an expression to represent how far he is currently from the village.

b. If he is currently 2.7km from the village, write an equation to represent the situation.
c. Solve the equation from part (b).

3. Ryan has *b* books. He sells half of his books then buys 13 more.
   a. Write an expression to represent half of the books.
   b. Write an expression to represent the situation after buying the extra books.
   c. If he now has 38 books, write an equation to represent the situation.
   d. Solve the equation from part (c).

Assessment A

4. 285 students go on a trip. 5 buses are filled, and 15 students travel by car.
   a. If each bus carries *p* passengers, write an expression for the number of passengers carried by 5 busses.
   b. Write an expression for the number of passengers carried by the 5 busses and the cars.
   c. Write an equation to represent the problem.
   d. Solve the equation from part (c).

   Which of the option below if the correct solution for question d?

   a) . 32 b). 60 c). 87 d). 54

5. Alisha has £24 to spend on notebooks. She buys 3 notebooks and has £12 left over.
   a. If each notebook costs *n* pounds, write an expression for the cost of three notebooks.
   b. Write an expression to represent how much money she had after buying the three notebooks.
   c. Set the expression equal to the amount she has left over to create an equation.
   d. Solve the equation from part (c).

   Which of the option below if the correct solution for question d?

   a) . 12 b). 4 c). 20 d). 8

Example 3 (Two-step equation)

1. A cycle shop rents bicycles for £18 plus £3 per hour. Jamie paid £39. For how long did he rent the bicycle?
   a) Define your variable.
   b) Write an equation to represent the problem.
   c) Solve the equation, showing all working out.

Why is this a two-step equation?

Model answer:

a) h = number of hours (variable)
b) 18 + 3h = 39

c) 18 **- 18** + 3h = 39 **-18** **(subtract 18 from both side) – STEP 1**

3h = 21

3h = 21 (**divide both sides by 3)**

h = 7

Practice B

1. Elisha bought a magazine for £2.25 and 5 candy bars. She spent £8.50. How much did each candy bar cost?

2. The sum of three consecutive numbers is 102. What is the smallest number?

3. Benny spent half of his allowance going to the movies. He washed the family car and earned £7. What is his weekly allowance if he ended with £16?

Assessment B

1. The sum of three consecutive even numbers is 150. What is the smallest of the three numbers?

Which of the option below if the correct solution?

a). 52 b). 48 c). 30 d). 10

2. The length of a football field is 30 yards more than its width. Express the length of the field in terms of its width w. If the perimeter of the field is 340 yards, find the width of the field.

Which of the option below if the correct solution?

a). 85 b). 50 c). 70 d). 100

Tutorial 2 – Write and manipulate expressions by collecting like terms

Objective: Collecting like-terms and simplifying expressions

Keywords: *expressions, like terms, simplify*

Objective: Collecting like terms and simplifying expressions

Example C

1. a + a + a + a = 4**a**

2. 4**a** + 2**b** + 3**a** + 5**b** (Group/collect *coefficient* with same *variables* together) – STEP 1

   4**a** + 3**a** + 2**b** + 5**b** (Then Simplify) – STEP 2

   7a + 7b

3. 2**b** + 2**a** + **c** - **b** (Group/collect *coefficient* with same *variables* together) -STEP 1

   2**b** – **b** + 2**a** + **c** (Then Simplify) – STEP 2

   2a + b + c

4. a^2 + **A** + **a** + a^2 (Group/collect *coefficient* with same *variables* together) -STEP 1

   a^2 + a^2 + **A** + **a** (Then Simplify) – STEP 2

   $2a^2$ + **A** + **a**

5. **ab** + **ba** = 2**ab**

Practice C

Predict what you think the answer will be and then *simplify the expression (if you can)*.

Developing – Collecting like terms

1) $a + 2a + 3a$
2) $3b + 4b + 5b - 3b$
3) $4c + 7c - 5c + 2c$
4) $6d - 4d + 3d + 6d$
5) $6e + 6e - 7e - 2e$
6) $f + f + 8f - 9f$
7) $9g - 5g + 2g + 7g$
8) $7h + 8h - 3h - 5h$
9) $9i + 10i - 7i$
10) $8j + 10j - 8j - 9j$
11) $7k + 7k - 14k$
12) $11l + 5l - 6l - 6l$
13) $9m - 8m + 10m + 5m$
14) $8n - 7n + 7n + 10n$

Securing – Collecting like terms with two different terms

1) $a + 5a - 3a + 4b + 3b$
2) $5c + 6c + 4d + 3d$
3) $7e - 5e + 4e - 5f + 8f + 2f$
4) $6g + 2h + 5g - 2g + 7h$
5) $8i - 9j - 5i + 10j$
6) $6k + 8l - 4k - 2k + 10l$
7) $7m + 2n + 7m - 2n$
8) $8p + 8p - 4p + 9q - 2p$
9) $10r - 8r + 5s - 3s + 3r + 4r$
10) $6s + 7s - 5t + 8t - 2s$
11) $9u - 7t + 2u - 11u + 7t$
12) $7x + 9x - 3x + y + 8y$

Example D

Introducing the multiplication operation:

Simplify

a) $2 \times 7 \times x = 14 \times x$ = **$14\,x$ (final solution)**

b) $5 \times y \times -3 \times y = 5 \times -3 \times y \times y = -15 \times y^2 =$ **$-15\,y^2$ (final Solution)**

Practice D

A) Simplify	B) Simplify	C) Simplify
a) $2 \times 10 \times x$	a) $4 \times -2 \times x$	e) $2x \times 7 \times x$
b) $5 \times 7 \times y$	b) $-3 \times y \times 5$	f) $5y \times y \times -3$
c) $8 \times 9 \times z$	c) $-7 \times -3 \times x$	g) $4x \times 7 \times 2x$
	d) $4 \times y \times -11$	h) $5y \times 9y \times -3x$

Assessment C and D

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Simplify the expression by combining like terms.

1) $7b - 3b + 4$ 1) _____
A) $10b + 4$ B) $8b$ C) $4b + 4$ D) $-4b + 4$

2) $9x + x - 4x + x$ 2) _____
A) $7x$ B) $x^2 + 5x$ C) $5x$ D) $-x^2 + 5x$

3) $6a - 3a - a - 12$ 3) _____
A) $3a - a - 12$ B) $3a - 12$ C) $2a - 12$ D) $3a - 13$

4) $8x - 3 + 4x - 3$ 4) _____
A) 6 B) $6x$ C) $4x - 6$ D) $12x - 6$

TRY THIS!

Which of these is the same as $2ab$?

A	B	C	D
$2 \times a \times b$	$2a - b$	$2a \times 2b$	$2a + b$

Give a reason for your choice:

_____

_____

_____

_____

Worksheet provided for additional PRACTICE AND ASSESSMENT questions

Tutorial 3 – Solving linear algebraic equations and substituting values

Objective A: Solving linear algebraic equations

Keywords: *equations, solve, solution*

Objective: Solving linear algebraic equations

Recap from Tutorial 1

Expressions vs. Equations

Expressions	Equations
$5x + 7$	$5x + 7 = 15$
$3x + 4y + 7$	$3x + 4y + 7 = 10$
$8 + 5$	$8 + 5 = 13x$
$4(x + 6)$	$4(x + 6) = 20x - 7$

Note the difference between the two; equations contain an equal sign and expressions do not.

Example E

Set 1

Problem **Solve $3y + 2 = 11$.**

$$\begin{aligned} 3y + 2 &= 11 \\ -2 \quad &-2 \\ \hline 3y \quad &= 9 \end{aligned}$$

Subtract 2 from both sides of the equation to get the term with the variable by itself.

$$\begin{aligned} \frac{3y}{3} &= \frac{9}{3} \\ y &= 3 \end{aligned}$$

Divide both sides of the equation by 3 to get a coefficient of 1 for the variable.

Answer $y = 3$

Set 2

Problem	**Solve $3x + 5x + 4 - x + 7 = 88$.**	
	$3x + 5x + 4 - x + 7 = 88$	There are three like terms $3x$, $5x$ and $-x$ involving a variable.
	$7x + 4 + 7 = 88$ $7x + 11 = 88$	Combine these like terms. 4 and 7 are also like terms and can be added. The equation is now in the form $ax + b = c$. So, we can solve as before.
	$7x + 11 = 88$ $-11 \quad -11$	Subtract 11 from both sides.
	$\frac{7x}{7} = \frac{77}{7}$ $x = 11$	Divide both sides by 7.
Answer	$x = 11$	

Practice E

:. Solve the following equations:

1. $x + 3 = 9$
2. $2x = 6$
3. $4 - x = 5$
4. $2x + 3 = 13$
5. $2x = 1$
6. $3x = 2$
7. $4x = 20$
8. $4x - 1 = 19$
9. $4x = -20$
10. $2x = -6$
11. $4x = -8$
12. $4x = -1$
13. $2x + 3 = -5$
14. $2x - 3 = 5$
15. $2x - 3 = x + 2$

Assessment E

16. $7x - 3 = 2x + 12$
17. $7y - 8 = 5y + 2$
18. $4x + 5 = 2x - 11$
19. $5x - 6 = 2x - 15$
20. $x + 2x = -15$
21. $3x - 5 = 4x - 7$
22. $2x + 7 = 5x - 3$
23. $2x + 7 = 12 - 3x$

Objective B: Substituting values in variables into an algebraic expression and equation

Keywords: *substitute, variable, equation, expression*

Objective B: Substituting values into variables in an equation

Example F

Questions	Model Answers
A1 $a=3, b=2, c=5$ Evaluate $3a+bc$	**A1** $a=3, b=2, c=5$ $3a+bc=3(3)+(2)(5)$ $=9+10$ $=19$
B1 $a=2, b=6, c=-3$ Evaluate $ab+2c$	**B1** $a=2, b=6, c=-3$ $ab+2c=(2)(6)+2(-3)$ $=12-6$ $=6$
C1 $a=-3, b=5, c=-2$ Evaluate a^2-bc	**C1** $a=-3, b=5, c=-2$ $a^2-bc=(-3)^2-(5)(-2)$ $=9+10$ $=19$

Practice F

A2 $d=7, e=4, f=13$ Evaluate $e(f-d)$	**A3** $x=5, y=3, z=6$ Evaluate $x^2-\frac{y}{z}$	**A4** $m=10, t=2$ Given that $G=\frac{m}{t^2-1}$ Find G
B2 $e=-1, f=4$ Evaluate $7(f-e)$	**B3** $p=-3, q=2, r=7$ Evaluate $p^2+2q-pr$	**B4** $p=2, q=8, r=-7$ Given that $t=pq+r$ Find t
C2 $a=3, b=-4, c=-1$ Evaluate $ab+bc-ac$	**C3** $p=-5, q=-4$ Evaluate $pq-\frac{p}{q}$	**C4** $a=-3, b=-8, c=-5$ Given that $M=a^2+\sqrt{\frac{4b-c}{a}}$ Find M

Assessment F

Evaluate 4a + 3b + 20 if a = 2 and b = 3

- 32
- 37
- 95
- 24

Evaluate 8x + 2y if x = 6 and y = 5

- 111
- 58
- 50
- 60

Evaluate $x^2 + 7y - z^2$ if x = 3, y = 5 and z = 10

- 13
- 23
- 34
- 28

Evaluate $a^2 - 15b - c^2$ if a = 5, b = 2 and c = 3

- – 35
- – 14
- – 18
- 14

Tutorial 4 – Expanding Brackets

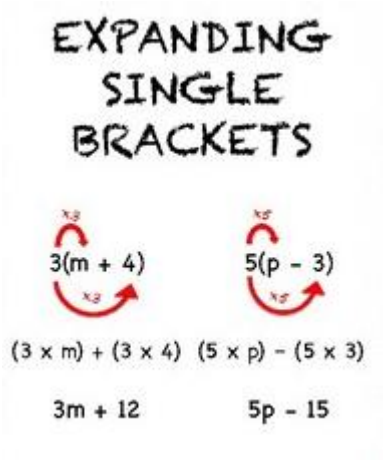
Objective 1: Expanding brackets single brackets with both positive and negative terms and simply

Keywords: *expanding, simplify, solve, like terms, coefficient*

Objective 2: Expanding brackets: Write down expressions and solve equations in application questions.

Example G

1.



2.

PRACTICE G

Q1, Expand the following with a positive term:

a) 2(x + 3) b) 5(x + 7) c) 3(2 + y)

d) 4(3 - x) e) 7(y - 6) f) 5(x + 2y)

g) 6(8 - 4y) h) 9(2x + 3y)

Q2, Expand the following with a negative term:

a) -2(x + 3) b) -4(x – 2) c) -5(2x + 4)

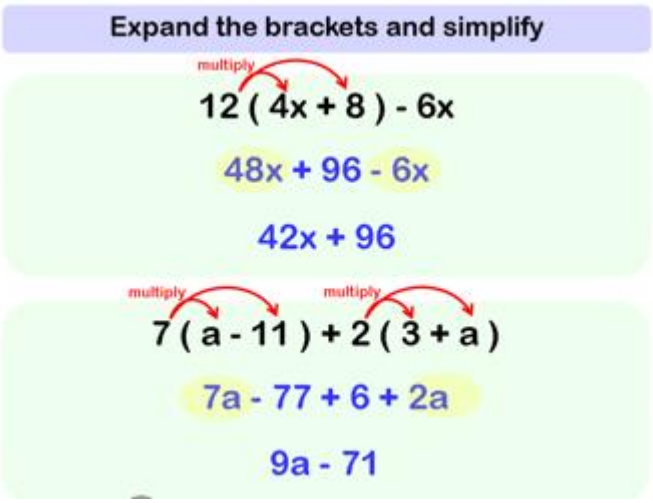
d) -6(y – x) e) -3(5 – 4y) f) -7x(2x + 4)

g) -2x(2 – x) h) -5y(x – 10)

Key

+ x + = +
+ x − = −
− x + = −
− x − = +

Example 2:



Practice H

Q1. Multiply out the following brackets with positive terms, and simplify:

a) 5(x + 4) + 3(x – 2)

b) 3(x – y) + 3(x + 2y)

c) 6(x – 2) + 3(y + 4)

d) 2(2x + 3y) + 5(x + y)

e) 3(3 + x) + 5(2x - 4)

f) 4(2x + 2) + 8(4 + x)

g) 7(2x – 1) + 3(10 + x)

h) 9(x + y) + 7(x + 5)

Q2. Multiply out the following brackets with negative terms, and simplify:

a) -2(3x – 5) - 4(5 + x)

b) 3(2y – x) - 7(x + 3y)

c) -6(2x + 2) + 3(y – 8)

d) -4(x + 8) - 9(6 – 2x)

e) 2y (6 – 5x) – 3(4x – 2y)

f) -5x(7x + 2) – 6(12 – 3x)

g) 8(2x + 8) – 4(4x – 6)

h) -3x(x – 12) + 9(x – 4)

ASSESSMENT G

Multiple choice questions:

1. **12(x + 3)** =

   a). 12x + 15 b). 12x + 4 c). 12x + 36 d). 12x + 9

2. **16(g - 1)** =

   a). 16g – 15 b). 16 + 16 c). 16g + 16 d). 16g – 16

3. **-(p + 2)** =

   a). p – 2 b). -p + 2 c). -p - 2 d). p + 2

4. **-6(q + 1)** =

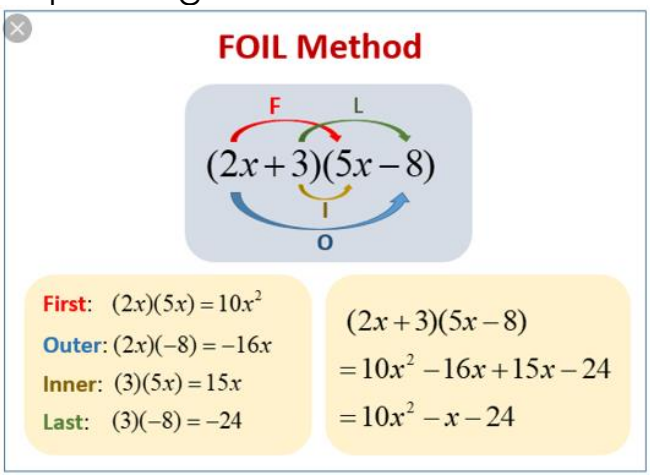
   a). 6q – 1 b). -6q + 6 c). -6q – 6 d). -6 – 1

5. **-7(m + 7)** =

   a). -7m + 7 b). 7m + 14 c). -7m – 14 d). -7m – 49

Other forms of expanding brackets (Double brackets):

3. Expanding double bracket



4. The Grid Method

Q1)Expand: $(x+3)(x-2)$

	x	-2
x	x^2	$-2x$
$+3$	$+3x$	-6

$(x+3)(x-2) = x^2 + x - 6$

ASSESSMENT H

Multiply out the brackets, collect like terms and solve the following equations:

Multiply.

5) $-3(25x)$ 5) _____
A) $-75x$ B) $25x - 3$ C) $-325 - 3x$ D) $22x$

6) $7(x + 5)$ 6) _____
A) $7x + 12$ B) $7x + 35$ C) $7x + 5$ D) $x + 35$

7) $8(y - 4)$ 7) _____
A) $8y - 32$ B) $8y - 12$ C) $8y - 4$ D) $y - 32$

Simplify the expression.

8) $-8(3y + 5) - 5$ 8) _____
A) $24y + 35$ B) $-24y - 80$ C) $-24y$ D) $-24y - 45$

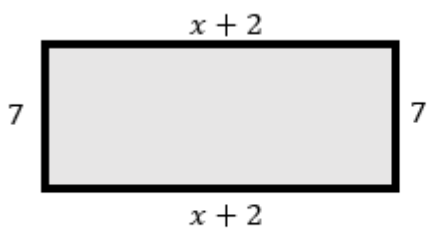
9) $8x + 4(x - 1)$ 9) _____
A) $32x + 3$ B) $12x - 4$ C) $12x + 4$ D) $9x - 4$

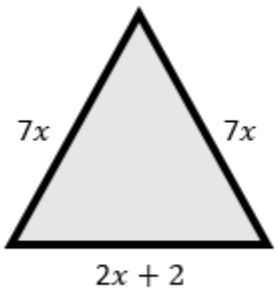
Objective 2: Expanding brackets: Write down expressions and solve equations in application questions.

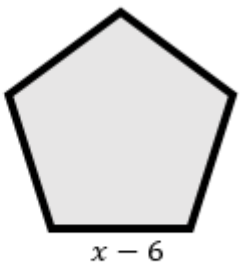
Mastering

1) $2(x + 3) + 3(x + 4)$
2) $3(x - 2) + 3(x - 4)$
3) $6x(x - 2) + 7(x - 2)$
4) $7(x + 3) + 2x(x - 2)$
5) $6(x - 2) + 2x(x - 3)$

Find the perimeter of these shapes:



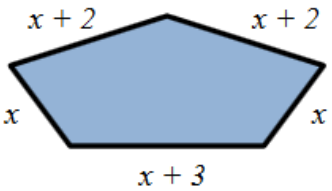




APPLICATIONS

Try this out!!

Problem Solving
Find the perimeter of the shape



1. The width of a rectangle is x centimeters and its length is $(x + 5)$ cm.

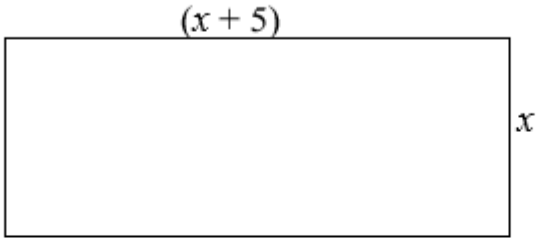


Diagram not drawn to scale

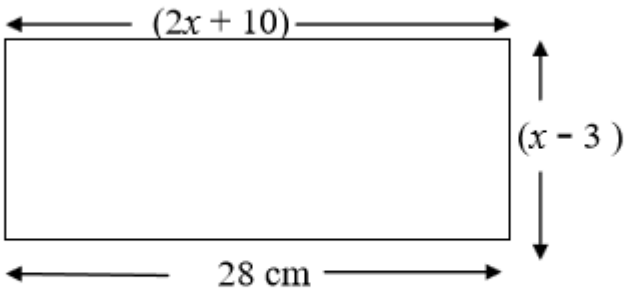
(a) Write down an expression for the perimeter of the rectangle, giving your answer in its simplest form.

The perimeter of the rectangle is 62 cm.

(b) Work out the length of the rectangle.

2. The diagram below is a rectangle. All measurements are in centimeters.

Diagram not drawn to scale



(a) Work out the value of x.

(b) Hence, work out the perimeter and area of the rectangle.

Tutorial 7 - Feedback

What is the Purpose of Tutorial 7?

- To receive feedback on final assessments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assessment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
• • •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Notes

Notes

Algebraic Expressions and Equations

Key Stage 4 Programme

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