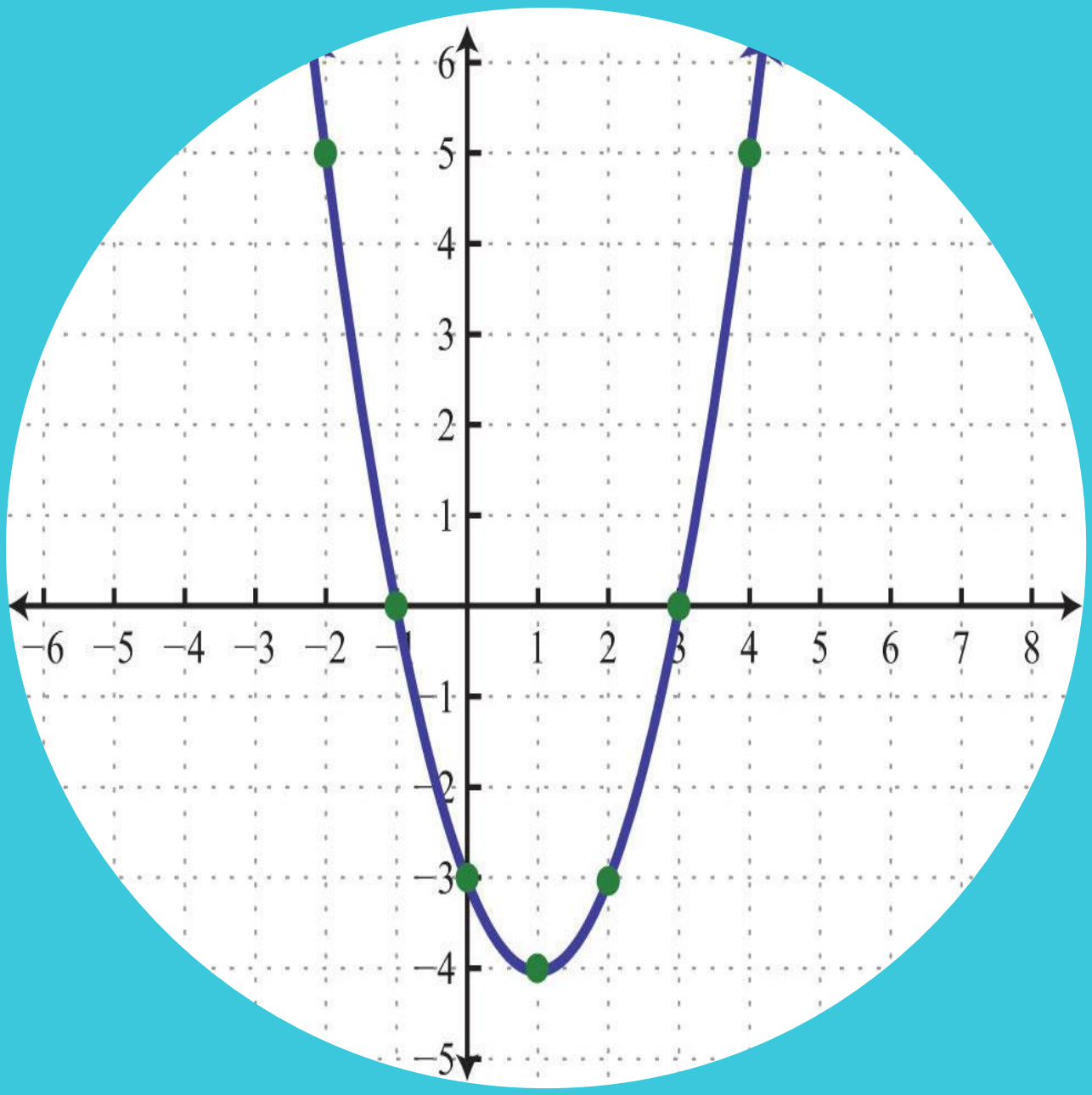

A Hard Grapht

Key Stage 4 Revision Programme

Pupil Name

Handbook designed by Dr McGaw

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1 (Baseline assessment)			
2			
3			
4			
5			
6 (Final assessment)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

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Tutorial 1	**P5**
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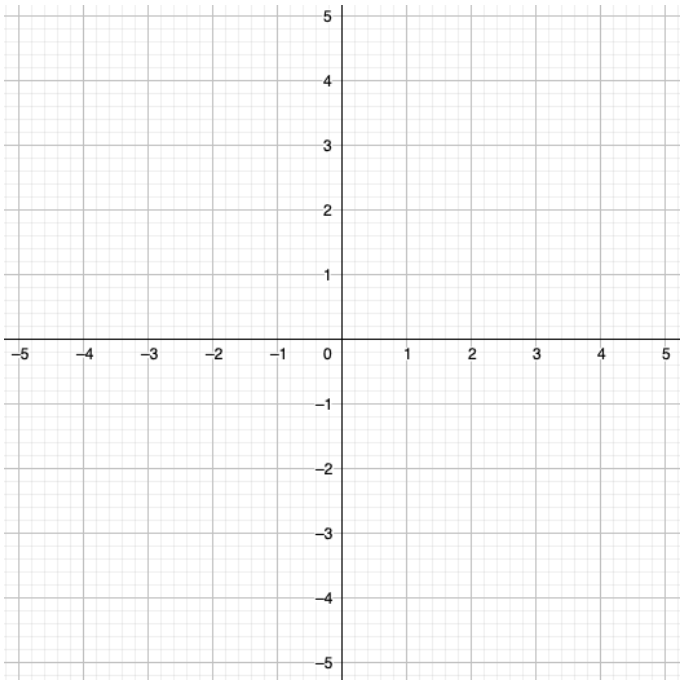
Course Rationale

Graphs are everywhere. You will encounter them in your studies in all areas of mathematics, from statistics to geometry. They are also integral in understanding calculus, a subject you will explore in your A-levels. More than that, you will find graphs everywhere in everyday life. It's hard to flick through a newspaper and *not* find a graph somewhere.

Why are graphs so common? One answer might be that they represent ideas in a visual way. In statistics, it is easier to see trends in a graph than in a set of data. Similarly, many algebraic concepts can be represented using graphs, and complex algebraic operations are simplified by studying the associated graphs.

However, as common as graphs are in academic and everyday life, they are commonly misunderstood and are rife with misconceptions. Mathematics is like a building – every new concept builds upon the last. If one floor of that building – or, worse, the foundation – is loose or shaky, it will be harder to grow. Lots of future mathematical concepts rely on having a secure understanding of graphs. By taking part in this course, your understanding of graphs will be solidified so that you will develop all future concepts securely and confidently. With such a robust understanding, there is no limit to the level your knowledge can grow!

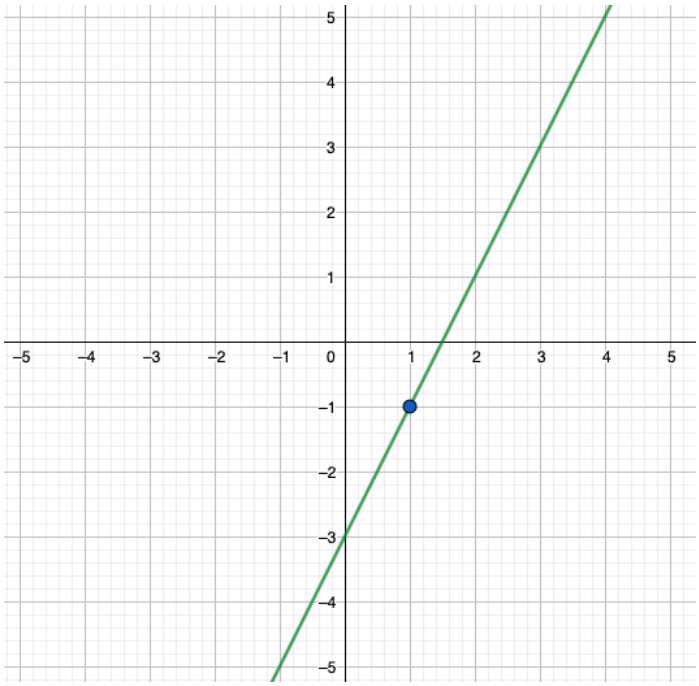
Tutorial 1 – Baseline Assessment



What is the Purpose of Tutorial 1?

- To complete the baseline assessment
- To complete the pre-SEF

Tutorial 2 – Lines and Points



Objective A – Points

- Identify whether points lie on a given line
- Find specific points on a line
- Find where lines intersect the x and y axes

An equation such as $y = 2x + 3$ represents a straight line. Another way to think about it is as follows: What are all the values of x and y which obey the rule that $y = 2x + 3$? For example, when $x = 2$ and $y = 6$, we can verify that the rule has not been followed by substituting these values into the equation:

Left	Right
y	$2x + 3$
6	$2(2) + 3$
6	7

Here we find that the left-hand side is not equal to the right-hand side, so the rule hasn't been obeyed. However, we can see that the rule is obeyed when $x = -1$ and $y = 1$, again by substituting the values into the equation:

Left	Right
y	$2x+3$
1	$2(-1)+3$
1	$-2+3$
1	1

Since both sides are the same, the rule has been followed. The line given by $y = 2x + 3$ represents all possible values of x and y that follow the rule.

This means we can check whether a point lies on a line by substituting values into the equation and seeing if both sides are equal. We also see that all the points on the y-axis are when $x = 0$ and all the points on the x-axis are when $y = 0$, so we can substitute $x = 0$ into our equation to find the y-intercept and substitute $y = 0$ to find the x-intercept.

Worked Examples A

1. Does the point (1,4) lie on the line $y = 2x + 2$?

Left	Right
y	2x+2
4	2(1)+2
4	4

Since both sides are equal, the point (1,4) lies on the line 2x+2.

2. A straight line has equation $y = 3x - 6$.
(a) Find the value of y when $x = 2$.
(b) Find the value of x when $y = 3$.

(a)
y=3x – 6
y=3(2) – 6 when x=2
y=6 – 6
y=0

(b)
y=3x – 6
3=3x – 6 when y=3
(+6) (+6)
9=3x
(÷3) (÷3)
x=3

3. A straight line has equation $y = -4x + 9$. Find the coordinates of the points where the line intersects the y-axis and the x-axis.

The line intersects the y-axis when x=0:
y=-4x+9
y=-4(0)+9
y=9
so the coordinate is (0,9).

The line intersects the x-axis when y=0:
y=-4x+9
0=-4x+9
4x=9
x=9/4=2.25
so the coordinate is (2.25,0).

Practice Questions A

1. Does the point (2,5) lie on the line $y = 2x + 1$?

2. Does the point (2,5) lie on the line $y = -3x + 5$?

3. Does the point (-2,11) lie on the line $y = -3x + 5$?

4. The point (2,k) lies on the line $y = 3x + 2$. Find the value of k.

5. The point (v,5) lies on the line $y = -x + 2$. Find the value of v.

6. A line has equation $y = 2x - 1$. Find
   (a) y when $x = 4$
   (b) x when $y = 3$
   (c) the coordinates of where the line intersects the y-axis
   (d) the coordinates of where the line intersects the x-axis.

7. A line has equation $y = \frac{1}{2}x + 5$. Find
   (a) y when $x = 6$
   (b) x when $y = 2$
   (c) the coordinates of where the line intersects the y-axis
   (d) the coordinates of where the line intersects the x-axis.

8. A line has equation $y = -4x - 11$. Find
   (a) y when $x = -\frac{1}{2}$
   (b) x when $y = 3$
   (c) the coordinates of where the line intersects the y-axis
   (d) the coordinates of where the line intersects the x-axis.

Assessment A

1. Which of the following points lie on the line $y = -3x + 2$?
   (a) (1,-1) (b) (-4,-10) (c) (-2,10) (d) (-3,11)

2. When $y = 4$, find the value of x on the line with equation $y = 8x - 6$.
   (a) 26 (b) 1.25 (c) -0.25 (d) 0.25

3. Which of the following are the x-intercept and y-intercept of the line with equation $y = 2x - 8$?
   (a) (4,0) and (0,-8) (b) (-8,0) and (0,4) (c) (-4,0) and (0,8) (d) (-4,0) and (0,-8)

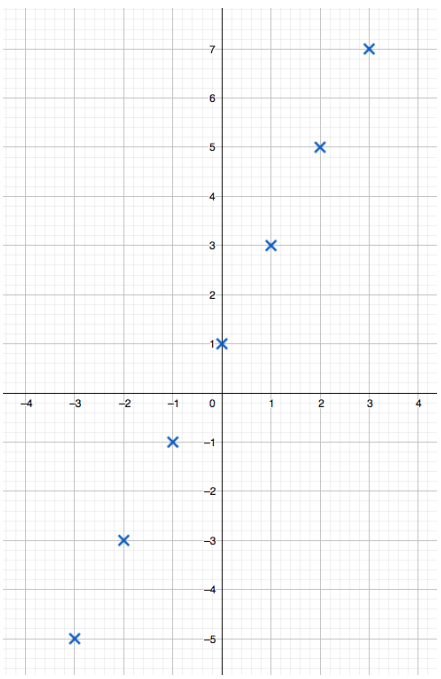
Objective B – Plotting

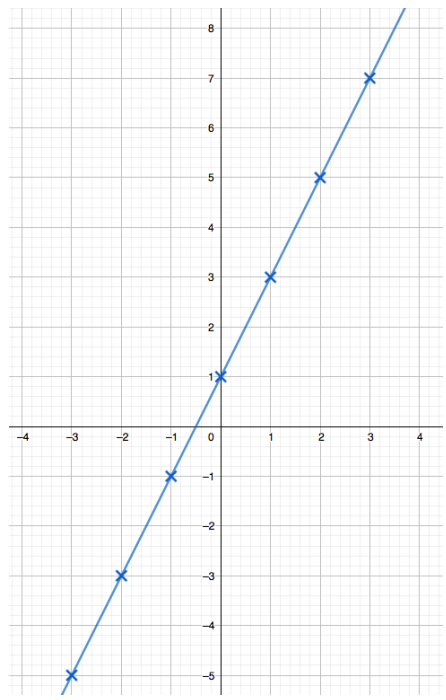
- Plot graphs of straight lines using a table of values
- Sketch graphs of straight lines

Now we will look at plotting a straight line. The easiest way to do this is by using a table of values. This is where we locate some coordinates the line must pass through, then join them up to form a line. For example, here is a table of values for the line with equation $y = 2x + 1$:

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

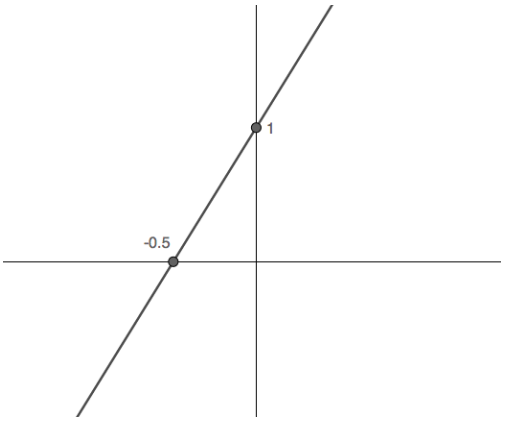
We obtain these values by substituting in the x coordinates and obtaining the y coordinates. For example, when $x = -2$ we find that $y = 2(-2) + 1$ and so $y = -3$ as shown in the table. This tells us that the line passes through the coordinates (-3,-5), (-2,-3), (-1,1), (0,1), (1,3), (2,5), and (3,7). We can plot these in our coordinate axes then draw the line with a ruler:





We must draw the line as long as we can, stretching as far as the axes are drawn. Note that we only actually need to plot two points before we can draw the line, but it is helpful to plot more point just in case a mistake has been made.

A **sketch** of a graph is something different. In a sketch, we draw the graph and only mark where it meets the axes, as below. After finding that the line $y = 2x + 1$ intersects the y-axis at 1 and the x-axis at -0.5, here is our sketch:

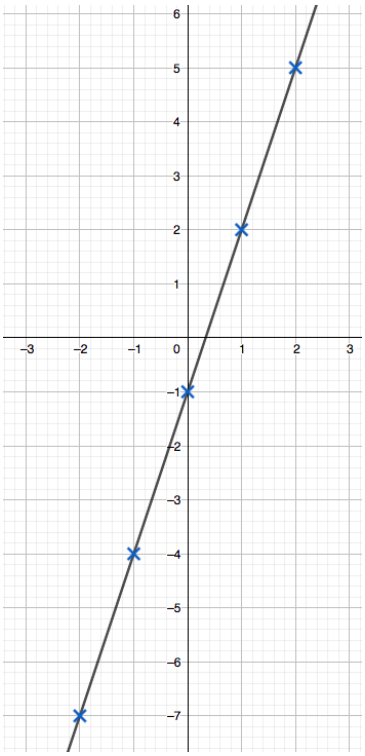


Notice that this isn't as precise as plotting. It doesn't matter – the purpose of a sketch is to give us a rough picture of what a graph looks like.

Worked Examples B

1. Plot the line with equation $y = 3x - 1$.

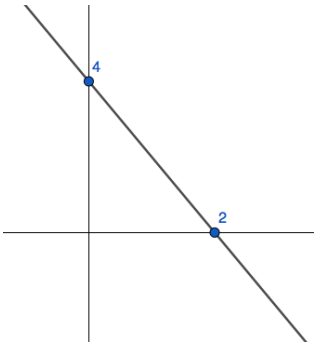
x	-2	-1	0	1	2
y	-7	-4	-1	2	5



2. Sketch the line with equation $y = -2x + 4$.

y-intercept is given when x=0:
y=-2x+4
y=-2(0)+4
y=4

x-intercept is given when y=0:
y=-2x+4
0=-2x+4
(+2x) (+2x)
2x=4
(÷2) (÷2)
x=2

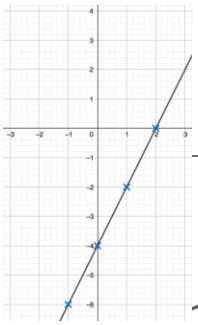


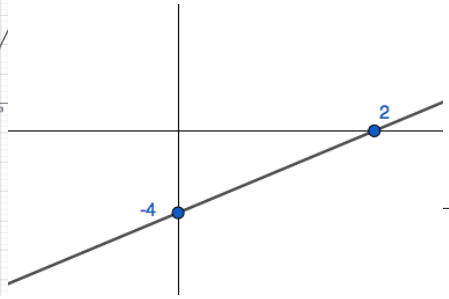
Practice Questions B

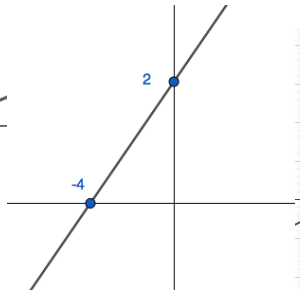
1. Plot the line with equation $y = 4x - 5$.
2. Plot the line with equation $y = -3x + 4$.
3. Sketch the line with equation $y = x + 4$.
4. Sketch the line with equation $y = -3x + 1$.
5. Sketch the line with equation $y = \frac{1}{3}x - 1$.

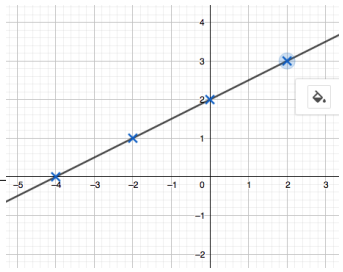
Assessment B

Which of these is a **plot**, and which of these is a sketch, of the line with equation $y = 2x - 4$?

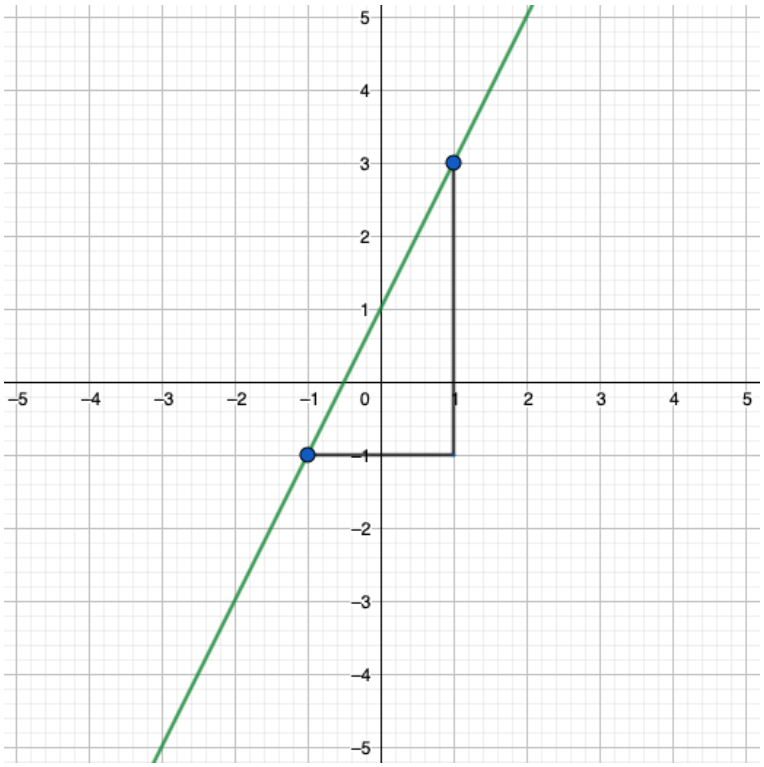








Tutorial 3 – Gradients and Equations



What is the Purpose of Tutorial 3?

- To find the gradient of a straight line
- To find and use the equation of a straight line

Objective C – Distances

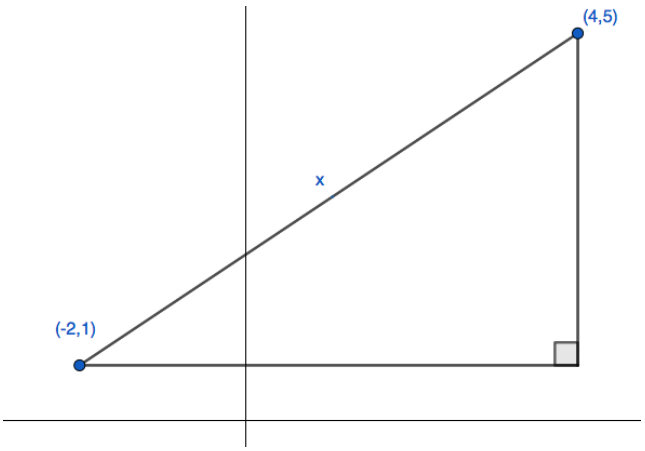
- Find the distance between two points
- Find the midpoint between two points

A common question we are asked is to find the distance or midpoint between two points. We find the distance by applying Pythagoras's Theorem, and find midpoints by finding the average between the points. Let's look at look at some examples.

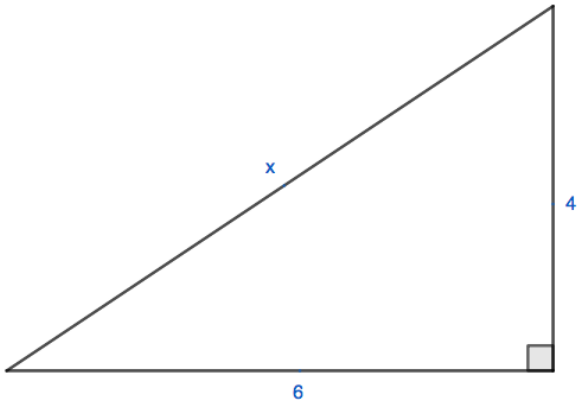
Worked Examples C

1. Find the distance between the points (-2,1) and (4,5).

It helps to sketch a picture, then produce a right-angled triangle:



Note that this is a **sketch**, so it doesn't need to be completely accurate. The distance between the points is labelled x. This is the hypotenuse of a right-angled triangle. Now, we need to find the short sides of the triangle using the coordinates. The base of the triangle travels from -2 to 4, so it has length 6. The height of the triangle travels from 1 to 5, so it has length 4. Let's add these to our triangle:



Now we can solve this using Pythagoras's Theorem:

$$x^2=4^2+6^2$$

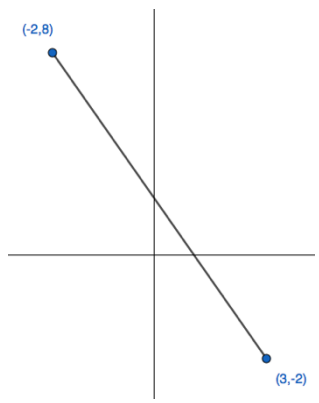
$$x^2=16+36$$

$$x^2=52$$

$$\therefore\ x=\sqrt{52}$$

$$x=7.11 \text{ to 2 d.p.}$$

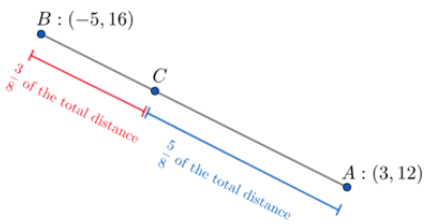
2. Find the midpoint between the points A (-2,8) and B (3,-2).



The x-coordinate of the midpoint will be halfway between the x-coordinates of A and B, while the y-coordinate will be halfway between the y-coordinates of A and B. To find these halfway values, we simply find the mean of the coordinates. Be careful with negatives!

x-coordinate of midpoint is the mean of -2 and 3. This is $\frac{-2+3}{2} = -0.5$.
y-coordinate of midpoint is the mean of 8 and -2. This is $\frac{8+-2}{2} = 3$.
Therefore, the midpoint is at (-0.5,3).

3. The point A has coordinates (3,12) and the point B has (-5,16). The point C lies on the line segment joining A and B such that AC:CB = 5:3. Find the coordinates of the point C.



So, we're still going to subtract the individual coordinates of A from B to find the distance in both x and y like we did in the previous example, but this time we're going to find $\frac{5}{8}$ of the total (as opposed to a half) before adding the values onto the coordinates of A.

First, x coordinates: $-5 - 3 = -8$, then $\frac{5}{8} \times -8 = -5$. Adding this to the x coordinate of A, we get

$$x \text{ coordinate of C } = 3 + (-5) = -2$$

Second, y coordinates: $12 - 16 = -4$, then $\frac{5}{8} \times -4 = -\frac{5}{2}$. Adding this to the y coordinate of A, we get

$$y \text{ coordinate of C } = 3 + \left(-\frac{5}{2}\right) = \frac{1}{2}$$

Therefore, the coordinates of C are $\left(-2, \frac{1}{2}\right)$.

Practice Questions C

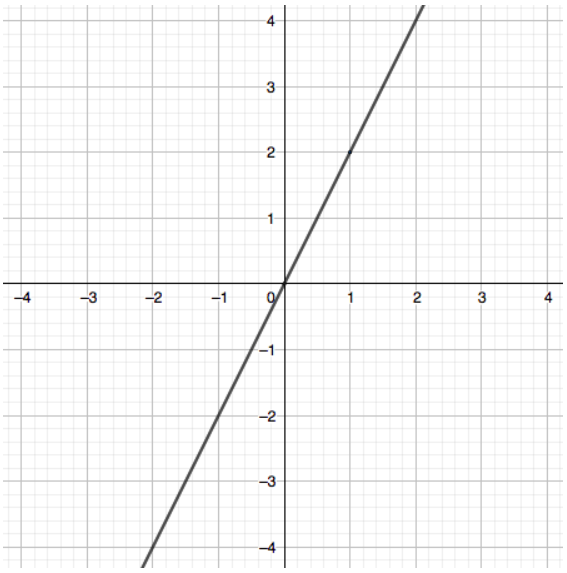
1. Find the distance and midpoint between points (-3, 1) and (5, 4).
2. Find the distance and midpoint between points (-5, -9) and (3, 8).
3. Find the distance and midpoint between points (1, 8.5) and (5.5, -3).
4. The point A has coordinates (-2,-3). The point M has coordinates (1,1) and is the midpoint between A and B. Find the coordinates of point B.
5. The point A has coordinates (-1,-1) and the point B has (5,2). The point C lies on the line segment joining A and B such that AC:CB = 1:2. Find the coordinates of C.
6. The point A has coordinates (-2,9) and the point B has (1,-3). The point C lies on the line segment joining A and B such that AC:CB = 5:1. Find the coordinates of C.

Assessment C

Find the distance and midpoint between points A (1, 8.5) and B (5.5, -9.5). Point C lies on the line segment between A and B such that AC:CB=4:5. Find the coordinates of C.

Objective D – Gradient

The gradient is how **steep** a line or curve is – for every unit across, how much do we go up or down? For example, if a straight line has gradient 2, that means that for every unit across, it goes two units upwards:



What makes a straight line special is that, unlike a curve, its gradient is the **same everywhere**. For this reason, the gradient is an important property of straight lines and we are required to be able to calculate it. If a straight line is given in the form $y = mx + c$, then the gradient is given by m. We will also look at calculating the gradient from a plot or sketch of the line.

Worked Examples D

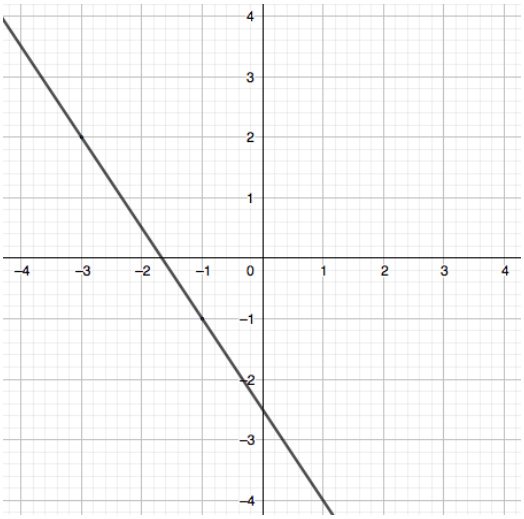
1. Find the gradient of the line given by the equation $2y + 5x - 8 = 0$.

First, we rearrange the equation so that it is in the form y=mx+c:

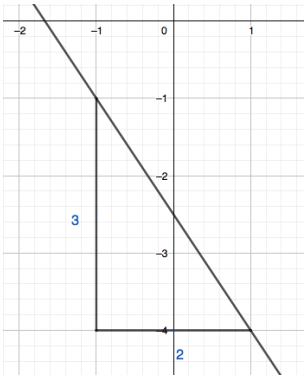
$$2y + 5x - 8 = 0$$
$$2y + 5x = 8$$
$$2y = -5x + 8$$
$$y = -\frac{5}{2}x + 4$$

The coefficient of x is -5/2, so that is the gradient.

2. Find the gradient of this straight line:



First, we draw a triangle on the graph. We can draw a triangle anywhere, but it helps to choose points on the line which intersect the gridlines. In this example, we will choose the points (-1,-1) and (1,-4). We then label the triangle's base and height.

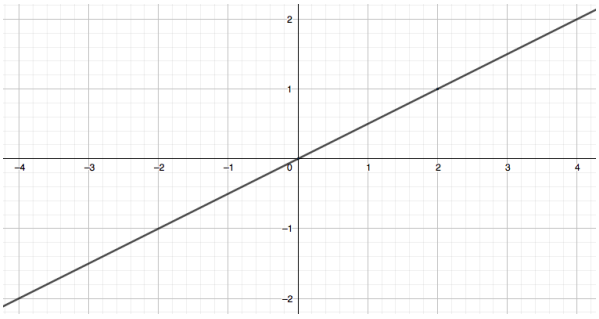


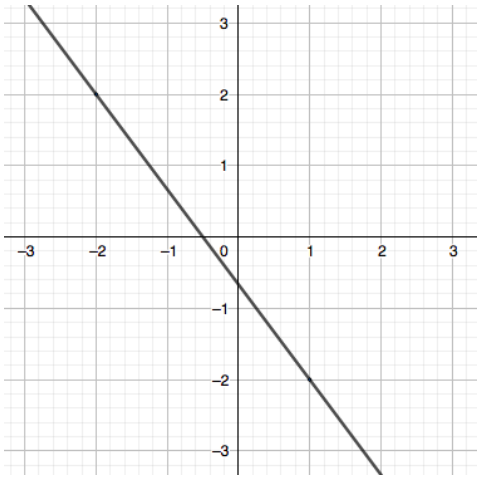
To find the gradient, we divide the height (the change in y) by the base (the change in x), then include a negative sign if the gradient is negative. In this example, 3/2=1.5, and because the gradient is negative, the gradient is -1.5.

Practice Questions D

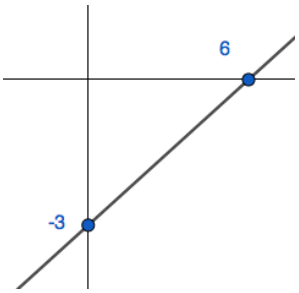
1. What is the gradient of the line with equation y=4x-5?
2. What is the gradient of the line with equation 3y-4x=-9

3. What is the gradient of these lines?





4. What is the gradient of the line sketched below?

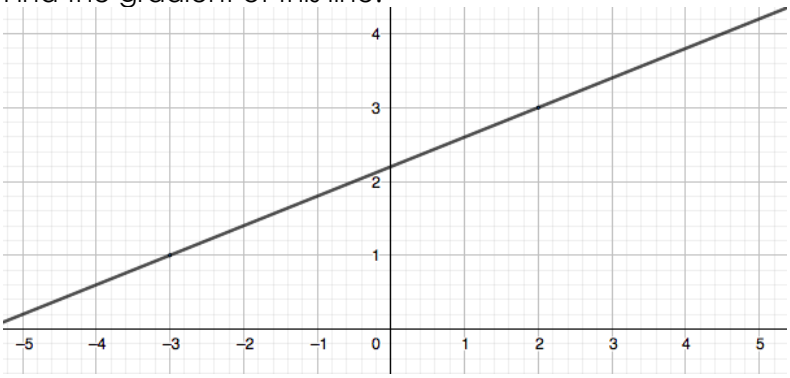


5. What is the gradient of the line passing through the coordinates (5,2) and (-1, 8)?

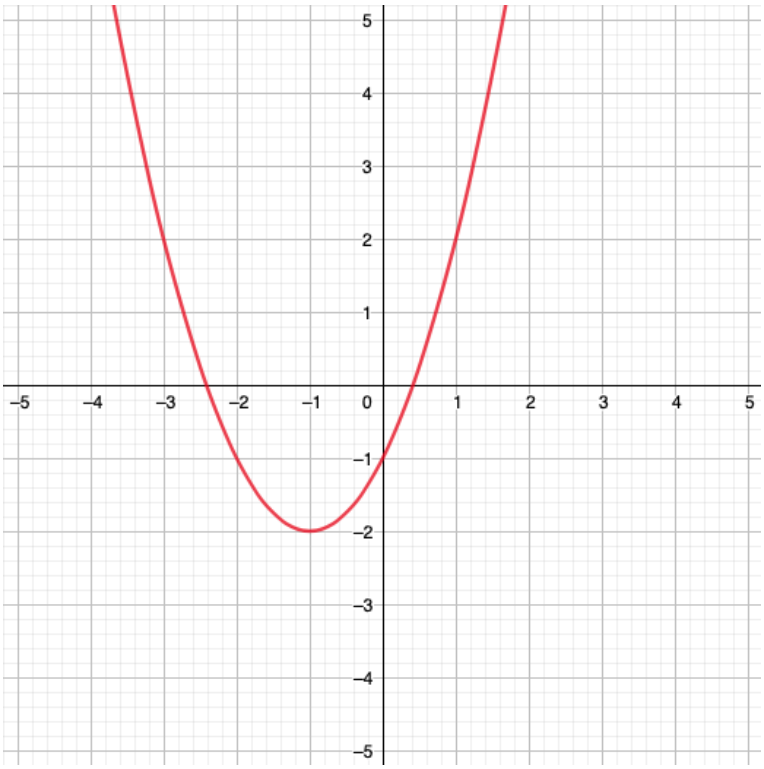
6. A line with gradient 2 passes through points A and B. Point A has coordinates (1,5), while point B has coordinates (3,k). Find the value of k.

Assessment D

Find the gradient of this line:



Tutorial 4 – Quadratic Graphs



What is the Purpose of Tutorial 4?

- Find the interception points of quadratic graphs
- Plot quadratic graphs

Quadratic Graphs

Now we will move onto graphs with quadratic equations. As opposed to straight lines, which have equations of the form $y = mx + c$, these graphs will have equations of the form $y = ax^2 + bx + c$. Also, instead of being straight lines, these graphs will be curved. Despite these differences, much of what we have learned about linear graphs will apply to quadratic graphs, including finding the points where these graphs intersect the axes, and how to plot them.

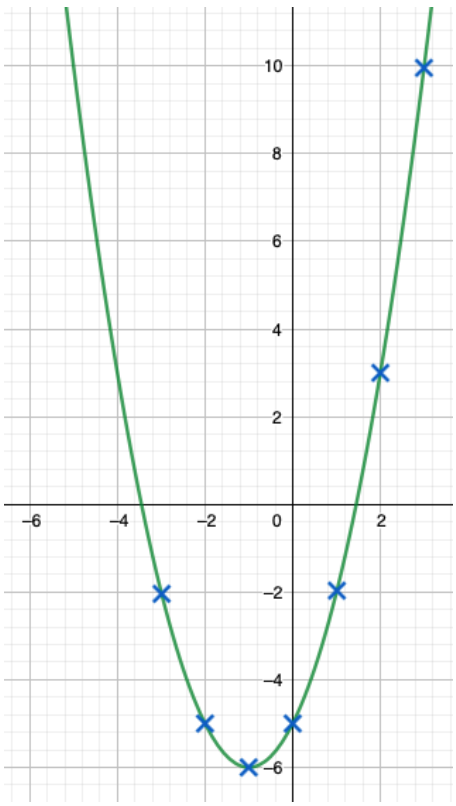
Objective E – Plotting quadratic graphs

As with linear graphs, we plot quadratic graphs using a table of values.

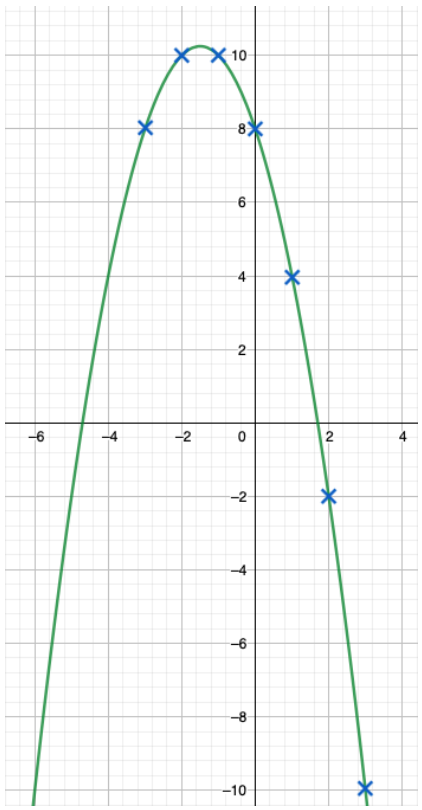
Worked Example E

1. Plot the graphs with equations $y = x^2 + 2x - 5$ and $y = -x^2 - 3x + 8$

x	-3	-2	-1	0	1	2	3
y	-2	-5	-6	-5	-2	1	10



x	-3	-2	-1	0	1	2	3
y	-2	-5	-6	-5	-2	1	10

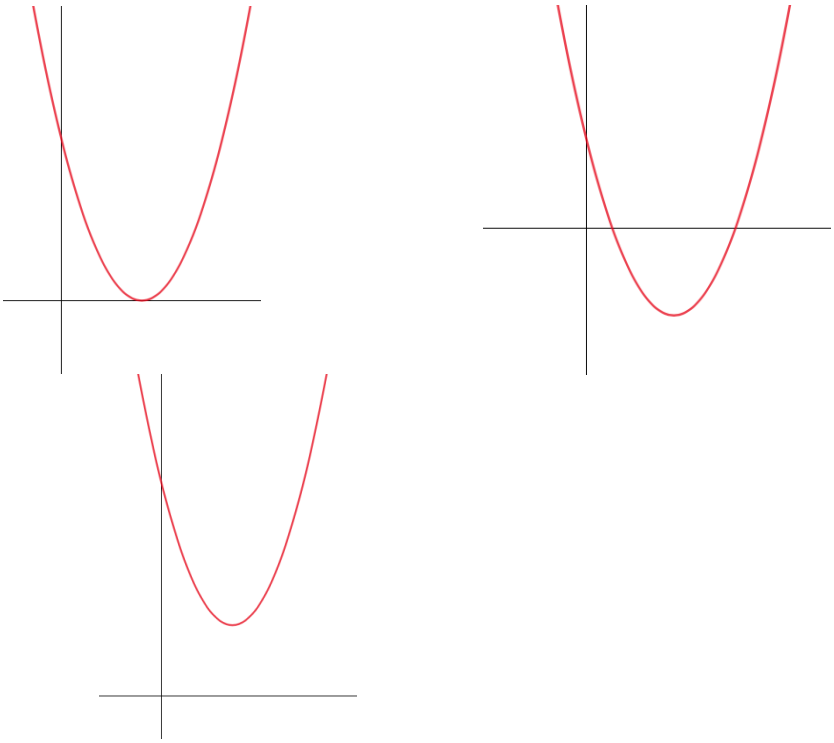


With both of these, we substituted values of x to obtain the y-coordinates, then plotted these points. Then we joined them up in a smooth curve. This curve shape is called a **parabola**. Note that when the coefficient of x^2 is positive, the graph forms a valley shape, and when the coefficient of x^2 is negative, the graph forms a hill shape. Knowing this helps us visualise what

the curve will look like. Finally, note that the graph is always symmetric – this helps us draw the curve if we know more coordinates on one side than the other.

Objective F – Interception points of quadratic graphs

The same reasoning we used with linear graphs applies to quadratic graphs. The point at which a graph meets the y-axis is when $x = 0$, and the point(s) at which a graph meets the x-axis is when $y = 0$. The biggest difference between quadratic and linear graphs is that while linear graphs always cross the x-axis exactly once, a quadratic graph can cross the x-axis once, twice, or not at all. To see this, consider the three parabolas sketched here:



Here we see that because of the valley shape, the curve must either miss the x-axis entirely, just touch it on one place, or cross it and cross it back. These lead to the cases where there are 1, 2, or 0 intercepts on the x-axis. This is true when the parabola is inverted – a hill shape. Let us now learn how to find these interception points.

Worked Examples F

Find the coordinates of where the graph with equation $y = x^2 - 6x - 16$ intersects the x-axis and y-axis.

y-intercept is given when x=0:
$y = x^2 - 6x - 16$
$y = 0^2 - 6(0) - 16$
$y = -16$
so (0,-16) is the point of intersection on the y-axis.

x-intercept is given when y=0:
$0 = x^2 - 6x - 16$

$0 = x^2 - 8x + 2x - 16$
$0 = x(x - 8) + 2(x - 8)$
$0 = (x + 2)(x - 8)$
$x + 2 = 0$ or $x - 8 = 0$
$x = -2$ or $x = 8$
so (-2,0) and (8,0) are the points of intersection on the x-axis.

Notice that finding the coordinates of intersection on the x-axis essentially means we have to solve the quadratic equation. Don't forget that there are many methods for this: factorisation, using the quadratic formula, or completing the square.

Practice Questions F

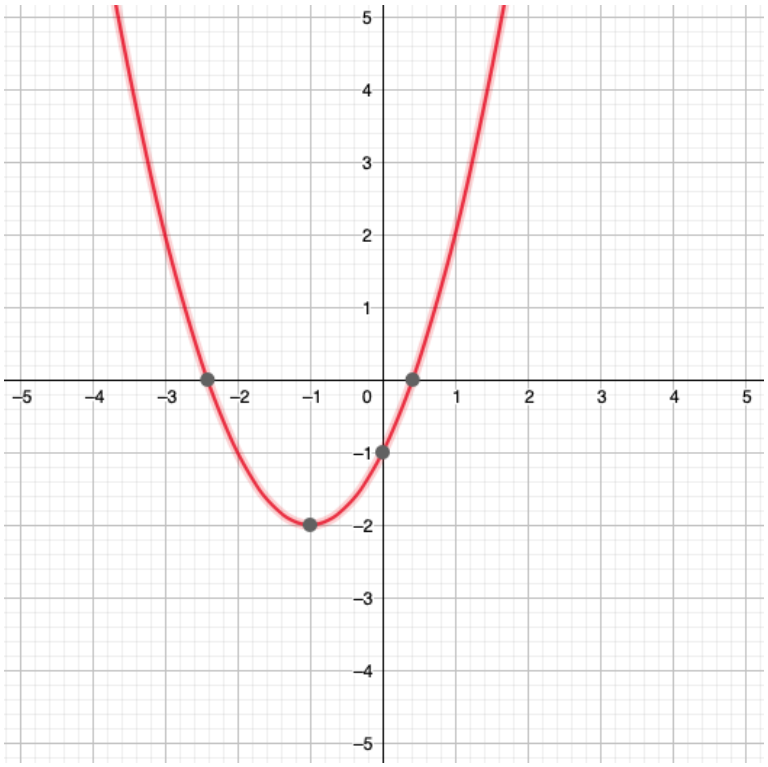
Find the points where the curves with these equations meets the x- and y-axis:

(a) $y = 2x^2 - 9x - 5$
(b) $y - x^2 = x - 12$
(c) $y = 6x^2 + 5x - 4$
(d) $y = 4x^2 - 4x + 1$

Assessment F

Find the points where the curve with equation $y = x^2 + 3x - 10$ meets the x- and y-axis.

Tutorial 5 – Turning Points and Sketches



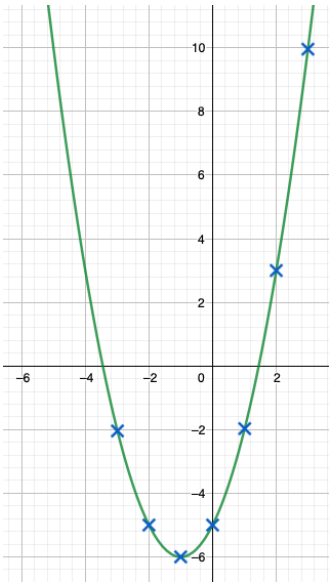
What is the Purpose of Tutorial 5?

- Find the turning points of a parabola
- Sketch a quadratic curve

Objective G – Finding the turning points of quadratic graphs

All quadratic graphs have the shape of a parabola. Because this shape is a hill or a valley shape, there is always a point where the graphs 'turns around'. More precisely, there is always a point where the gradient of the graph switches from negative to positive, or vice versa. This is called the **turning point** of the graph.

Previously, we plotted the graph with equation $y = x^2 + 2x - 5$. We can see that the turning point of this graph is at the point (-1,-6). But how do we calculate it without plotting the graph?



Worked Example G

Find the turning point of the curve with equation $y = x^2 + 4x - 5$.

First, we complete the square: $y = (x + 2)^2 - 9$. Now, remember that the turning point is the bottom point of the graph – it's where y is at its smallest. To make y as small as possible, we have to make $(x + 2)^2 - 9$ as small as possible. This means we have to make $(x + 2)^2$ as small as possible – since this is a square, it is always zero or positive, which means it is at its smallest when it is zero. Hence x=-2 and when we substitute that value back in, we obtain y=-9. Hence the coordinate of the turning point is (-2,-9).

Practice Questions G

Find the turning points of curves with equation:

(a) $y = x^2 + 4x - 8$
(b) $y = 2x^2 - 4x + 1$
(c) $y + x^2 = -6x - 6$
(d) $y = x^2 + 5x + 9$

Assessment G

Find the turning point of the curve with equation $y = x^2 - 4x + 5$

Objective H – Sketching quadratic graphs

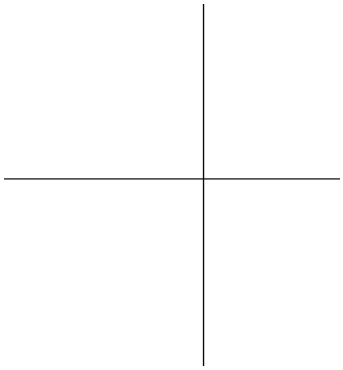
In this last objective, we will sketch quadratic graphs. As with linear graphs, a sketch is a rough drawing that need not be completely accurate. However, certain key features must be labelled clearly and in the correct position. These are the intercepts with the axes and the turning point.

Worked Examples H

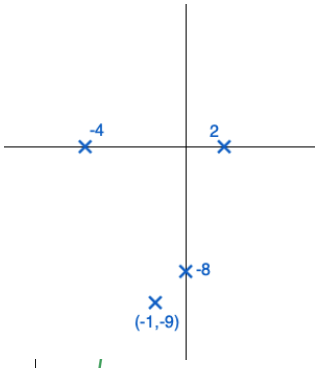
Sketch the curve given by the equation $y = x^2 - 2x - 8$. We will skip some steps here – for a recap, see objectives F and G.
x-intercepts: (-2,0) and (4,0).
y-intercept: (0,-8).
Turning point (-1,-9).

Because these features need to be in approximately the correct place, it is easier to draw the points first before sketching the curve, like so:

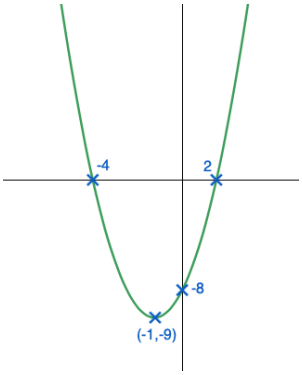
Start with the axes



Mark the key features



Sketch the parabola



Practice Questions H

Sketch the curves based on the values in the table:

Question	y-intercept	x-intercept(s)	Turning point
(a)	-3	-2 and 5	(3,-5)
(b)	4	-2	(-2,0)
(c)	6	-1 and 8	(3,11)
(d)	2	None	(1,1)

Sketch the curve $y = x^2 + 6x + 8$

Assessment H

Sketch the curve $y = x^2 - 8x + 12$

Tutorial 6 – Final Assessment

Tutorial 7 - Feedback

What is the Purpose of Tutorial 7?

- To receive feedback on final assessments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assessment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
• • •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Notes

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Notes

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