Researchers in Schools

Everything is Algebra

Key Stage 4 Programme

Pupil Name

Handbook Designed by Dr Durston

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
Baseline Assessment	9th Dec.	3pm	A14
1	13th Jan.		
2	20th Jan.		
3	27th Jan.		
4	3rd Feb.		
Final Assessment	10th Feb.		
Feedback Session	24th Feb.		

Timetable – Homework Assignments

Assignment	Description	Due Date
1	Complete tutorial 1 practice exam questions.	17th Jan.
2	Complete tutorial 2 practice exam questions.	24th Jan.
3	Complete tutorial 3 practice exam questions.	31st Jan.
4	Complete tutorial 4 practice exam questions.	7th Feb.

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Course Rationale

"Mathematics is a creative and highly interconnected discipline..."

The above statement is the very first line of the latest Key Stage 4 National Curriculum for Mathematics. Mathematics is unique as a subject in the way that its topics overlap and flow seamlessly into each other. Surds are learnt in the context of number problems, but give meaning to the geometric figures following the golden ratio. Statistics is generalised by the algebraic measures and variables used with data. When applying maths to other subjects or industries, it is rare to require an isolated topic. Indeed, the National Curriculum goes on to encourage pupils to "solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication".

"The letter is susceptible of operations which enables one to transform literal expressions and thus to paraphrase any statement into a number of equivalent forms. It is this power of transformation that lifts algebra above the level of a convenient shorthand."

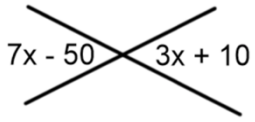
Tobias Dantzig

This course is about taking advantage of the power of algebra when facing questions about other mathematical topics and areas. The skills developed in this course will allow students to further develop and apply their mathematical knowledge outside of the subject, in areas such as Physics, Computer Science and Geography.

Key Vocabulary

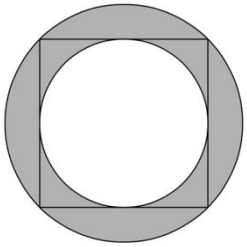
explicit – fully and clearly expressed or demonstrated.

Questions which **explicitly** require algebra will either state that algebra must be used, or will give you algebra to use in the question. Typically, these questions involve *forming an equation* from expressions given to you.



implicit – implied, rather than expressly stated.

Questions which **implicitly** require algebra do not clearly state that they require algebra to solve. The first step in these questions is to *assign variables* to elements of the question.



Not drawn accurately

Prove that the shaded area between the two circles is the same as the area enclosed by the inner circle.

Tutorial 1 – Algebra in Geometry

Objectives
A. Form and solve equations in geometry questions which explicitly require algebra. B. Assign variables and solve geometry questions which implicitly require algebra.

A. Form and solve equations in geometry questions which explicitly require algebra.

Knowledge

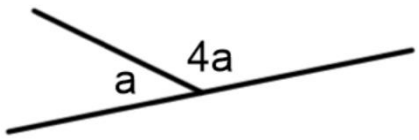
In the GCSE Mathematics exam, it is common for geometry problems to involve algebra. Variables (the letters used in algebra) often appear as angles, lengths areas or volumes of shapes.

When tackling a geometry problem which explicitly uses algebra, consider two things:

1. Can I form a sentence describing this situation?
2. Can that sentence be converted to an equation?

Example 1

Find the value of a.



Angle fact: The sum of angles on a straight line is 180°.

"The sum of a and 4a is 180 degrees."

$A + 4a = 180°$

$5a = 180°$

$a = 36°$

Example 2

The area of the triangle is $24cm^2$. Find the value of z.

Shape fact: The area of a triangle is half x base x height.

"Half multiplied by 3z multiplied by z is $24cm^2$."

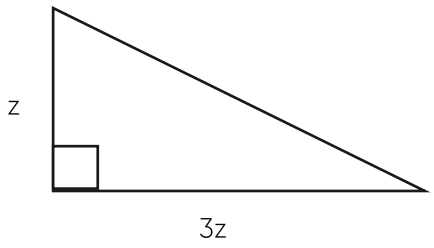
$\frac{1}{2} \times 3z \times z = 24cm^2$

$\frac{3}{2} z^2 = 24cm^2$

$3z^2 = 48cm^2$

$z^2 = 16cm^2$

$z = 4cm$

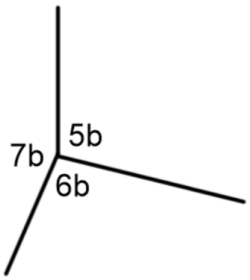


Practice

Exercise 1

Find the values of the variables:

1.



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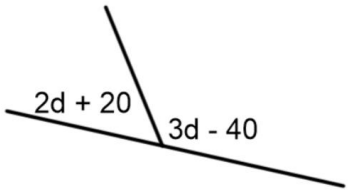

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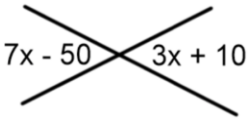

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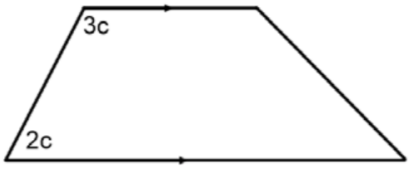

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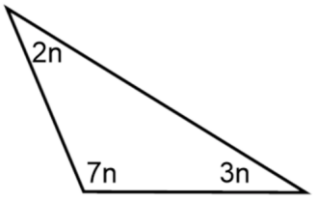

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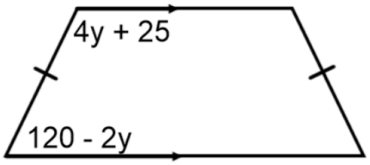
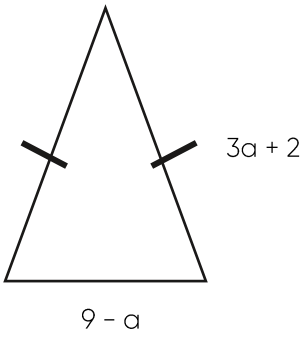


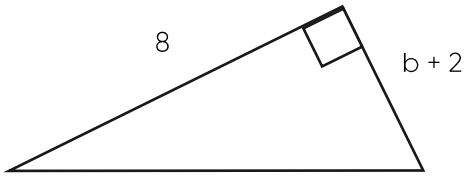
1. b = 20°, 2. d = 40°, 3. x = 15°, 4. c = 36°, 5. n = 15°, 6. y = 17.5°.

Exercise 2

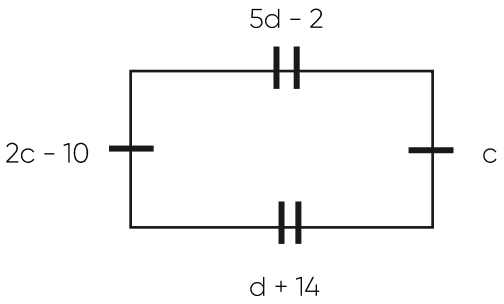
1. The perimeter of the triangle is 23. Find a.



2. The area of the triangle is 12. Find b.



3. Find the area of the rectangle.



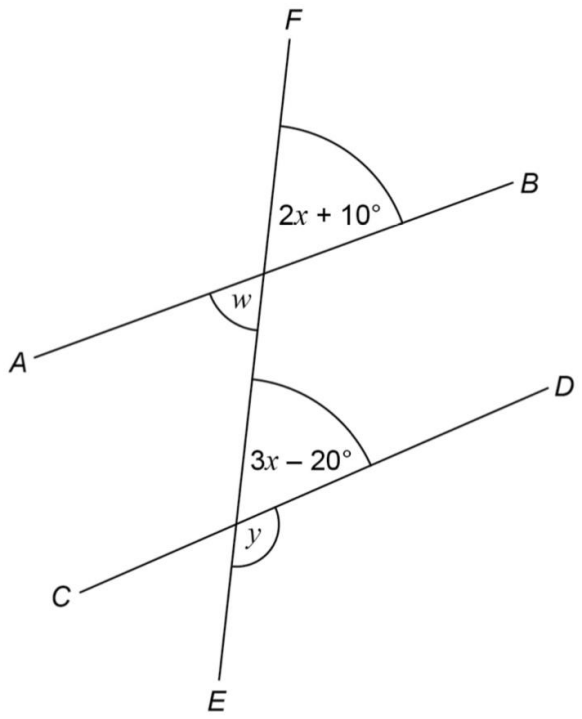
4. The shape below is made from five identical squares.

The area of the shape is 180cm².

Work out the length of side y.

1. a = 2, 2. b = 1, 3. 180, 4. y = 6cm.

10 *AB*, *CD* and *EF* are straight lines.



Not drawn accurately

10 (a) Ava assumes that *AB* and *CD* are parallel.

What answer should she get for the size of angle y?

[4 marks]

10 (b) In fact,

AB and *CD* are **not** parallel

angle w is 60°

What effect does this have on the size of angle y?

Tick a box.

☐ y is bigger

☐ y is the same

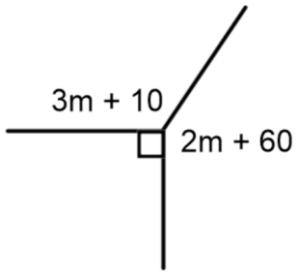
☐ y is smaller

Show working to support your answer.

[3 marks]

Assessment

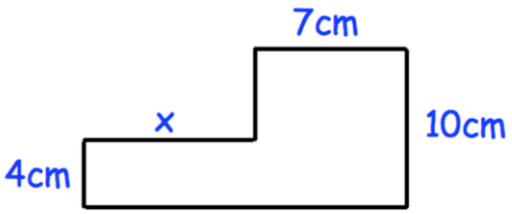
1.



m =

A	B	C	D
4°	22°	40°	None of the above

2.



The area of the compound shape is 106cm².
Work out the size of x.

A	B	C	D
3.6cm	9cm	10.6cm	None of the above

B. Assign variables and solve equations in geometry questions which implicitly require algebra.

Knowledge

Some questions require the use of algebra, even though the question does not mention algebra or variables at all. In the following section we will *assume* all of the questions require algebra. We should then consider:

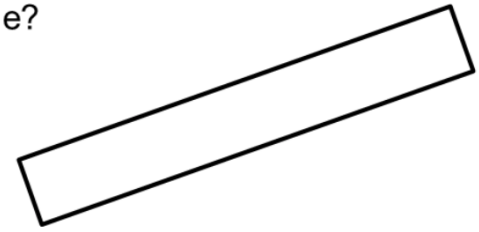
- Which element of the question can I represent using algebra?

Example 1

the length of a rectangle is 2cm less than 7 × width

the perimeter is 60 cm

what are the dimensions of the rectangle?



"I will represent the width of the rectangle by w."

"The length of the rectangle is 2cm less than 7 x w."

Length of rectangle = 7w – 2

Shape fact: The perimeter is the sum of each side.

"The perimeter is the length + the length + the width + the width of the rectangle."

60 = 7w – 2 + 7w – 2 + w + w

60 = 16w – 4

16w = 64

w = 4

Example 2

Length of original rectangle = l
Width of original rectangle = w

Shape fact: The perimeter is the sum of each of the sides.

"The first new perimeter is twice the original length + twice the original length + half the original width + half the original width of the rectangle."

48 = 2l + 2l + 0.5w + 0.5w

4l + w = 48

"The second new perimeter is half the original length + half the original length + twice the original width + twice the original width of the rectangle."

42 = 0.5l + 0.5l + 2w + 2w

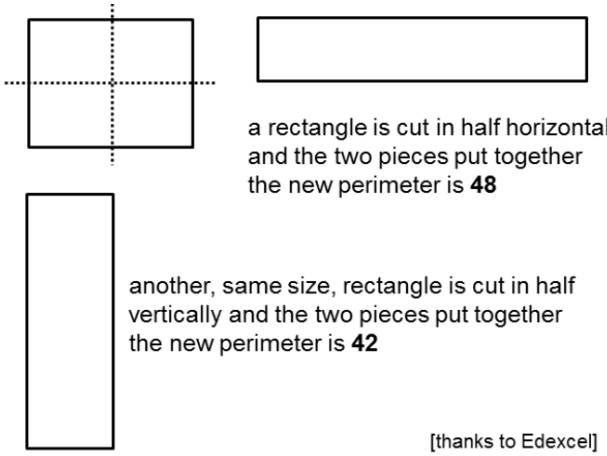
l + 4w = 42

Solve simultaneous equations:

-15w = -120

w = 8

l = 10



Practice

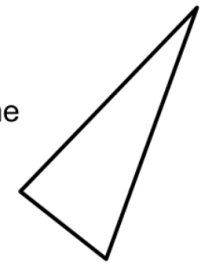
Exercise 1

1. the first side of a triangle is 8cm shorter than the second side

   the third side is 4 × the first side

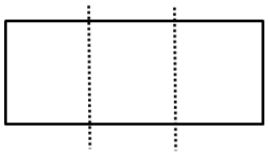
   the perimeter is 26cm

   what are the lengths of the triangle?



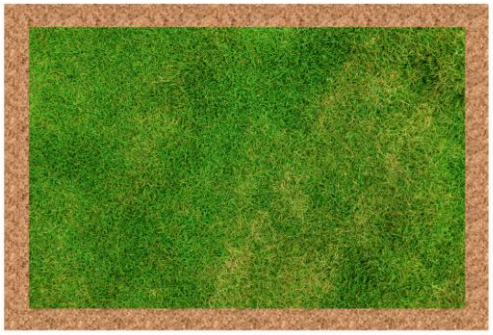
2.

   the rectangle is cut in thirds horizontally and the three pieces put together the new perimeter is **188**



   the rectangle is cut in thirds vertically and the three pieces put together the new perimeter is **92**

3. Draw a trapezium. Split it into two triangles. Find an expression for the area of the trapezium.

4. 

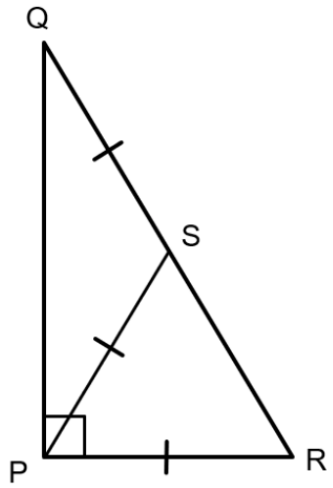
   the area of the path around a lawn is 32 m^2

   the perimeter of the lawn is 28 m

   how wide is the path?

Exercise 2

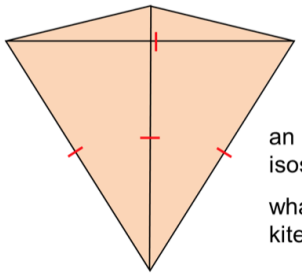
1.



a right angled triangle is split into two isosceles triangles

show that the bottom isosceles triangle (PSR) is equilateral

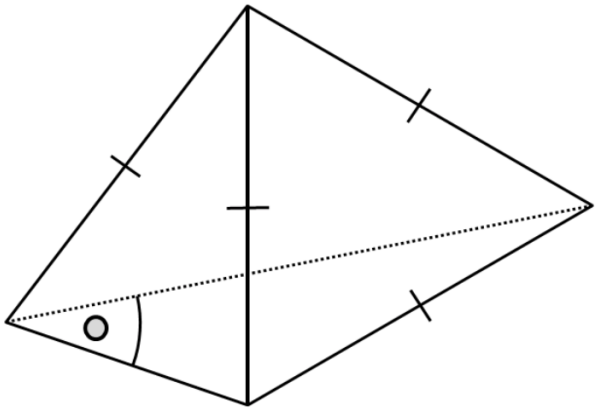
2.



an equilateral triangle with two isosceles triangles –

what are the angles inside the kite?

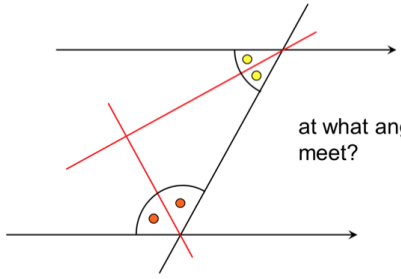
3.



an equilateral triangle with an isosceles triangle

what is the angle shown?

4.



at what angle do the red lines meet?

11 A large rectangle is made by joining three identical small rectangles as shown.

Not drawn accurately

The perimeter of one small rectangle is 15 cm

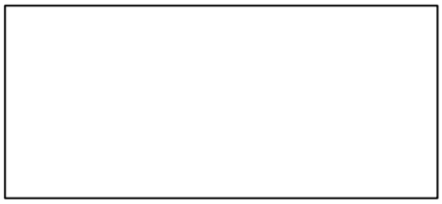
Work out the perimeter of the large rectangle.

[4 marks]

19 The length of a rectangle is five times the width.

The area of the rectangle is 1620 cm^2

Not drawn accurately

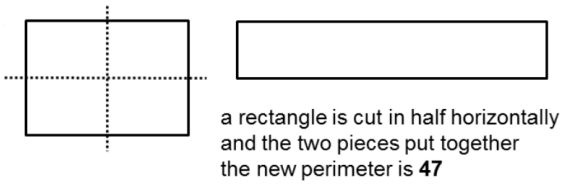


Work out the width of the rectangle.

[3 marks]

Assessment

1.

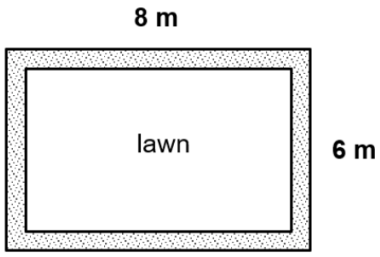


The length (horizontal) of the original rectangle is 10.

What is the width of the original rectangle?

A	B	C	D
7	8	9.25	None of the above

2. The area of the path is always the same.



the perimeter of the lawn is **18.4 m**

how wide is the path?

A	B	C	D
< 1.5m	1.5m – 2m	2m – 2.5m	2.5m >

Tutorial 2 – Algebra in Ratio & Proportion

Objectives
A. Form and solve equations in proportion questions which explicitly require algebra. B. Assign variables and solve proportion questions which implicitly require algebra.

A. Form and solve equations in proportion questions which explicitly require algebra.

Knowledge

Similarly, to algebra in geometry, it is common to find ratio problems which involve algebra in GCSE Mathematics exams. Variables typically represent one or more of the quantities being held in proportion to each other.

In ratio and proportion questions, the key consideration is the relationship between the involved quantities. Consider:

1. What is the relationship between two quantities?
2. How is this written as an equation?

Example 1

e is 150% more than *d*
what is the ratio *d* : *e* in its simplest form ?
what is the ratio *e* : *d* in the form 1 : *n* ?

e = 1.5 d. If d = 2, then e = 3.

So d : e = 2 : 3

e : d = 1 : $\frac{2}{3}$

Example 2

the ratio *a* : *b* = 9 : 4
and 7*b* = 3*c*

what is the ratio *a* : *c* ?

a : b = 9 : 4 means if a = 9, b = 4. So a = $\frac{9}{4}$ b.

7b = 3c means b = $\frac{3}{7}$ c.

So a = $\frac{9}{4}$ x $\frac{3}{7}$ c.

a = $\frac{27}{28}$ c

If c is 28, a is 27. So...

a : c = 27 : 28

Practice

Exercise 1

1. the ratio $d : e = 11 : 8$
and $7e = 5f$

what is the ratio $d : f$?

2. the prices of two phones are in the ratio $a : b$

when the prices are both increased by £20
the ratio becomes 5 : 2

when the prices are both decreased by £5 the
ratio becomes 5 : 1

express the ratio $a : b$ in lowest terms

3. the ratio **a : b** = 3 : 4

what is the ratio

(i) b : 3a – b in its simplest form?

(ii) 2a + b : 4a – b in its simplest form?

4. bag A contains 7n counters
bag B contains 3n counters

3 counters are taken from bag A
and put in bag B

the ratio of counters in bag A to
bag B is now 5 : 3

how many counters were in the
bags initially?

5. P is a rectangle with length 60 cm and width x cm
Q is a rectangle with width y cm

the length of Q is 20% more than the length of P

the area of P is 15% less than the area of Q

work out the ratio of $x : y$

13 $x : y = 7 : 4$

$x + y = 88$

Work out the value of $x - y$

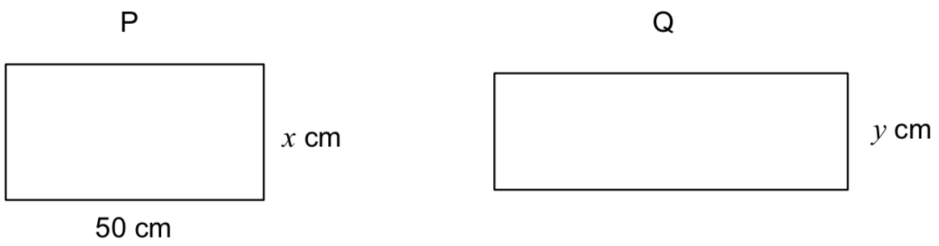
[3 marks]

Answer _____

17 P is a rectangle with length 50 cm and width x cm

Q is a rectangle with width y cm

Not drawn accurately



The length of Q is 20% **more** than the length of P.

The area of Q is 10% **less** than the area of P.

Work out the ratio $x : y$

Give your answer in its simplest form.

[4 marks]

Assessment

1. the ratio $g : h = 5 : 9$
and $4h = 7i$

what is the ratio $g : i$?

A	B	C	D
5 : 4	7 : 4	20 : 63	None of the above

2. a and b are in the ratio 9 : 4

$a + b = 65$

what is the value of $a - b$?

A	B	C	D
−25	25	Infinitely many solutions	None of the above

B. Assign variables and solve equations in proportion questions which implicitly require algebra.

Knowledge

Solutions to ratio and proportion questions rarely require a single approach, particularly in the GCSE exams. However, it is often convenient to implement algebra in these situations. In the following section we will *assume* all of the questions require algebra. We should then consider:

- Which element of the question can I represent using algebra?

Example 1

two whole numbers are such that their difference
and their sum are in the ratio 1 : 7

what could they be?

Let the two whole numbers be a and b, with a being the larger number.

The difference between the two numbers is a – b. Their sum is a + b.

difference : sum = a – b : a + b = 1 : 7

If the sum is 7, the difference will be 1. So, sum = 7 x difference.

So, a + b = 7 (a – b).

$$a + b = 7a - 7b$$

$$6a = 8b$$

$$a = \frac{4}{3}b$$

So any a and b which satisfies the above relationship will have difference : sum = 1 : 7, e.g. a = 4 and b = 3.

Example 2

Anna and Bea have sweets in the ratio 7 : 3
Anna gives Bea 3 sweets and the ratio is now 5 : 3
how many sweets did each have initially?

Let initial number of sweets be S. At the beginning, Anna has $\frac{7}{10}$ S and Bea has $\frac{3}{10}$ S.

After Anna gives Bea 3 sweets, Anna has $\frac{7}{10}$ S – 3 and Bea has $\frac{3}{10}$ S + 3.

The ratio is now 5 : 3. In other words, if you divide the number of sweets Bea has by 3 and multiply that by 5, you get the number of sweets Anna has. So:

Anna's sweets = $\frac{5}{3}$ Bea's sweets

$$\frac{7}{10}S - 3 = \frac{5}{3} \times \left(\frac{3}{10}S + 3\right)$$

$$\frac{7}{10}S - 3 = \frac{5}{10}S + 5$$

$$\frac{1}{5}S = 8$$

$$S = 40.$$

So the original number of sweets was 40.

Practice

Exercise 1

1. two whole numbers are such that their difference and their sum are in the ratio 2 : 5

   what could they be?

2. Cara and Dylan have sweets in the ratio 5 : 1
   Cara gives Dylan 14 sweets (she had a lot) and the ratio is now 3 : 2
   how many sweets did each have initially?

3. the ratio of Ann's age to Bob's age is 3 : 4

   in 7 years time this ratio will be 4 : 5

   (i) what are their ages now?
   (ii) after how many years (from now) will the ratio be 5 : 6?

4. two numbers are such that their difference, their sum and their product are in the ratio 1 : 9 : 20

   what are they?

5. the ratio of pairs of white socks to pairs of red socks is 9 : 5

   9 pairs of the white socks are dyed red

   the ratio of pairs of white socks to pairs of red socks is now 3 : 2

   how many pairs of each colour were there initially?

6. a school has 270 students with the ratio of boys : girls = 5 : 4

   another school has 180 students with the ratio of boys : girls = 4 : 5

   if the two schools were combined what fraction would be girls?

7. the ratio of geese to ducks on a lake is 4 : 9

when 10 extra geese land on the lake (and there are no duck changes) the ratio is 2 : 3

how many of each were there initially?

8. the ratio of men : women : children at a Shrewsbury football match is 15 : 7 : 3

there were 2016 more men than women at this match

how many children were there?
how many people altogether?

Exam Questions

8 Theo starts with savings of £18
James starts with no savings.

Each week from now,

Theo will save £4.50 and James will save £4

In how many weeks will Theo and James have savings in the ratio 15 : 8 ?

[3 marks]

11 Ed and Fay shared £330 in the ratio 7 : 4

Ed gives Fay some of his money.

Fay now has the same amount as Ed.

How much does Ed give Fay?

[3 marks]

Assessment

1. Jan and Kim own numbers of marbles that are in the ratio 5 : 6

Jan gains 2 more marbles and the ratio is now 7 : 8

In total, how many marbles did they originally own?

A	B	C	D
< 50	50 – 100	100 – 200	200 >

2. Rune sold 216 cakes in the ratio small : medium : large = 5 : 8 : 11

the profit for one medium cake was twice the profit for one small cake

the profit for one large cake was three times the profit for one small cake

the total profit was £340.20

work out the profit for one small cake

A	B	C	D
70p	£1.58	£6.30	None of the above

Tutorial 3 – Algebra in Other Topics

Objectives
A. Form and solve equations in questions which explicitly require algebra. B. Assign variables and solve questions which implicitly require algebra.

A. Form and solve equations in questions which explicitly require algebra.

Knowledge

Algebra can come up in a variety of topics outside of those we've looked at already (Geometry and Ratio/Proportion). On the topic of Number, algebra often is required for questions regarding proof, and is closely related to the operations used on surds. In Statistics algebra can be used to simplify reverse-mean questions and frequently occurs in probability problems.

Due to the wide range of contexts algebra may appear in, it is difficult to formulate a single strategy for implementing algebra in a range of mathematical topics. However, in general we should consider:

1. What is the relationship between two quantities?
2. How is this written as an equation?

Example 1

Find the value of b, such that the following sequence is quadratic:

$b - 2 \qquad b + 1 \qquad 3b \qquad 2b + 10$

In quadratic sequences, the difference between differences is the same. Let's calculate the differences:

$$\begin{array}{ccccccc} b-2 & & b+1 & & 3b & & 2b+10 \\ & 3 & & 2b-1 & & -b+10 & \\ & & 2b-4 & & -3b+11 & & \end{array}$$

If the difference between differences is the same, then we must have:

$$2b - 4 = -3b + 11$$

$$5b - 4 = 11$$

$$5b = 15$$

$$b = 3$$

Let's check our answer. Substituting b = 3 into the sequence gives:

$$1 \qquad 4 \qquad 9 \qquad 16$$

which is a quadratic sequence!

Example 2

In a wallet there are n 5p coins, 2n 10p coins and 4 20p coins. When taking a coin at random, the probability of getting a 5p coin is $\frac{1}{5}$.

How much money is in the wallet?

In total, there are 3n + 4 coins in the wallet. As there are n 5p coins, the probability of getting one is $\frac{n}{3n+4}$. We are told that this is equal to $\frac{1}{5}$. So:

$$\frac{n}{3n+4} = \frac{1}{5}$$

$$\frac{5n}{3n+4} = 1$$

$$5n = 3n + 4$$

$$2n = 4$$

$$n = 2$$

Substituting n = 2 into the number of each coin, we have:

2 x 5p coins
4 x 10p coins
4 x 20p coins

So, in total there is £1.30 in the wallet.

Practice

Exercise 1

1. $(\sqrt{x})^4 = 121$. Find x.

2. $\frac{\sqrt{96}}{\sqrt{x}} = 3\sqrt{2} + 3$. Find x.

3. set A contains the four numbers
104 , 120, 160 and 180

set B contains the five numbers
75 , 78 , 90 , 120 and *n*

mean set A : mean set B = 4 : 3

find *n*

4. n is a whole number.

- Multiply n by 6
- Add 12
- Multiply by the original number (n)
- Divide by 3
- Add 2

Show that the result is even and a multiple of n+1.

5. A bank pays 10% simple interest. After 5 years, Alex and Brian receive £x in their shared account. Initially, Alex had invested £$\frac{x}{3}$ and Brian £18.

Who contributed more?

6. (Following Q5). If Brian had initially invested £50, who would have contributed more?

Can you explain your result?

7. there are 5 people in a room:
A , B, C, D and E

their ages: C is 7, D is 17 and E is 20

the mean age of A and D is equal to the mean age of B and C

the mean age of all 5 people is 12

what ages are A and B?

8. in another room there are also 5 people:
P , Q, R, S and T

their ages: R is 8, S is 13 and T is 20

the mean age of P and R is equal to the mean age of Q and T

the mean age of P, Q and S is one more than the mean age of P and Q

what are the ages of P and Q?

9. a set of four positive whole numbers is:

$b - a$, a , b , $b + a$

not necessarily in this order

the mean of all the numbers is less than the median

what values can *a* and *b* have?

10. find the value of 'n' that makes these four expressions the consecutive terms in an arithmetic sequence

(a)	2n + 1	4n – 1	5n	19
(b)	n + 1	4n	7n – 1	9n

11. A bag contains x blue tokens and 5 red tokens.

If Ahmed takes 1 token, what is the probability (in terms of x) of:

a) Getting a red token?
b) Getting a blue token?

If Ahmed takes 1 token, puts it back, then takes another token, what is the probability (in terms of x) of:

c) Getting 2 red tokens?
d) Getting 2 blue tokens?
e) Getting 1 red and 1 blue token?

If Ahmed takes 2 tokens (without putting any back), what is the probability (in terms of x) of:

f) Getting 2 red tokens?
g) Getting 2 blue tokens?
h) Getting 1 of each token?

In scenario h), what is the probability when there is originally 1 blue token in the bag?

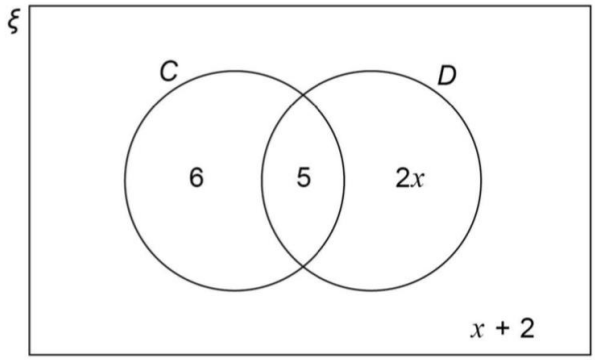
Ahmed takes a token,then puts it back. He does this 100 times. He records 80 blue tokens. Estimate the number of blue tokens in the bag.

14 In the Venn diagram

ξ represents 31 students in a class

C is students who have a cat

D is students who have a dog



14 (a) One student from the class is picked at random.

Work out the probability that the student has a dog.

[3 marks]

14 (b) One of the students who has a cat is picked at random.

Work out the probability that this student has a dog.

[1 mark]

20 A linear sequence starts

$a + 2b$ $a + 6b$ $a + 10b$ …….. ……..

The 2nd term has value 8

The 5th term has value 44

Work out the values of a and b.

[4 marks]

Assessment

1. Find the value of c such that the following sequence is quadratic:

c – 1 2c – 1 3c + 3 6c – 1

A	B	C	D
$-5 < c$	$-5 < c < 0$	$0 < c < 5$	$c > 5$

2. The set:

2 5 a 9 10 b

has a mean of 9 and a range of 18.

What is the median?

A	B	C	D
8	8.5	20	None of the above

B. Assign variables and solve equations in questions which implicitly require algebra.

Knowledge

Similarly, to the previous section, there are a wide range of contexts in which algebra may be convenient to use. This section contains many questions which may or may not require the use of algebra to solve. However, in all of these questions, the use of algebra provides a convenient route to a solution. Always consider:

- **Which element of the question can I represent using algebra?**

Example 1

Find 5 consecutive integers with a mean of 8.

Let the middle consecutive integer be n.

The 5 consecutive integers are then n−2, n−1, n, n+1, n+2.

"The mean is found by adding each point, then dividing by the number of points."

$$\text{Mean} = \frac{n-2 + n-1 + n + n+1 + n+2}{5}$$

$$\text{Mean} = \frac{5n}{5}$$

$$\text{Mean} = n$$

"The mean is 8."

$$N = 8$$

So, the consecutive numbers are 6, 7, 8, 9, 10.

Important Note: For the above example, it doesn't matter which consecutive number you choose to be "n". You'll find the same result. Choosing the middle number makes it a bit quicker though!

Example 2

a person is taking 6 exam papers and they have taken 5 already

they want to attain an overall average mark of 80% or over

on the first 5 papers their average is 78%

what mark (at least) must they try to obtain on their last paper?

Let the score on each paper be $x_1, x_2, x_3, x_4, x_5, x_6$.

"The average of the first 5 papers is 78."

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 78$$

So, $x_1 + x_2 + x_3 + x_4 + x_5 = 390$

"The overall average must be 80 or over."

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 80$$

So, $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 480$

But, $x_1 + x_2 + x_3 + x_4 + x_5 = 390$

So, $x_6 = 480 - 390$

$x_6 = 90.$

So they must obtain at least 90% on the last paper.

Practice

Exercise 1

1. Find 4 numbers with:
mean = 9, mode = 7, range = 6.

2. 10 students in a history class of 25 had a mean score of 80%

the other students had an average score of 60%

what is the average score of the whole history class?

3. Find 3 consecutive even numbers which add up to 42.

Find four consecutive even numbers which add up to 60.

Find five consecutive even numbers which add up to 60.

4. the mean of 5 numbers is 10

another number is included and the mean is now 11

what number was included?

5. Find 5 numbers with mode, median, mean and range being consecutive numbers in that order.

(Hint: mode = n, median = n+1, mean = n+2, range = n+3).

6. in one week in June one year, the mean temperature on Sun, Mon and Tues was 15°

the mean temperature on Mon, Tues and Wed of the same week was higher, 16°

if the temperature on that Sun was 15° what was the temperature on that Wed?

7. two consecutive odd numbers multiply to make 143
what are they?

two consecutive even numbers multiply to make 168
what are they?

two consecutive numbers multiply to make 210
what are they?

Exam Questions

9 The cost of a ticket increases by 10% to £19.25

Work out the original cost.

[3 marks]

Answer £ _____

14 The mean mass of a squad of 19 hockey players is 82 kg

A player of mass 93 kg joins the squad.

Work out the mean mass of the squad now.

[3 marks]

Answer _____ kg

Assessment

1. Using an algebraic method, find three consecutive odd numbers which add up to 57.

   What is the lowest number?

A	B	C	D
17	18	19	None of the above

2. Find 5 numbers whose:

   range = 10
   mean = 4
   mode = 1
   median = 2

   What is the second largest number?

A	B	C	D
1	4	11	None of the above

Tutorial 4 – Algebra in Exam Questions

Objectives
A. Form and solve equations in exam questions which explicitly require algebra. B. Assign variables and solve exam questions which implicitly require algebra.

A. Form and solve equations in exam questions which explicitly require algebra.

Knowledge

So far, we've practised using algebra in a variety of Mathematical contexts, such as a number, shape, ratio and probability questions. We've also had some experience answering the more advanced questions which appear in GCSE examinations.

It is always important to remember our main two considerations:

1. What is the relationship between two quantities?
2. How is this written as an equation?

However, there is one simple thing we need in order to answer exam questions with algebra: practice! In this section we will practise answering a variety of past exam questions which explicitly require the use of algebra.

Example

The table shows information about some CDs.

Type	Rock	Pop	Jazz
Number of CDs	2	x	$2x + 5$

A CD is chosen at random.

The probability it is **rock** is $\frac{1}{20}$

Work out the probability it is jazz.

[4 marks]

The total number of CDs is 2 + x + 2x + 5. Collecting like-terms gives 3x + 7.

Of those 3x + 7, 2 are "rock", so the probability of getting rock is $\frac{2}{3x+7}$.

However, we're told that the probability of getting rock is $\frac{1}{20}$. So,

$$\frac{2}{3x+7} = \frac{1}{20}$$

$$40 = 3x + 7$$

$$33 = 3x$$

$$x = 11$$

Substituting x = 11 into our original information tells us there are 11 pop CDs and 27 jazz CDs, making 40 total.

So the probability of getting a jazz CD is $\frac{27}{40}$.

3 y is $1\frac{1}{2}$ times x.

Circle the ratio that is equivalent to $y : x$

[1 mark]

2 : 5 5 : 2 3 : 2 2 : 3

4 $a : b = 4 : 3$

Circle the correct statement.

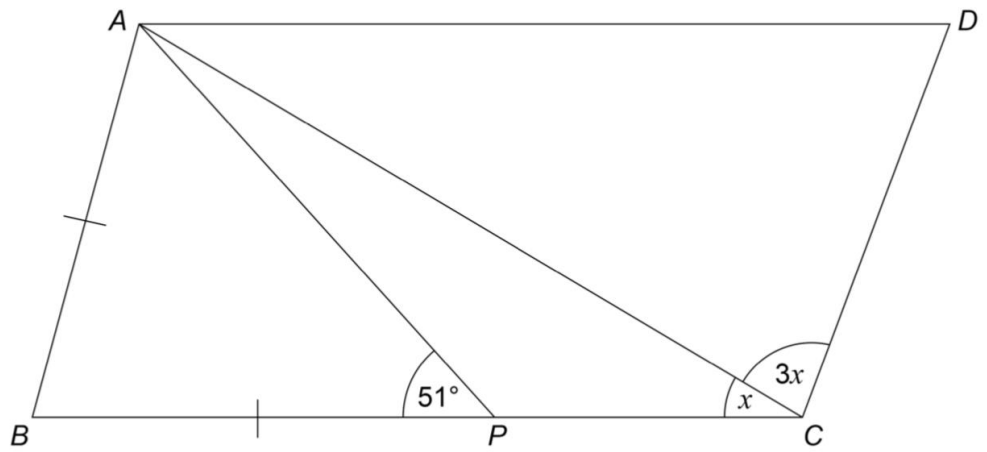
[1 mark]

b is $\frac{4}{7}$ of a b is $\frac{3}{7}$ of a b is $\frac{4}{3}$ of a b is $\frac{3}{4}$ of a

8 *ABCD* is a parallelogram.

AB = *BP*

Not drawn accurately



Work out the size of angle x.

[4 marks]

8 x km/h = y mph

Use 8 km/h = 5 mph to write a formula for y in terms of x.

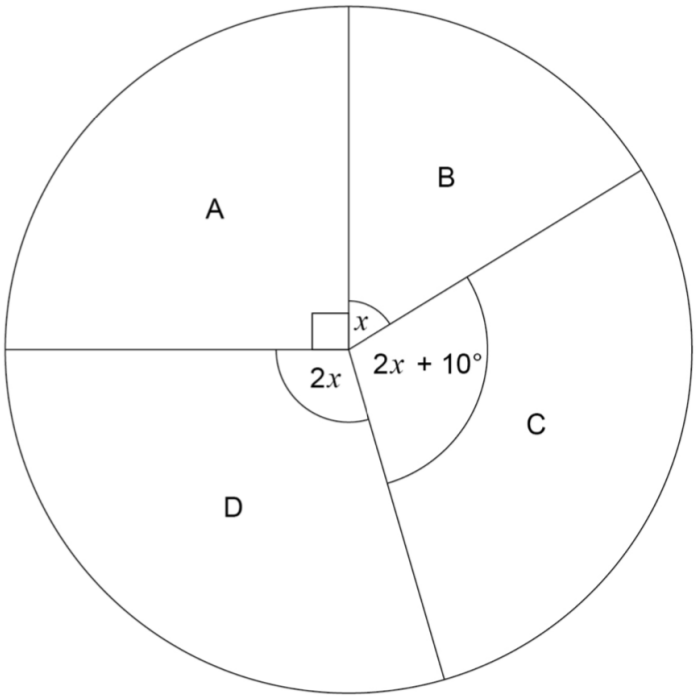
[2 marks]

11 The four candidates in an election were A, B, C and D.

The pie chart shows the proportion of votes for each candidate.

Proportion of votes

Not drawn accurately



Work out the probability that a person who voted, chosen at random, voted for C.

[4 marks]

11 Here are two sets of numbers, A and B.

Set A

200	160
104	100

Set B

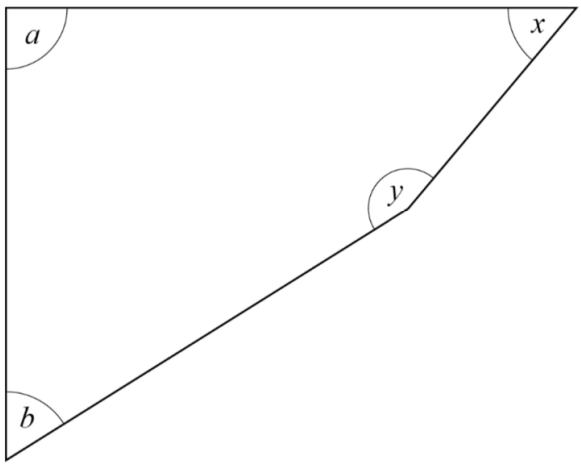
270	400	483
300	x	

mean of Set A : mean of Set B = 3 : 8

Work out the value of x.

[4 marks]

14 Here is a quadrilateral.



Not drawn accurately

$a = 90°$ and $a : b = 5 : 3$

$x : y = 1 : 3$

Show that $b = x$

[3 marks]

24 $a : b = 9 : 4$ and $10b = 7c$

Work out $a : c$ in its simplest form.

[3 marks]

24

Volume of a sphere = $\frac{4}{3}\pi r^3$	where r is the radius
Volume of a cone = $\frac{1}{3}\pi r^2 h$	where r is the radius and h is the perpendicular height

A sphere has radius $2x$ cm

A cone has

radius $3x$ cm

perpendicular height h cm

The sphere and the cone have the same volume.

Work out radius of cone : perpendicular height of cone

Give your answer in the form $a : b$ where a and b are integers.

[4 marks]

16 The value of y is 20% more than the value of x.

Circle the ratio $x : y$

[1 mark]

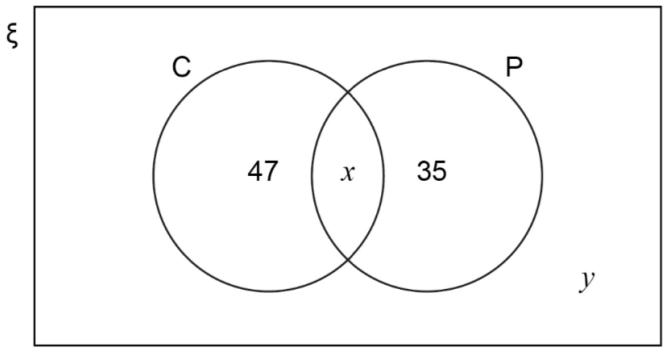
5 : 6 6 : 5 4 : 5 5 : 4

25 The Venn diagram shows some information about 150 students.

ξ = 150 students

C = students who study Chemistry

P = students who study Physics



The probability that a Physics student, chosen at random, also studies Chemistry is $\frac{5}{12}$

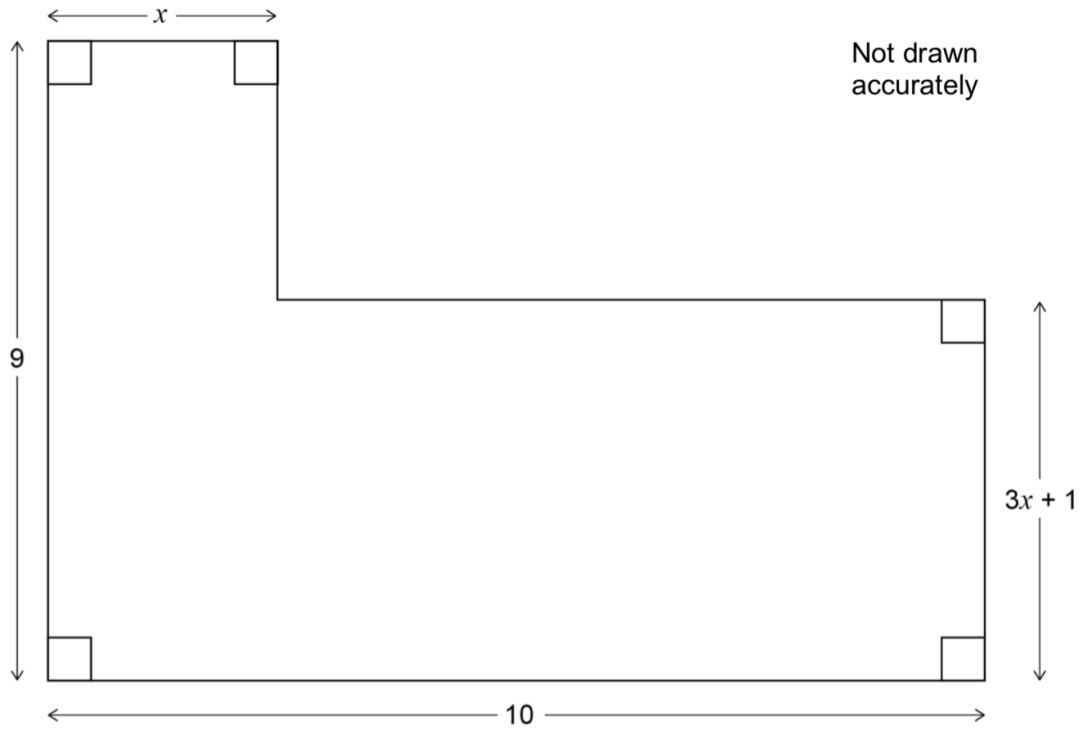
One of the 150 students is chosen at random.

Work out the probability that the student does **not** study either Chemistry or Physics.

[4 marks]

26 Here is an L-shape.

All dimensions are in centimetres.



The area of the L-shape is 65 cm^2

Work out the value of x.

[6 marks]

B. Assign variables and solve equations in exam questions which implicitly require algebra.

Knowledge

There are often multiple methods by which we can solve exam questions which do not explicitly involve algebra. However, if we are confident with our use of algebra, assigning variables to unknowns and forming equations is often a reliable way to find the answers to some of the most difficult questions in GCSE Maths; the kinds of questions which will earn you a grade 9.

Whenever we come across a question and we can't immediately see the route to a solution, we should immediately start thinking about what information we have. Highlighting this information or writing it down is a great way to organise your thoughts. Once we've done this, we should consider:

- **Which element of the question can I represent using algebra?**

Once we've decided this, we can go on to form and solve equations using the skills we've developed so far. Again, the best knowledge we can have of this technique is the practical knowledge which comes with experience. This is what we will develop in this section.

Example

In an office there are twice as many females as males.

$\frac{1}{4}$ of the females wear glasses.

$\frac{3}{8}$ of the males wear glasses.

84 people in the office wear glasses.

Work out the number of people in the office.

[4 marks]

For this question, we have a large choice of elements which we could assign a variable to. We're trying to find the number of people in the office, so let's call that P.

"There are twice as many females as males" means females to males are in the ratio 2 : 1. If we split the office into 3 parts, 2 will be female and 1 male.

In other words, the number of females is $\frac{2}{3}P$ and males are $\frac{1}{3}P$.

"$\frac{1}{4}$ of the females wear glasses" means $\frac{1}{4} \times \frac{2}{3}P$ which gives $\frac{1}{6}P$.

"$\frac{3}{8}$ of the males wear glasses" means $\frac{3}{8} \times \frac{1}{3}P$ which gives $\frac{1}{8}P$.

So in total, $\frac{1}{6}P + \frac{1}{8}P$ people wear glasses, which gives $\frac{7}{24}P$.

We're told that 84 people wear glasses, so

$$\frac{7}{24}P = 84$$

$$P = 288.$$

So, there are 288 people in the office.

7 Three friends arrive at a party.

Their arrival increases the number of people at the party by 20%

In total, how many people are now at the party?

[2 marks]

Answer _____

13 In a class of 28 students

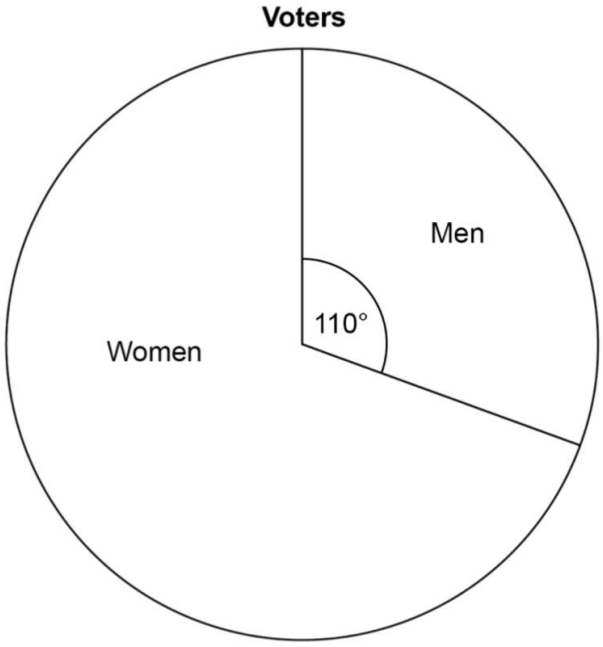
the mean height of the 12 boys is 1.58 metres

the mean height of all 28 students is 1.52 metres.

Work out the mean height of the girls.

[4 marks]

10 The pie chart shows information about voters in an election.



3360 **more** women voted than men.

Work out the total number of voters.

[3 marks]

24 (b) Work out $\sqrt[3]{16}$ as a power of 2

[2 marks]

19 Rana sells 192 cakes in the ratio small : medium : large = 7 : 6 : 11

The profit for one medium cake is twice the profit for one small cake.

The profit for one large cake is three times the profit for one small cake.

Her total profit is £532.48

Work out the profit for one small cake.

[5 marks]

Feedback

Objectives
• Receive feedback on final assessments. • Share examples of best practice with the other pupils in your group. • Write targets for improvement in school lessons. • Reflect on the programme including what was enjoyed and what was challenging.

Final Assessment Feedback

What I did well...	What I could have improved on...

My target for future work is...

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?

What did you find challenging about the programme?	How did you overcome these challenges?

Notes

Notes

Notes

Notes

Notes

Everything is Algebra

Key Stage 4 Programme

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