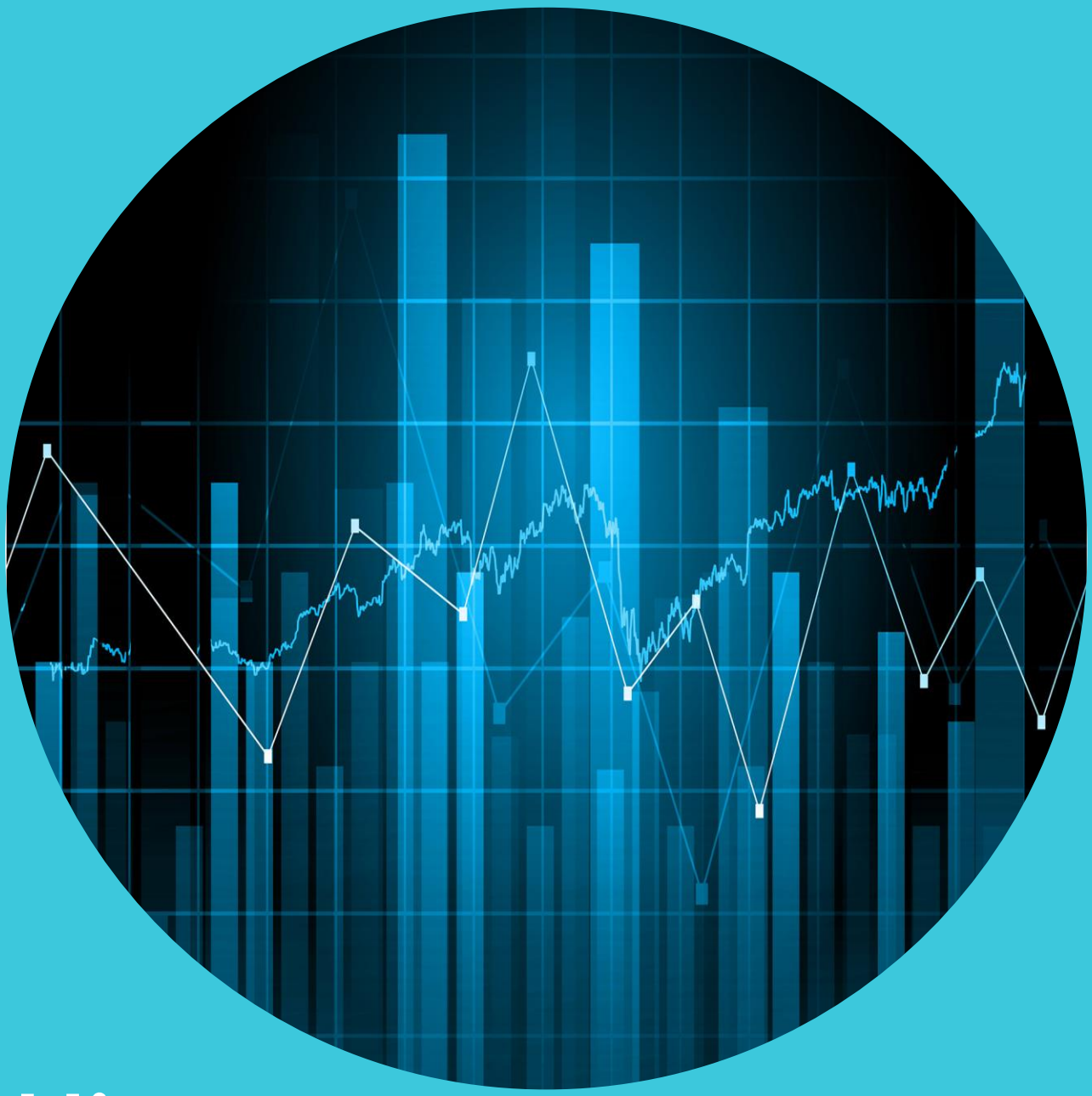

Histograms

Key Stage 4 Revision Programme

Pupil Name	
Handbook Designed by	Dr O King

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1 (SEF & Baseline)			
2			
3			
4			
5			
6 (Exam Technique & Final Assignment)			
7 (Feedback)			

Timetable – Homework Assignments

Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

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Tutorial 7 – Feedback	**P29**
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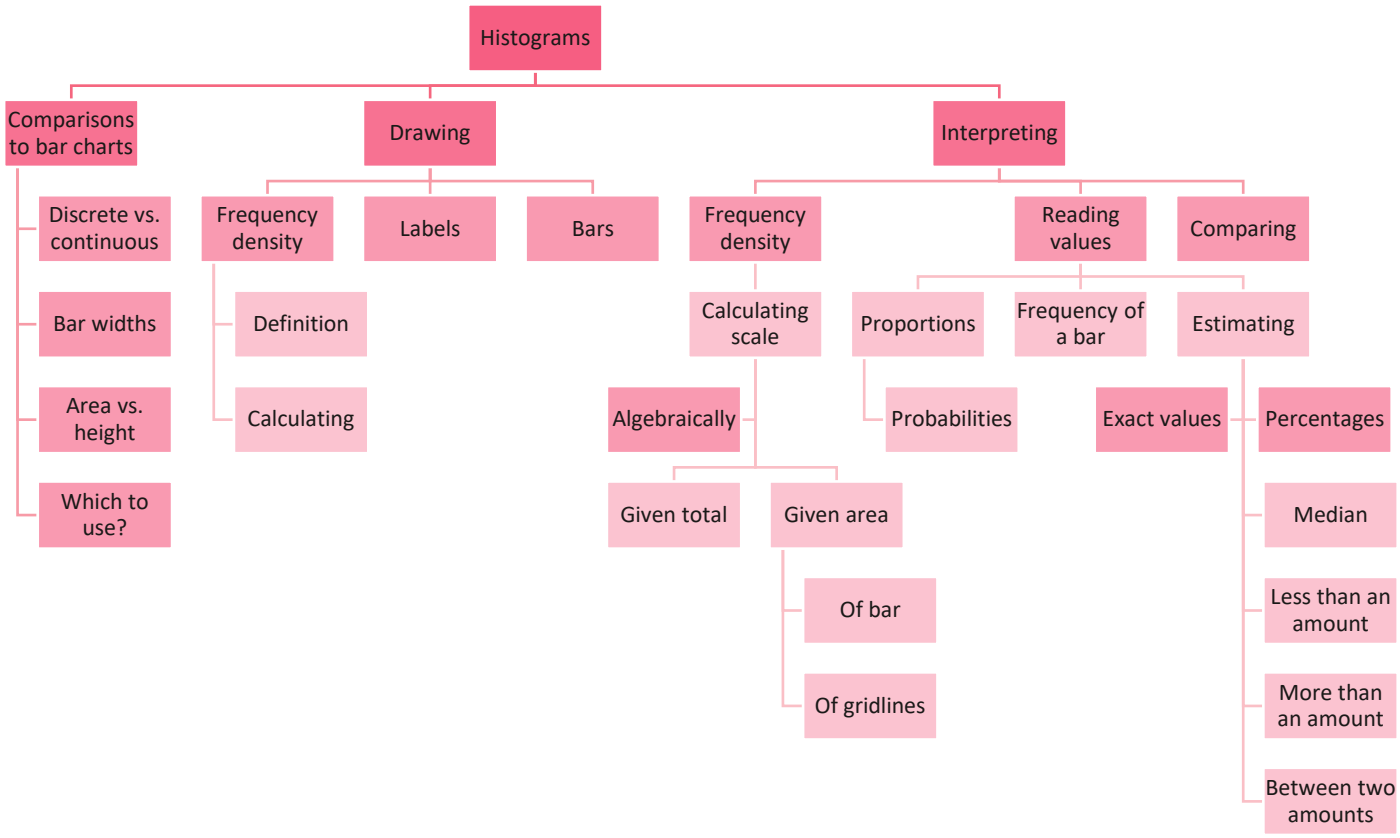
Graph paper	**P31**

Course Rationale

Each topic in the GCSE course can appear in many forms. In order to master a particular skill you will need to be able to deal with any type of question that could be related to that skill. In order to do this, we must practice each aspect of this point, and see how it relates to the others. Think of learning a piece of maths as like learning a piece of music – you would not try to play the entire song from start to end over and over again. Instead, you focus on a particular part until you have mastered it, then move on to the next, returning occasionally to link the steps together. This technique is called "deliberate practice".

In this booklet we will be focussing on histograms, although the principle applies to all areas of mathematics. I've chosen histograms as it frequently occurs on multiple GCSE papers, and the questions usually carry a large number of marks. However, they are usually poorly answered, so any gain in these areas will give you a big advantage over the rest of the students taking the exam!

Below is the "concept map" relating to histograms – a diagram showing each of the skills required and how they relate to one another. Over the course of this booklet we will practice each of these sections so that you will have a much greater understanding of the topic and be able to maximise these marks in the exams. You may wish to compare how well you understand each of these points now to the end of the course.



Tutorial 1 – Drawing a histogram

Objectives this tutorial:

A) Calculate the quantities needed to draw a histogram
B) Accurately draw a histogram

Starter:

Draw a frequency diagram (bar chart) for the following data set:

Weight (kg)	Frequency
$35 < x \leq 40$	10
$40 < x \leq 50$	50
$50 < x \leq 65$	90
$65 < x \leq 70$	20
$70 < x \leq 80$	30

Objective A: Calculating quantities

In order to draw a histogram, we must first calculate the **frequency density** of each class in the data.

Frequency density = Frequency ÷ Class width

Worked example:

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 8$	3	$8 - 0 = 8$	$3 \div 8 = 0.375$
$8 < x \leq 16$	6	$16 - 8 = 8$	$6 \div 8 = 0.75$
$16 < x \leq 24$	7	$24 - 16 = 8$	$7 \div 8 = 0.875$
$24 < x \leq 32$	4	$32 - 24 = 8$	$4 \div 8 = 0.5$

We can also work backwards to find the frequencies if we are given the frequency density.

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 8$	$0.375 \times 8 = 3$	$8 - 0 = 8$	0.375
$8 < x \leq 16$	$0.75 \times 8 = 6$	$16 - 8 = 8$	0.75
$16 < x \leq 24$	$0.875 \times 8 = 7$	$24 - 16 = 8$	0.875
$24 < x \leq 32$	$0.5 \times 8 = 4$	$32 - 24 = 8$	0.5

Finally, we can find the class intervals by calculating the class width.

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 0 + 8$	3	$3 \div 0.375 = 8$	0.375
$8 < x \leq 8 + 8$	6	$6 \div 0.75$	0.75
$16 < x \leq 16 + 8$	7	$7 \div 0.875 = 8$	0.875
$24 < x \leq 24 + 8$	4	$4 \div 0.5 = 8$	0.5

Practice A:

Fill in the missing values in the following tables.

1)

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 10$	1		
$10 < x \leq 20$	2		
$20 < x \leq 30$	4		
$30 < x \leq 40$	3		

2)

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 10$	2		
$10 < x \leq 20$	4		
$20 < x \leq 30$	3		
$30 < x \leq 40$	6		

How do your answers compare to Question 1? Why is this?

3)

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 5$	2		
$5 < x \leq 10$	4		
$10 < x \leq 15$	3		
$15 < x \leq 20$	6		

How do your answers compare to Question 2? Why is this?

4)

Mass, x (kg)	Frequency	Class width	Frequency density
$5 < x \leq 10$	2		
$10 < x \leq 15$	4		
$15 < x \leq 20$	3		
$20 < x \leq 25$	6		

How do your answers compare to Question 3? Why is this?

5)

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 10$			0.2
$10 < x \leq 15$			0.8
$15 < x \leq 20$			0.6
$20 < x \leq 35$			0.4

How is this question different to the others?

6)

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq$	3		0.2
$< x \leq$	4		0.8
$< x \leq$	6		0.6
$< x \leq$	6		0.4

What is different about this question?

Assessment A:

Which of these statements are true?

1. Frequency density is always less than frequency.
2. To calculate frequency density, you divide the class width by the frequency.
3. If you make the class width bigger, the frequency density also gets bigger.
4. If you double the frequency, you double the frequency density.

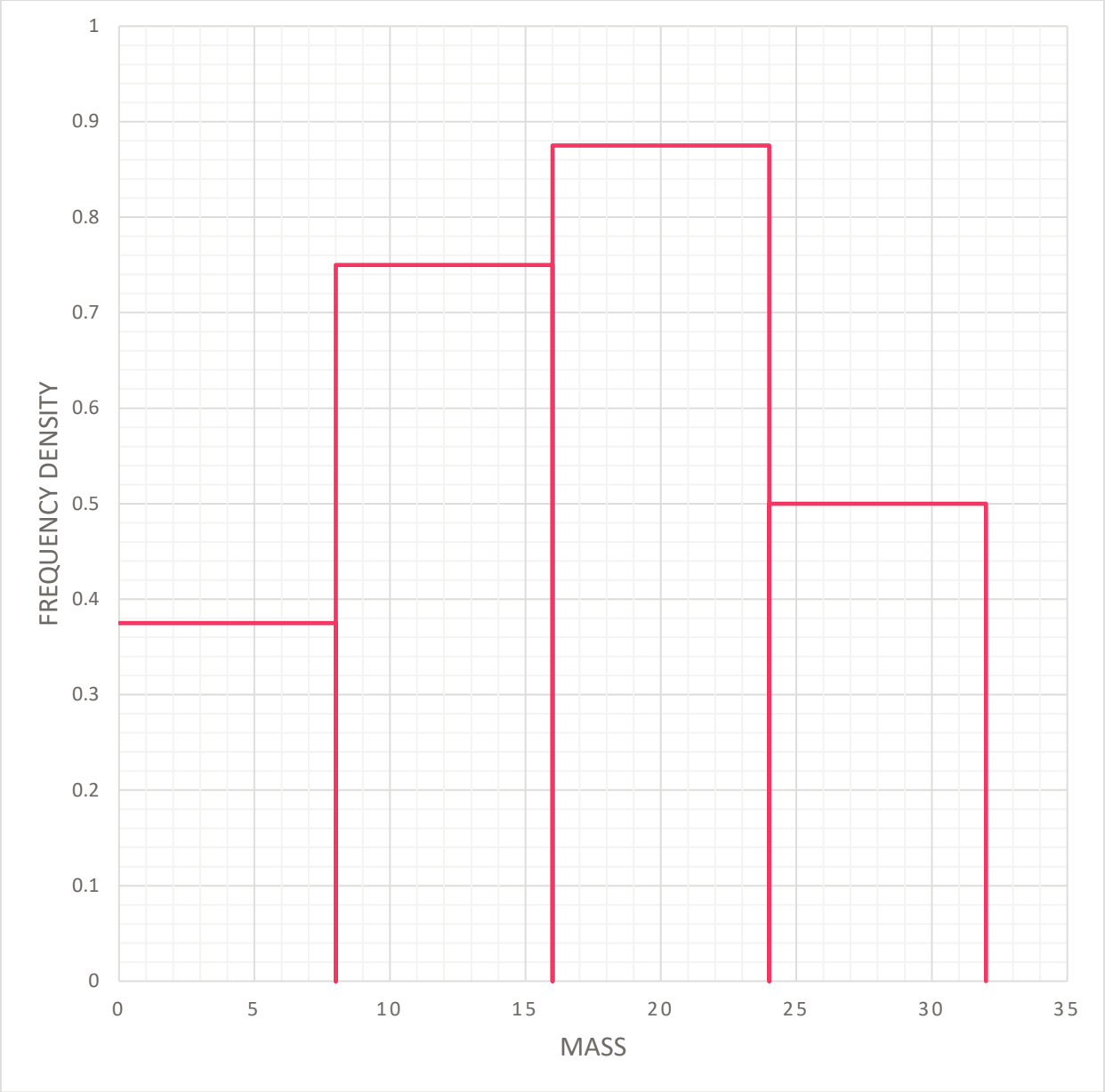
For those that are not true, use an example to show why.

Objective B: Drawing a histogram

We draw a histogram in much the same way as a frequency diagram, but we plot frequency density on the y-axis instead of frequency.

Worked example:

Mass, x (kg)	Frequency	Class width	Frequency density
$0 < x \leq 8$	3	$8 - 0 = 8$	$3 \div 8 = 0.375$
$8 < x \leq 16$	6	$16 - 8 = 8$	$6 \div 8 = 0.75$
$16 < x \leq 24$	7	$24 - 16 = 8$	$7 \div 8 = 0.875$
$24 < x \leq 32$	4	$32 - 24 = 8$	$4 \div 8 = 0.5$



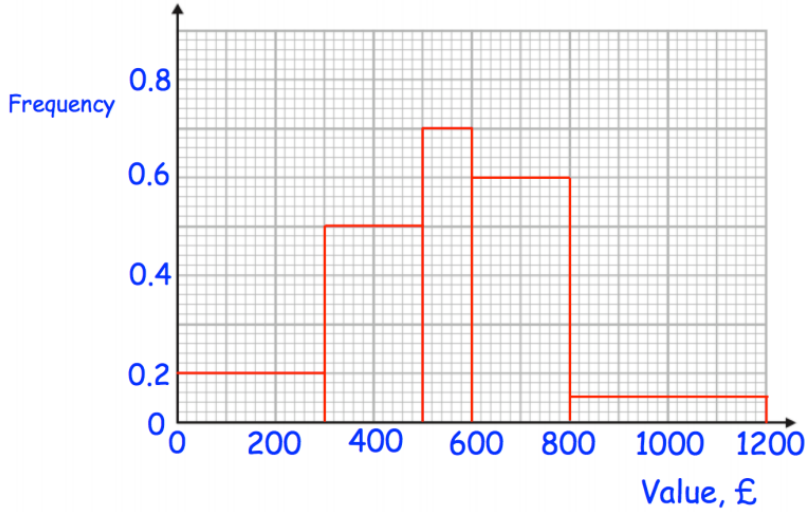
Practice B:

Draw the histograms for the tables in Practice A, Questions 1-4.

Compare the histograms you have drawn. What is the same, and what is different? Why is this?

Assessment B:

Look at the following attempt at drawing a histogram. What mistakes have been made?



Value, v pounds	Frequency
$0 \leq v < 300$	60
$300 \leq v < 500$	100
$500 \leq v < 600$	70
$600 \leq v < 800$	60
$800 \leq v < 1200$	40

Draw the histogram as it should be for this data set.

To finish...

Go back to the concept map on Page 4. Consider the following questions:

- What parts have we looked at in this tutorial?
- How well did you understand each part?
- Do you understand the links between them?
- What could you do to strengthen the areas you had trouble with?

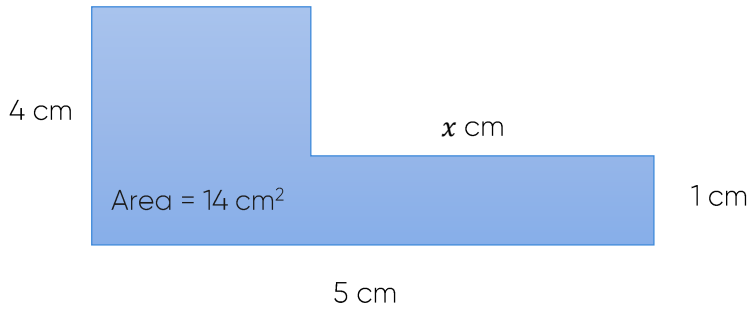
Tutorial 2 – Finding frequencies and completing histograms

Objectives this tutorial:

A) Calculate the frequency of a bar in a histogram
B) Calculating the scale for a histogram

Starter:

Find the missing value in the figure below.

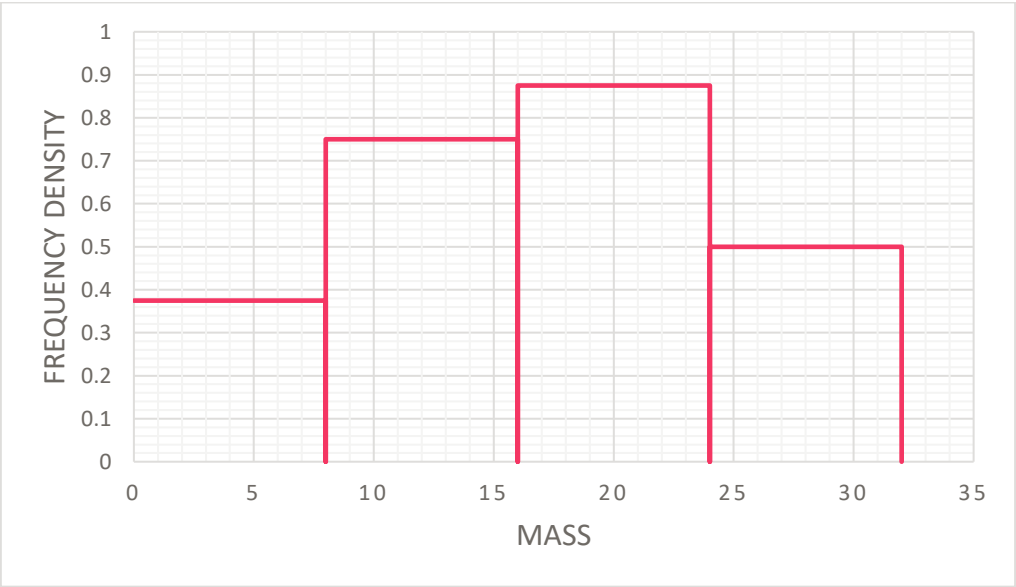


Objective A: Calculating the frequency of a bar

Recall from Tutorial 2 that we can calculate the frequency if we know the class width and the frequency density using the following formula:

$$\text{Frequency} = \text{Frequency density} \times \text{class width}$$

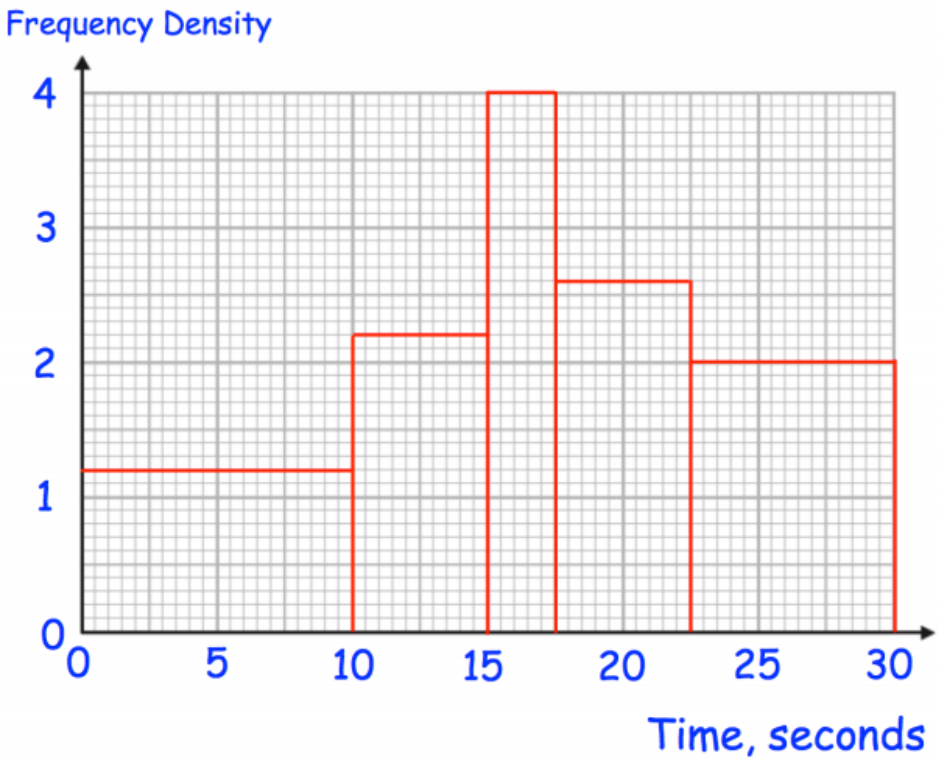
If we look at a histogram, then this formula gives us the area of one of the bars.

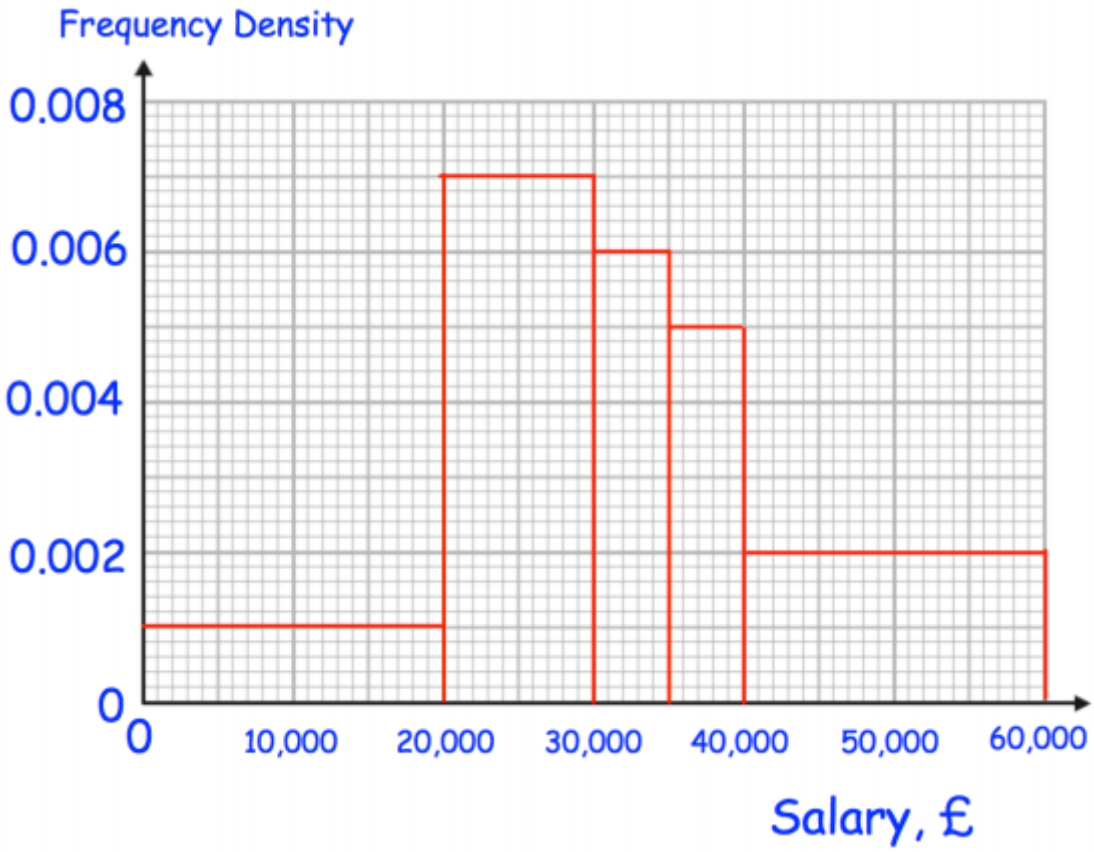


The second bar has width $16 - 8 = 8$ and height 0.75 so the frequency represented by the bar is $8 \times 0.75 = 6$

Practice A:

Find all the frequencies represented by the following histograms.


Frequency Density
4
3
2
1
0
0
5
10
15
20
25
30
Time, seconds


Frequency Density
0.008
0.006
0.004
0.002
0
0
10,000
20,000
30,000
40,000
50,000
60,000
Salary, £

Assessment A:

A histogram is drawn to represent the weights of adult males visiting a doctor during one week. One bar has width 50 and height 0.4. Another bar represents the same frequency and has height 0.2. What is the width of the bar?

20 25 100 625

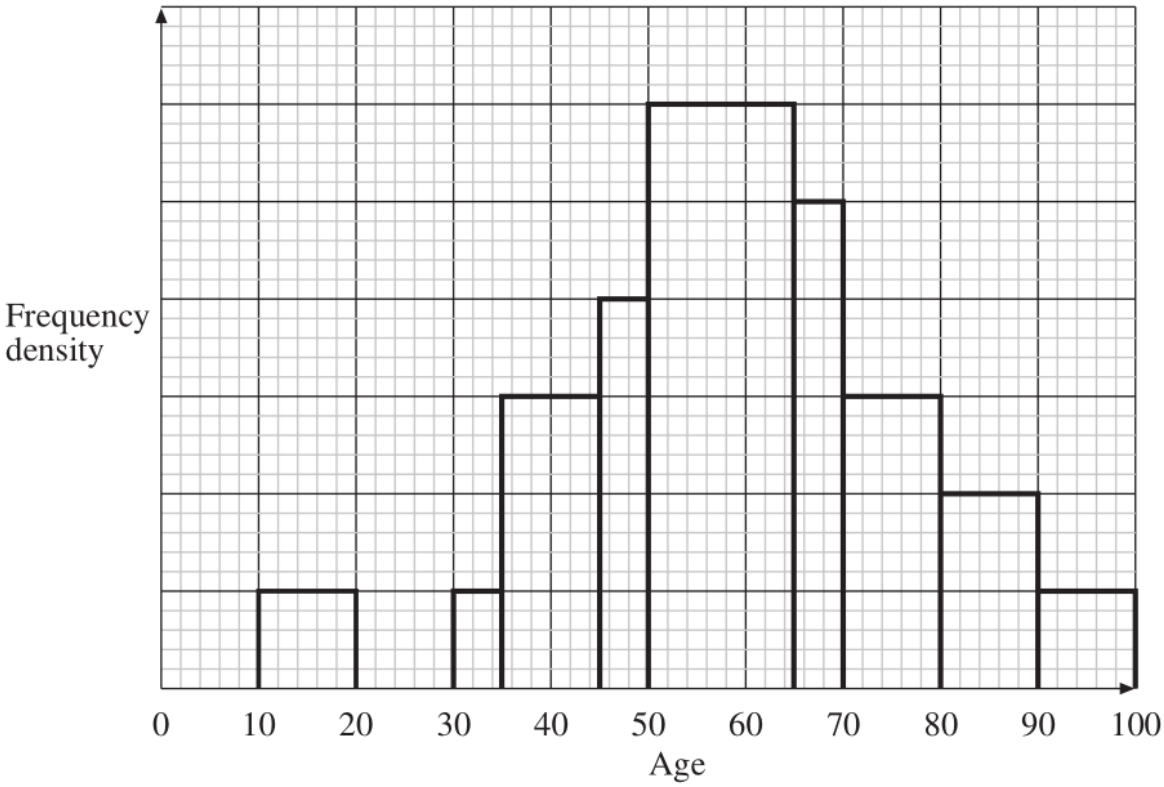
State your reasoning here:

Objective B: Calculating the scale of a histogram

In some cases, we will be given some information about the frequency or area of a bar and will need to use this to work out the scale for frequency density.

Worked example:

Look at the following histogram. We are told that there are 60 people in the 70-80 age bracket.



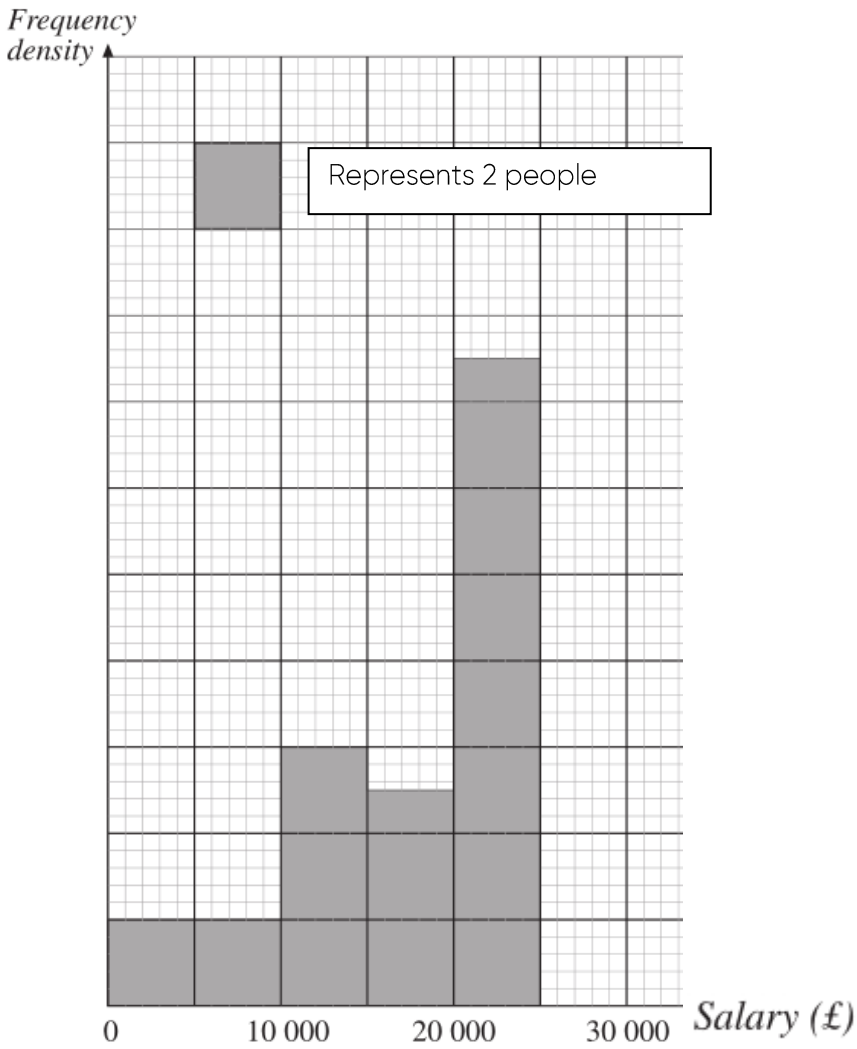
Recall that in a histogram, the frequency of a bar is given by its area. Since the 70-80 bar has a width of 10, the height must therefore be

$$60 \div 10 = 6$$

So three large squares up the y-axis represents a frequency density of 6, so each large square must represent a frequency density of 2. From this, we can calculate all other frequencies:

Age, x (years)	Frequency density	Frequency
$0 < x \leq 10$	0	$0 \times 10 = 0$
$10 < x \leq 20$	2	$2 \times 10 = 20$
$20 < x \leq 30$	0	$0 \times 10 = 0$
$30 < x \leq 35$	2	$2 \times 5 = 10$
$35 < x \leq 45$	6	$6 \times 10 = 60$
$45 < x \leq 50$	8	$8 \times 5 = 40$
$50 < x \leq 65$	12	$12 \times 15 = 180$
$65 < x \leq 70$	10	$10 \times 5 = 50$
$70 < x \leq 80$	6	$6 \times 10 = 60$
$80 < x \leq 90$	4	$4 \times 10 = 40$
$90 < x \leq 100$	2	$2 \times 10 = 20$

Rather than being given the frequency of an entire bar, we may also be given the frequency of a small area of the graph (usually one of the squares made by the grid).



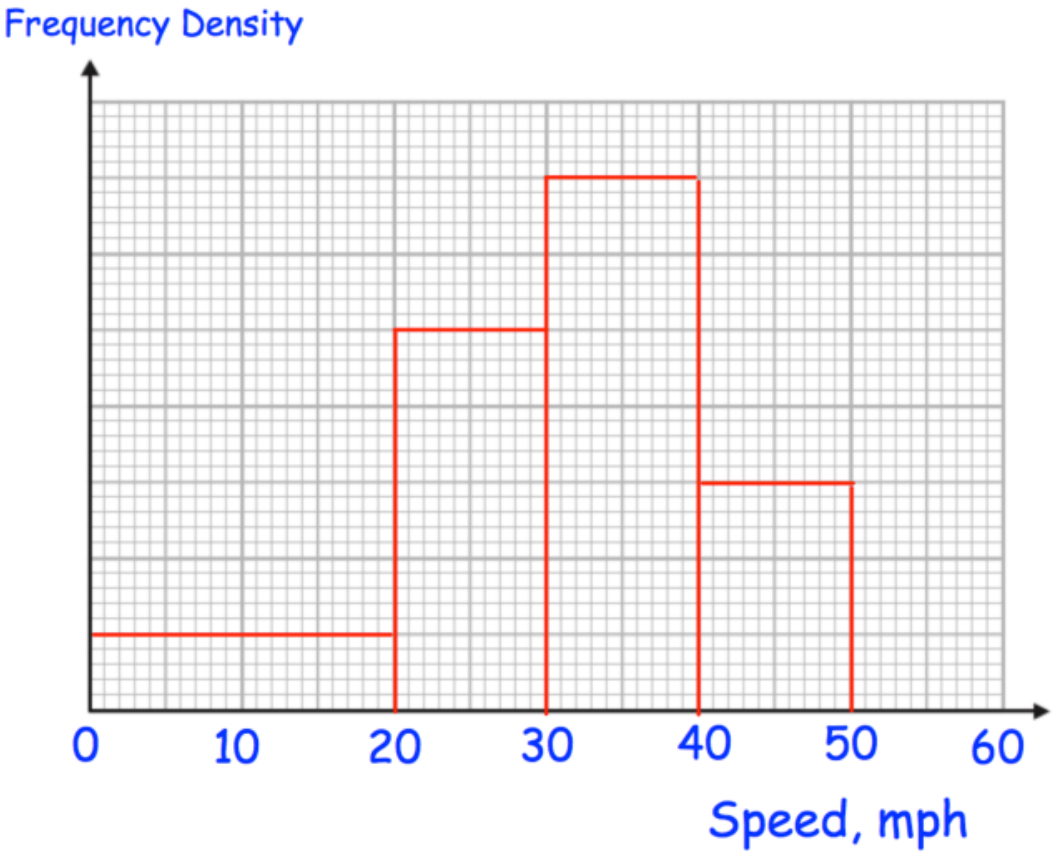
In this case, each large square represents two people, so the 0-10000 bar represents 4 people. Again, this is the area of the given bar so we must divide this by the class width to find the frequency density:

$$4 \div 10000 = 0.0004$$

This means that the scale of the frequency density axis is one large square to 0.0004.

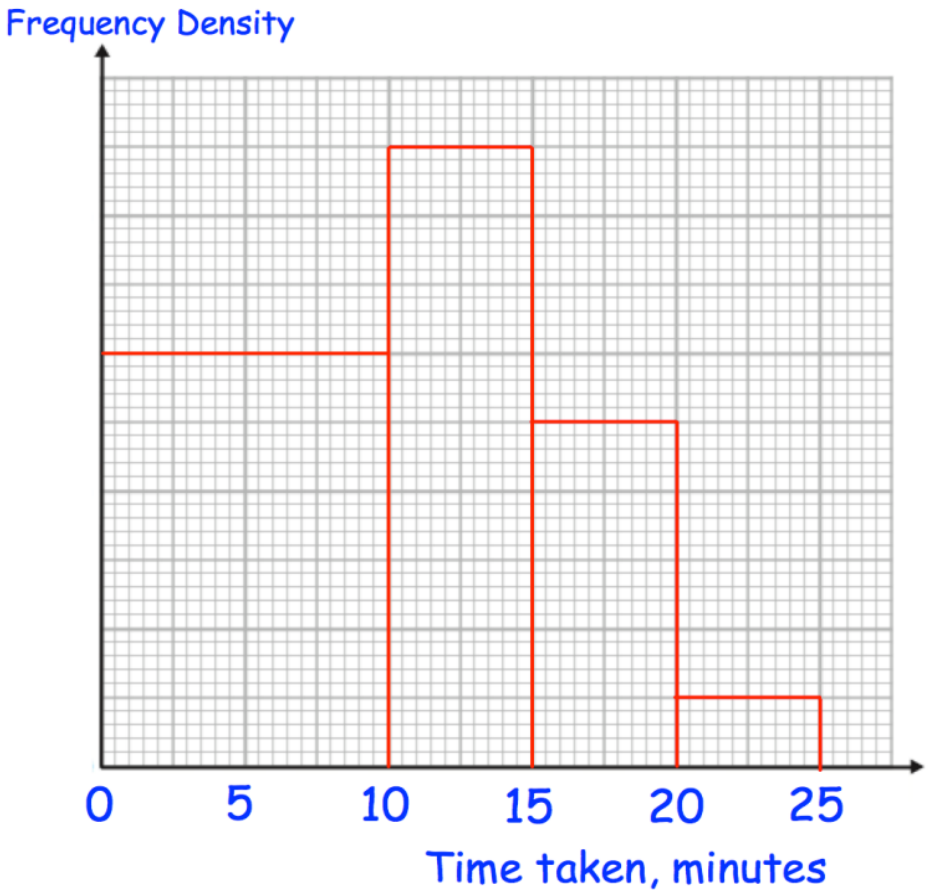
Practice B:

1) In the following histogram, 24 cars travelled faster than 40mph.
   a. Work out the scale of the histogram.
   b. Work out the frequencies of the other bars.



2) In the following histogram, the total frequency is 270. Work out the scale from this information.

(Hint: try to work out the frequency represented by one of the large squares)



Assessment B:

One bar of a histogram represents a frequency of 50. What frequency will a bar twice the width and twice the height be?

To finish...

Go back to the concept map on Page 4. Consider the following questions:

- What parts have we looked at in this tutorial?
- How well did you understand each part?
- Do you understand the links between them?
- How did this tutorial build on the previous one?
- What could you do to strengthen the areas you had trouble with?

Tutorial 3 – Estimating values from histograms

Objectives this tutorial:

A) Estimating frequencies from a histogram
B) Estimating the median from a histogram

Starter:

Look at the following grouped frequency table:

Weight (kg)	Frequency
$35 < x \leq 40$	10
$40 < x \leq 50$	50
$50 < x \leq 65$	90
$65 < x \leq 70$	20
$70 < x \leq 80$	30

1) How many objects weighed less than 50kg?
2) How many weighed more than 65kg?
3) How many weighed between 40 and 70kg?

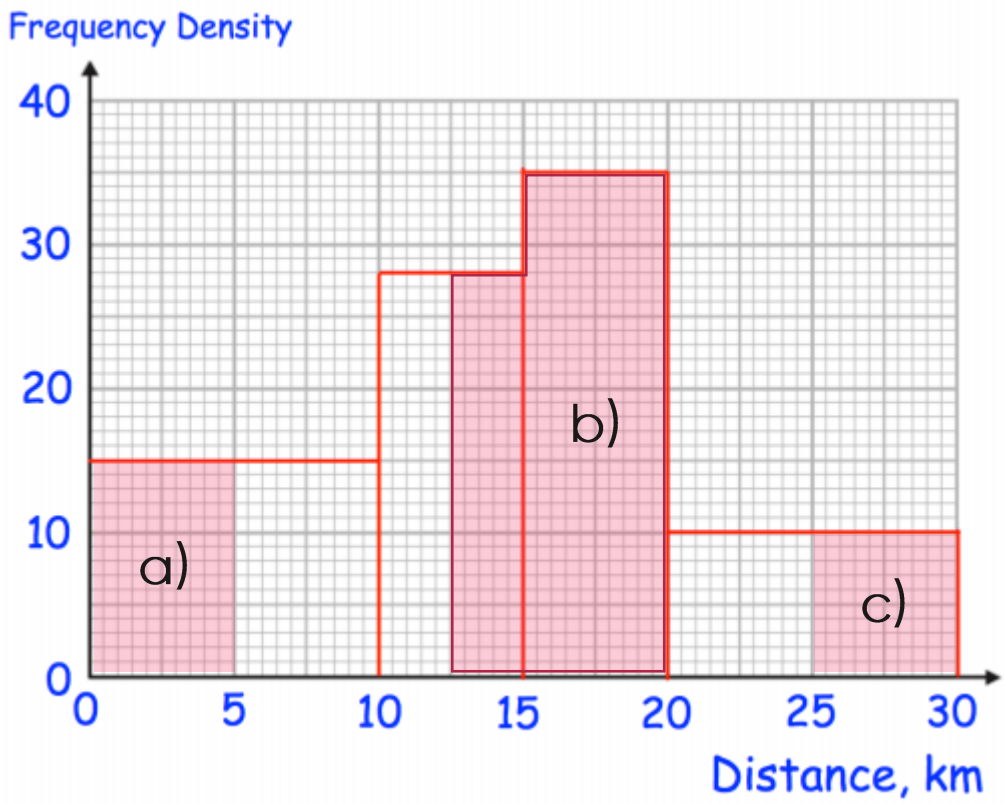
Objective A: Estimating frequencies from a histogram

To estimate frequencies from a histogram we find the area of regions of the chart.
A histogram is drawn from the following frequency table:

Distance, $\boldsymbol{x}$ (km)	Frequency
$0 < x \leq 10$	150
$10 < x \leq 15$	140
$15 < x \leq 20$	175
$20 < x \leq 30$	100

We will estimate the frequencies for

a) $x < 5$
b) $12.5 < x < 20$
c) $x > 25$



a) Find the area of the part of the bars between 0 and 5

Width $= 5$, Height $= 15$

Frequency = Area $= 5 \times 15 = 75$

b) Find the area of the part of the bars between 12.5 and 15, and add this to the area of the bars between 15 and 20

First area: Width $= 2.5$, Height $= 28$

Frequency = Area $= 2.5 \times 28 = 70$

Second area: Width $= 5$, Height $= 35$

Frequency = Area $= 5 \times 35 = 175$

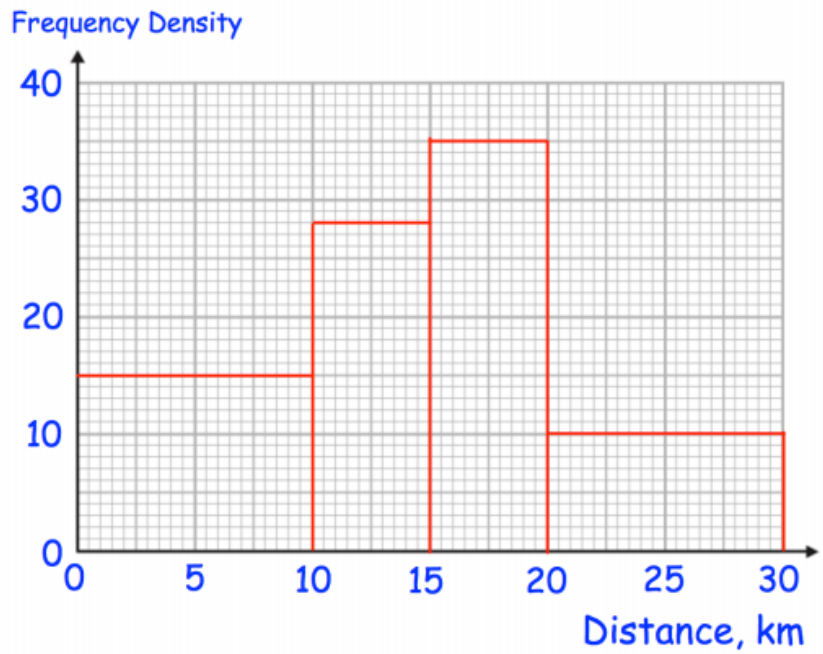
Total frequency = Total area $= 70 + 175 = 245$

c) Find the area of the part of the bars between 25 and 30

Width $= 5$, Height $= 10$

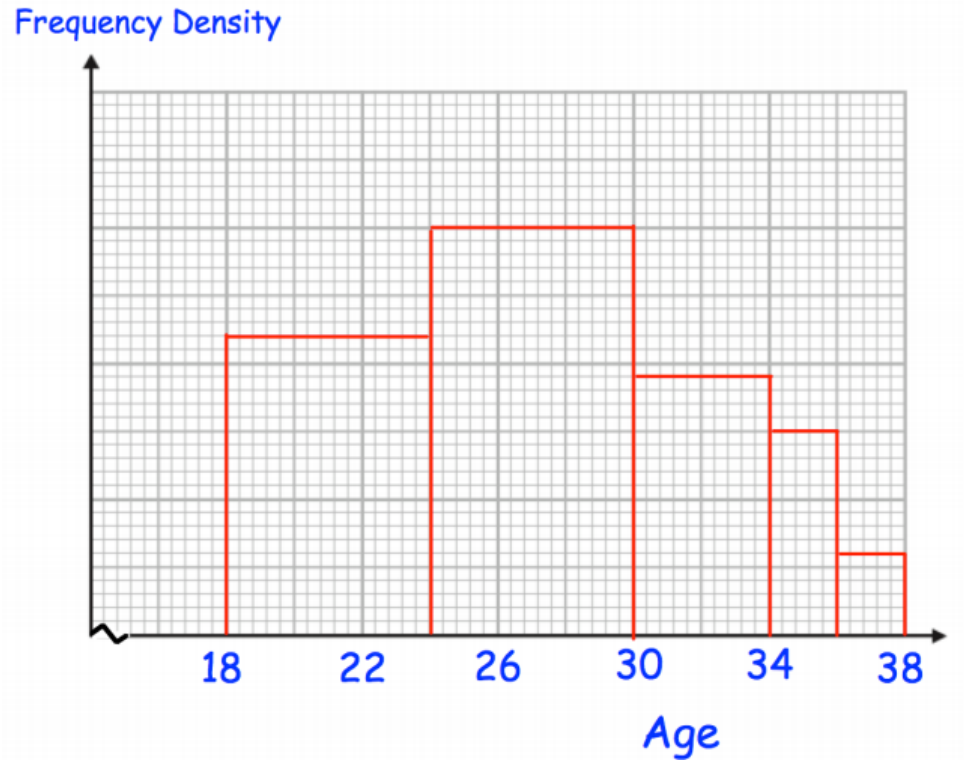
Frequency = Area $= 5 \times 10 = 50$

Practice A:



1) Estimate the frequency within the following ranges:
   a. $5 < D < 12.5$
   b. $10 < D < 14$
   c. $D > 16$

2) In the following histogram, there are 768 people under the age of 26. Estimate how many players are over 32.



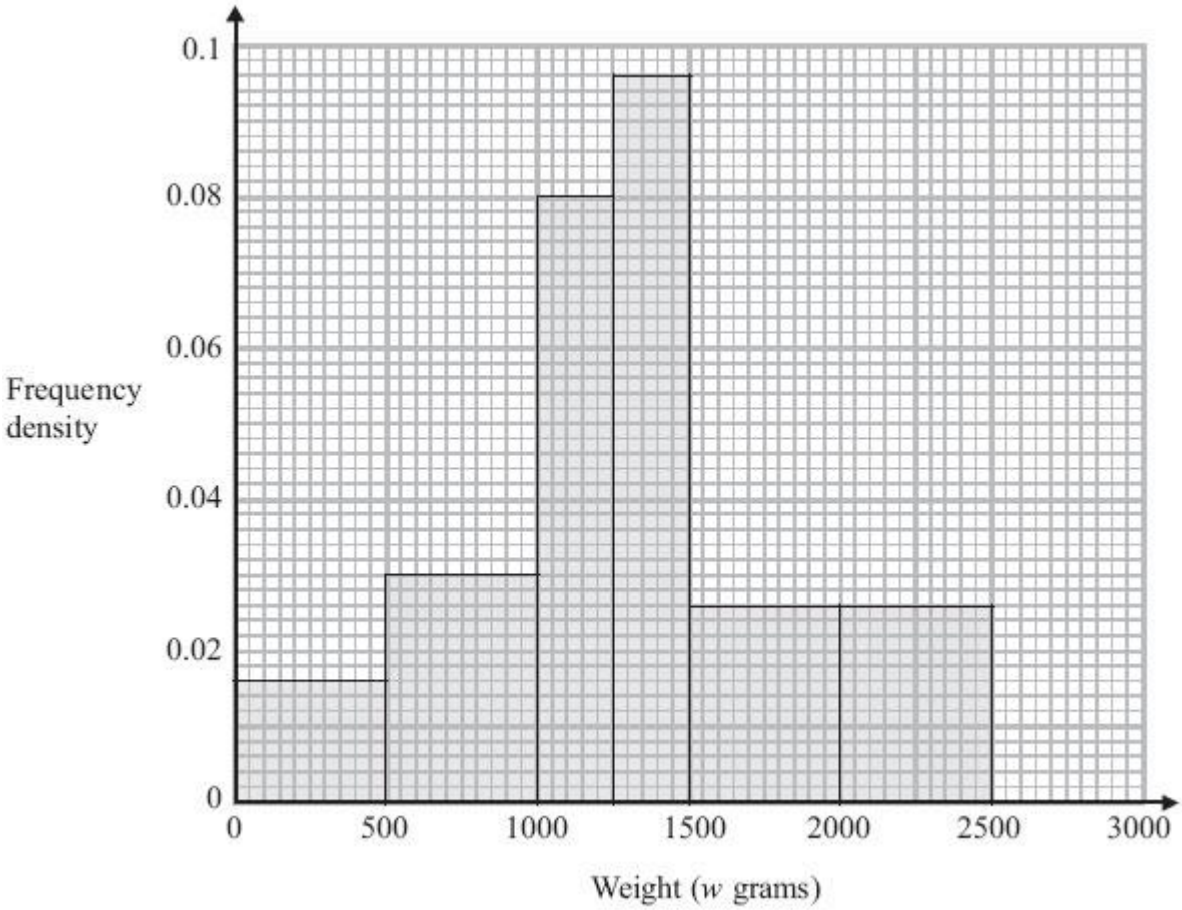
What other skills does this question require?

Assessment A:

When is your frequency an estimate, and when is it an exact value?

Objective B: Estimating the median from a histogram

We can use histograms to find an estimate for the median. This will be a value on the x-axis, and it the point at which the total area of the histogram is divided in half.



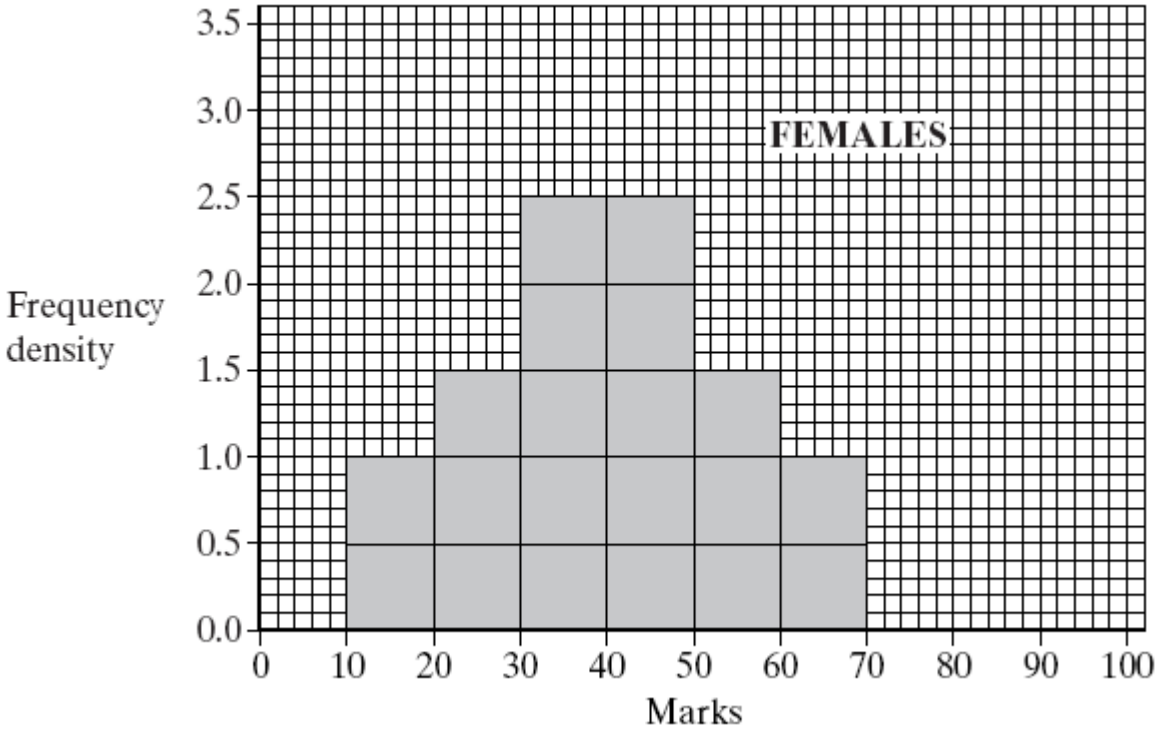
The area of each bar (frequency) is the given by multiplying the width by the height:

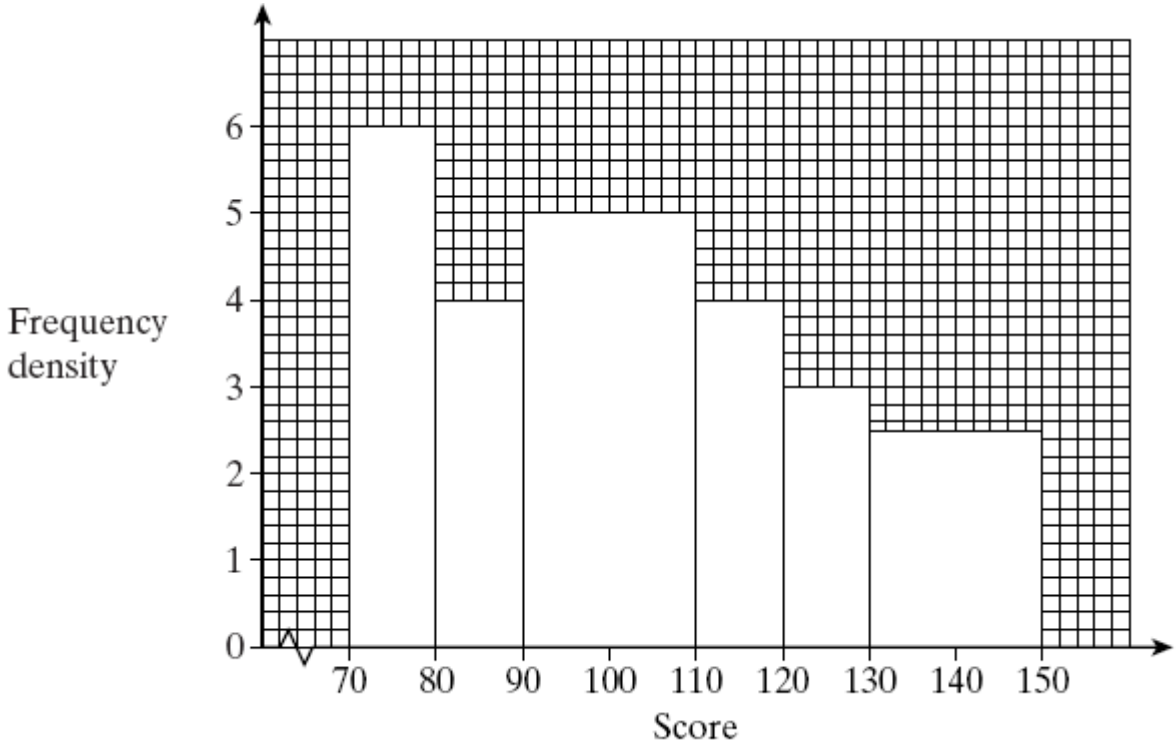
$$\begin{aligned} 500 \times 0.018 &= 9 \\ 500 \times 0.03 &= 15 \\ 250 \times 0.08 &= 20 \\ 250 \times 0.096 &= 24 \\ 500 \times 0.026 &= 13 \\ 500 \times 0.026 &= 13 \\ \text{Total} & \ \ 94 \end{aligned}$$

The total area is 94, so half of this is $94 \div 2 = 47$.
The area of the first three bars adds up to 44, so the median will lie in the fourth bar.
Since we need an extra area of 3, and the height of this bar is 0.096, the extra width needed is $3 \div 0.096 = 31.25$
Therefore the median is $1250 + 31.25 = 1281.25$

Practice B:

Find the median for the following histograms

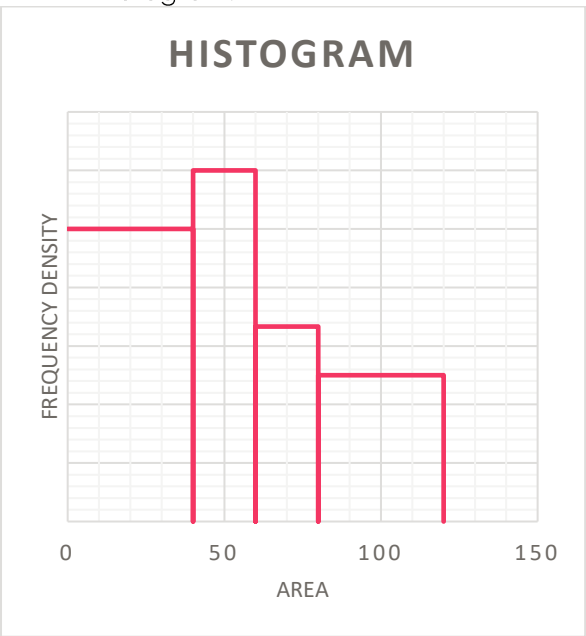

FEMALES
Frequency density
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
50
60
70
80
90
100
Marks

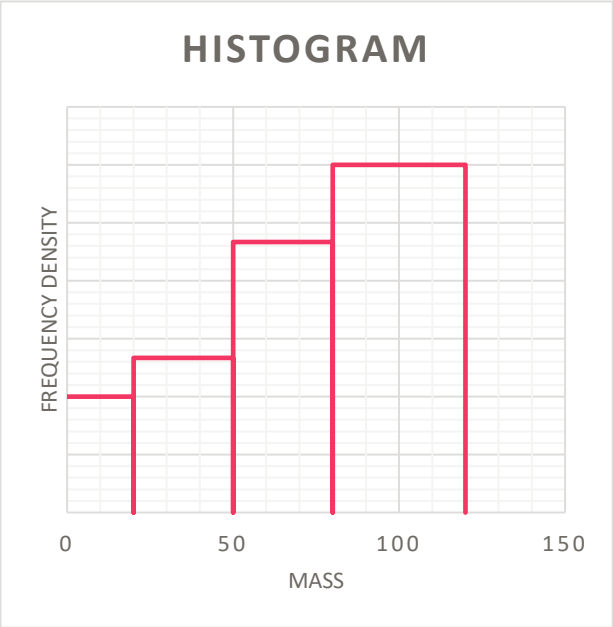

Frequency density
6
5
4
3
2
1
0
70
80
90
100
110
120
130
140
150
Score

What does the zig-zag on the x-axis mean?

Assessment B:

A) Which of these histograms has the greater median, and why?
B) The first bar in the first graph has a frequency of 400. Use this to work out the scale of the histogram.





To finish...

Go back to the concept map on Page 4. Consider the following questions:

- What parts have we looked at in this tutorial?
- How well did you understand each part?
- Do you understand the links between them?
- How did this tutorial build on the previous ones?
- What could you do to strengthen the areas you had trouble with?

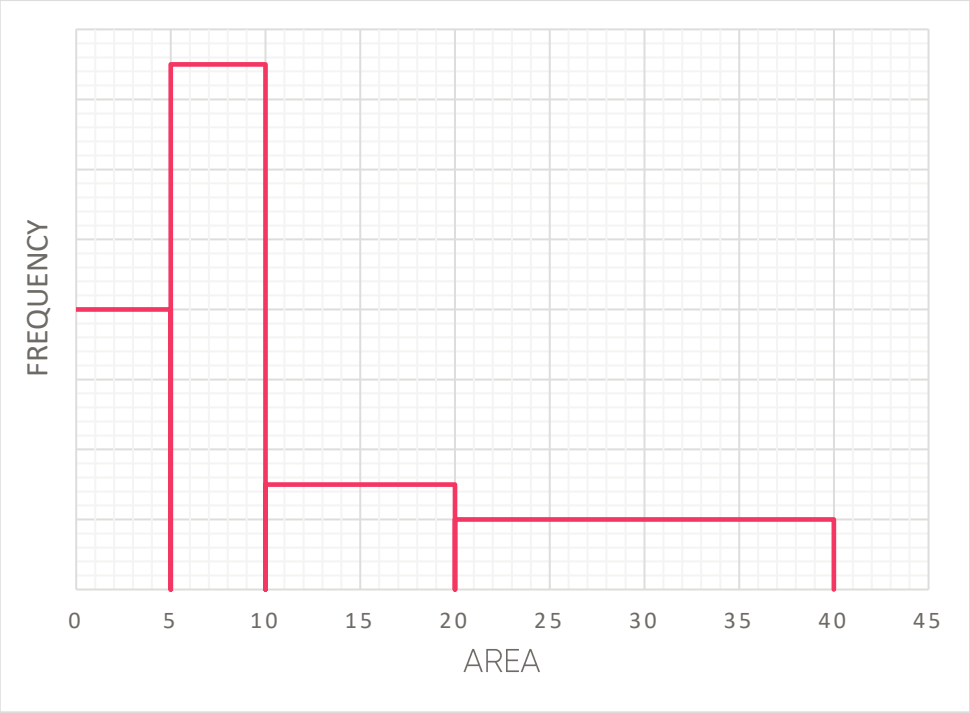
Tutorial 4 – Why use histograms?

Objectives this tutorial:

A) Understand differences between a histogram and a frequency diagram (bar chart), and compare two histograms.

Starter:

Some people took part in a charity race, and their times were recorded in the following diagram. 8 people finished in less than 5 minutes. How many ran the race in total?



Look carefully at the axes! What kind of chart is this?

Objective A: Histograms and frequency diagrams

Histograms and frequency diagrams are both drawn from a grouped frequency table, and the axes denoting the class intervals will be the same. However as we have seen, histograms use frequency density on their other axis, whereas frequency diagrams plot frequency.

In a frequency diagram, the height of each bar gives the frequency. In a histogram, the frequency is given by the area of each bar.

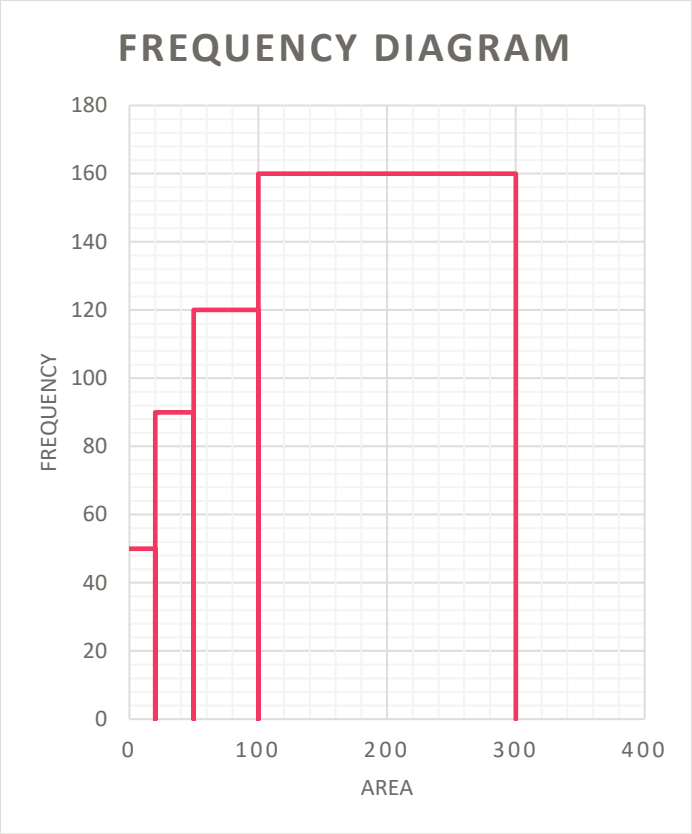
Practice A:

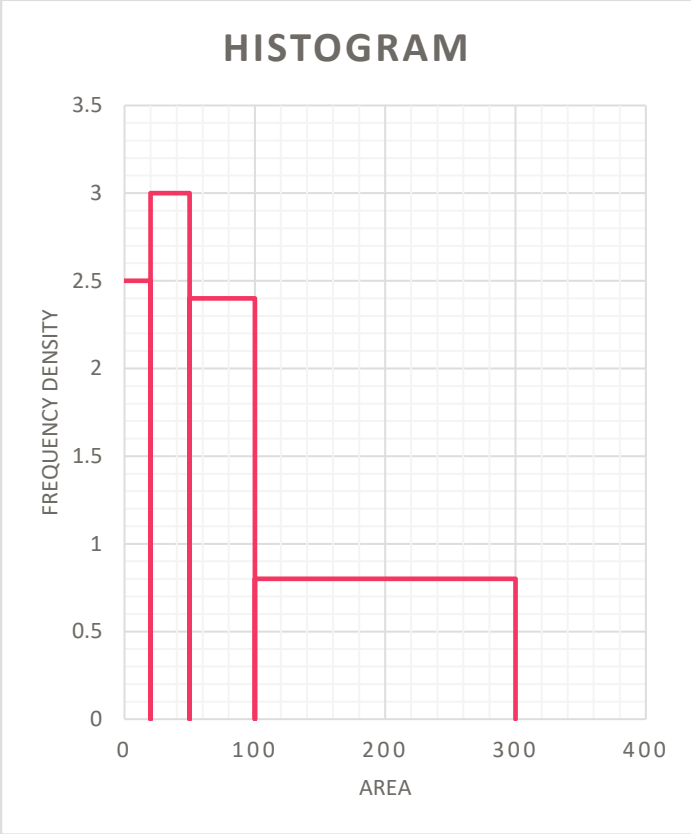
The following diagrams both represent the same data set.

What do you notice about them?

How do they compare?

Find the original data table. Which do you think represents the data the best?

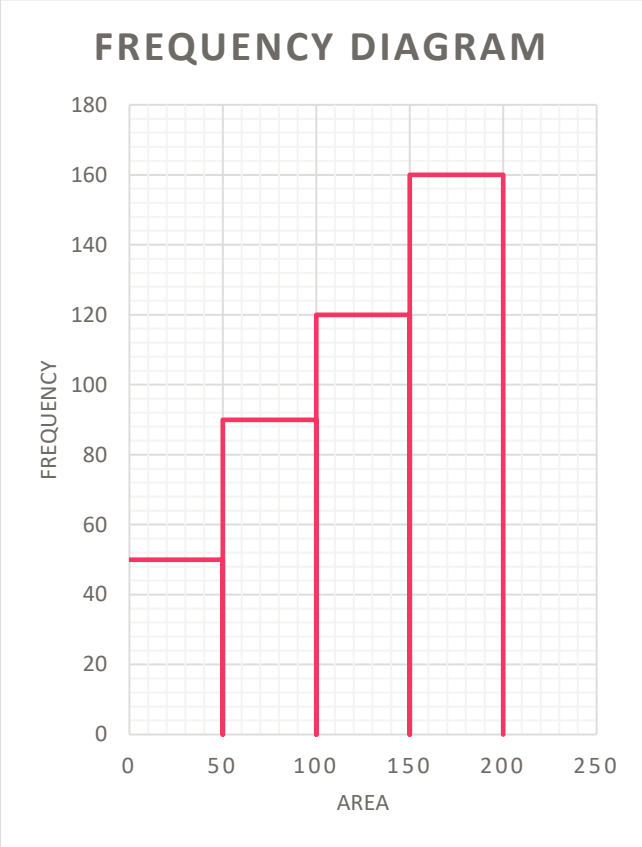

FREQUENCY DIAGRAM
FREQUENCY
180
160
140
120
100
80
60
40
20
0
0
100
200
300
400
AREA

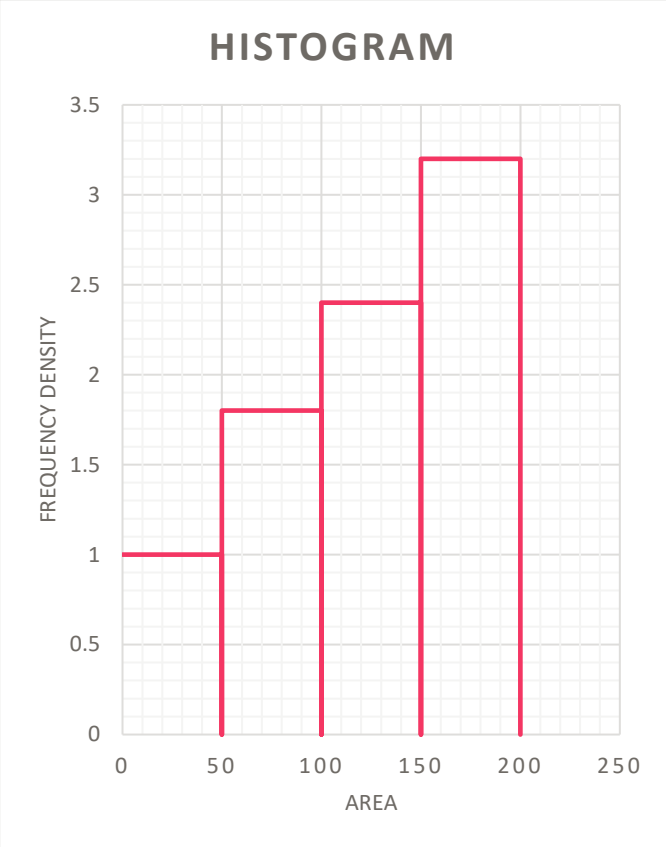

HISTOGRAM
FREQUENCY DENSITY
3.5
3
2.5
2
1.5
1
0.5
0
0
100
200
300
400
AREA

Now compare these two charts. Again, they are using the same data set.

What do you notice this time?

Why might this be the case?

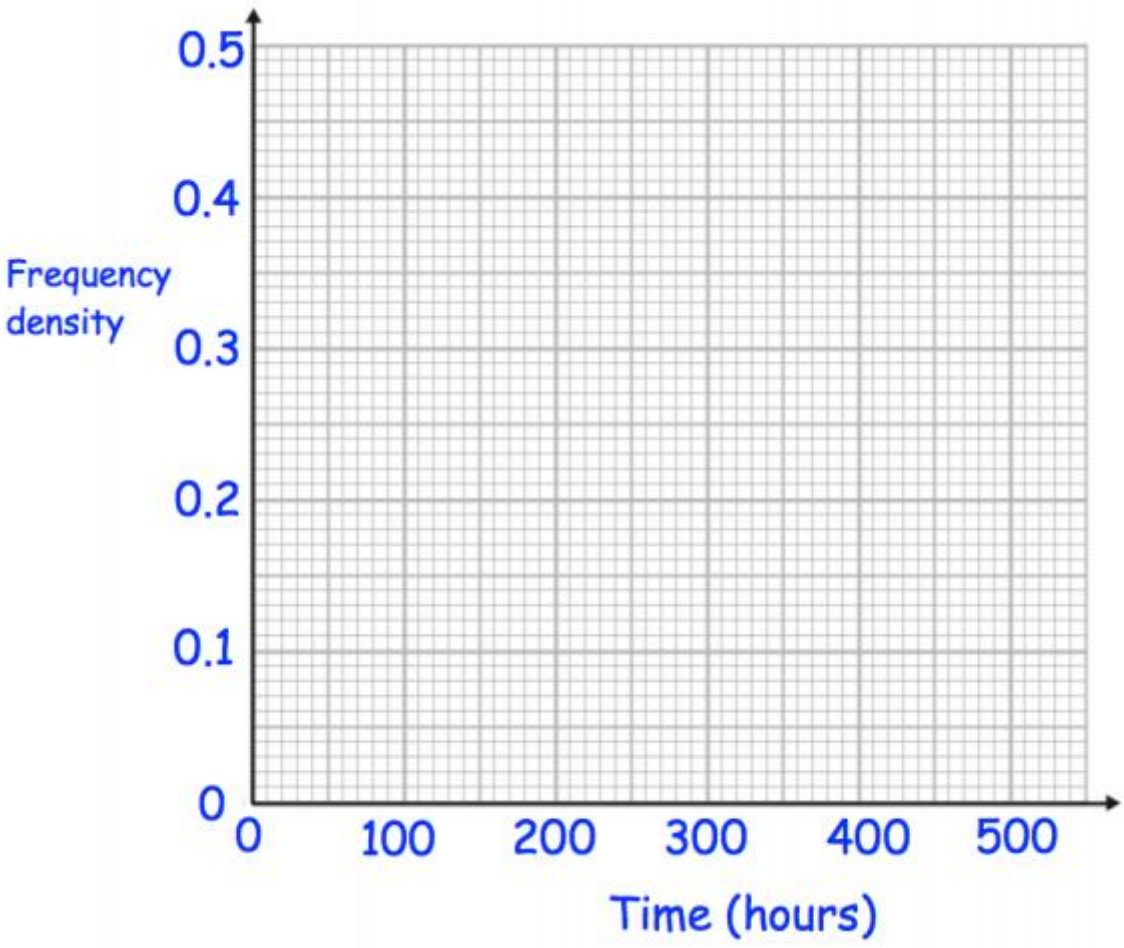




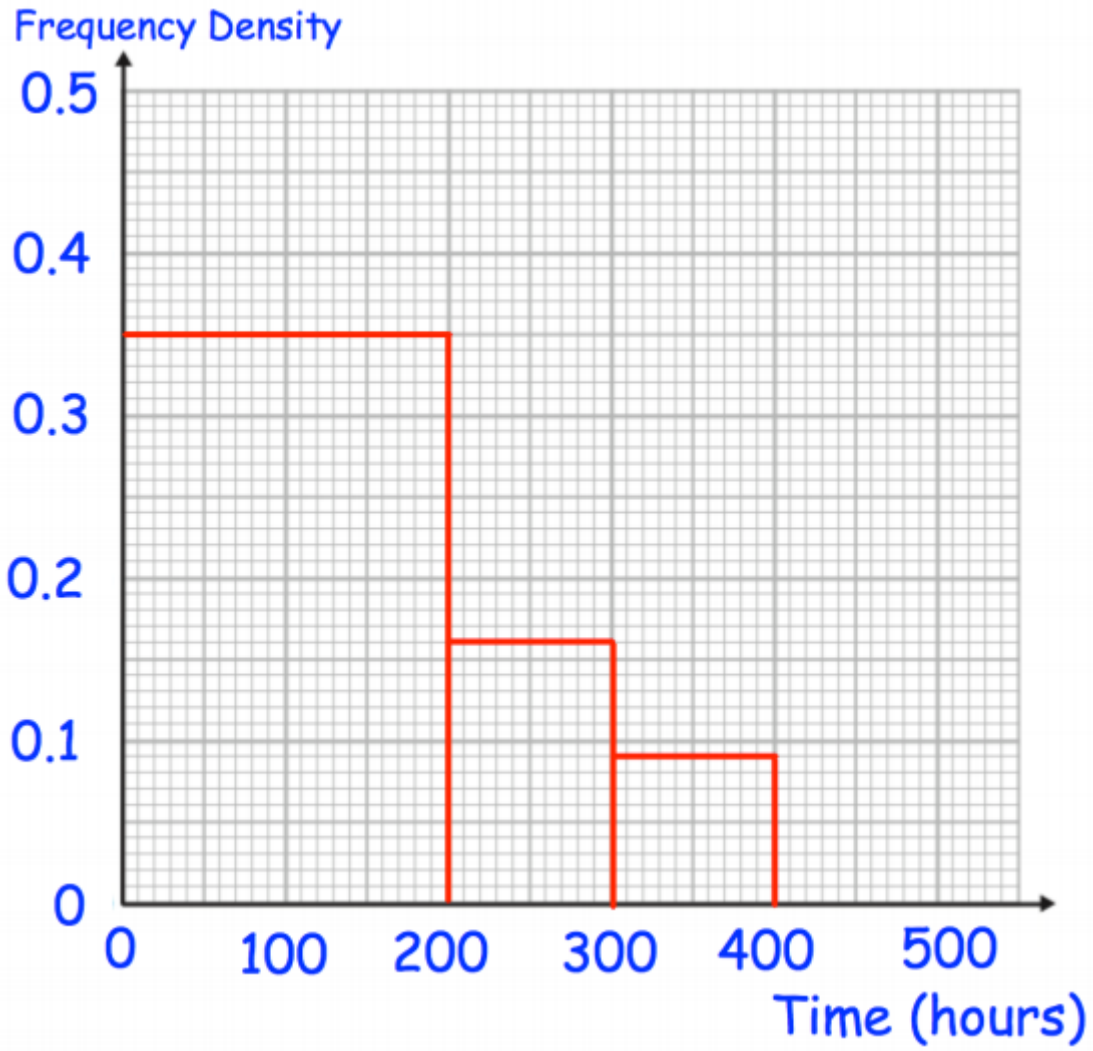
Assessment A:

This table gives information about the hours that one airline company have spent flying. Draw a histogram to show this information.

Time, $\boldsymbol{t}$ (hours)	Frequency
$0 < t \leq 100$	24
$100 < x \leq 150$	21
$150 < x \leq 200$	17
$200 < x \leq 350$	24
$350 < t \leq 500$	9



This histogram shows the distribution of time spent by pilots of another company. What comparisons can you make between the two?


Frequency Density
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
Time (hours)

To finish...

Go back to the concept map on Page 4. Consider the following questions:

- What parts have we looked at in this tutorial?
- How well did you understand each part?
- Do you understand the links between them?
- How did this tutorial build on the previous ones?
- What could you do to strengthen the areas you had trouble with?

Tutorial 7 - Feedback

What is the purpose of this tutorial?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well...	What I could have improved on...
•	•
•	•
•	•

My target for future work is...

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
•   •   •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Graph paper

Graph paper

Graph paper

Graph paper

Histograms

Key Stage 4 Programme

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