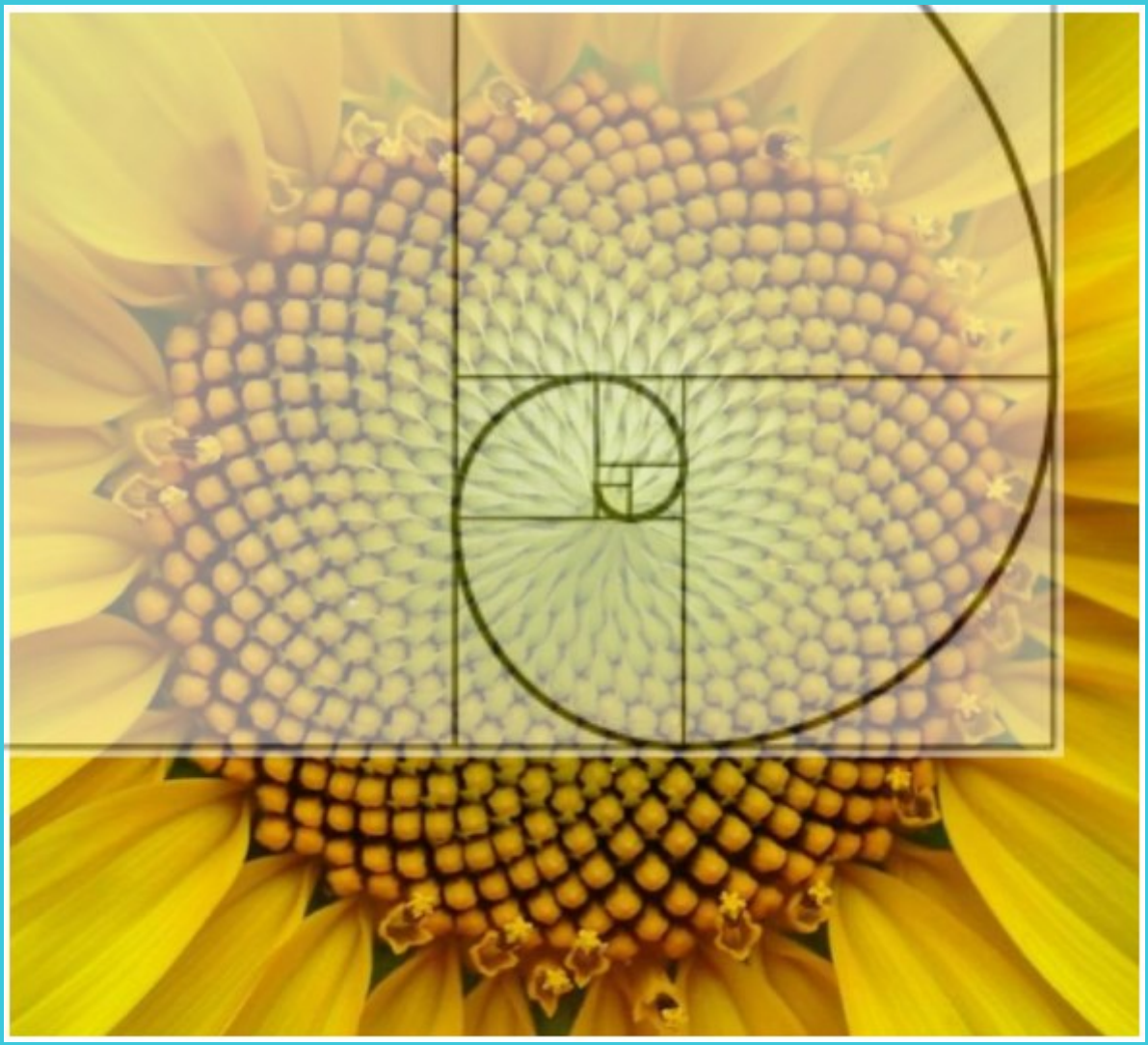

Sequences

Pupil Name	
Handbook Designed by	Dr Pearce

Timetables

Tutorials

Tutorial	Date	Time	Location
1 (Baseline)			
2			
3			
4			
5			
6 (Final assignment)			
7 (feedback)			

Homework

Homework	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

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Course Rationale

To gain a more in depth understanding and appreciation for sequence in maths and science through deliberate practice. We will go through all the topics on the GCSE syllabus that involve sequences, which include:

- What is a **sequence**?
- Generating terms of a sequence from **term-to-term**
- Generating terms of a sequence from **position-to-term**
- Recognise and use sequences of **triangular**, **square**, and **cube** numbers
- Recognise and use sequences of **Fibonacci**-type sequences and **geometric** progressions
- Recognise and use **linear** (arithmetic) sequences
- Deduce expressions to calculate the **nth-term** of a **liner** (arithmetic) sequence
- Recognise and use **quadratic** sequences
- Deduce expressions to calculate the **nth-term** of a **quadratic** sequence
- **Revise** everything you need to know about sequences

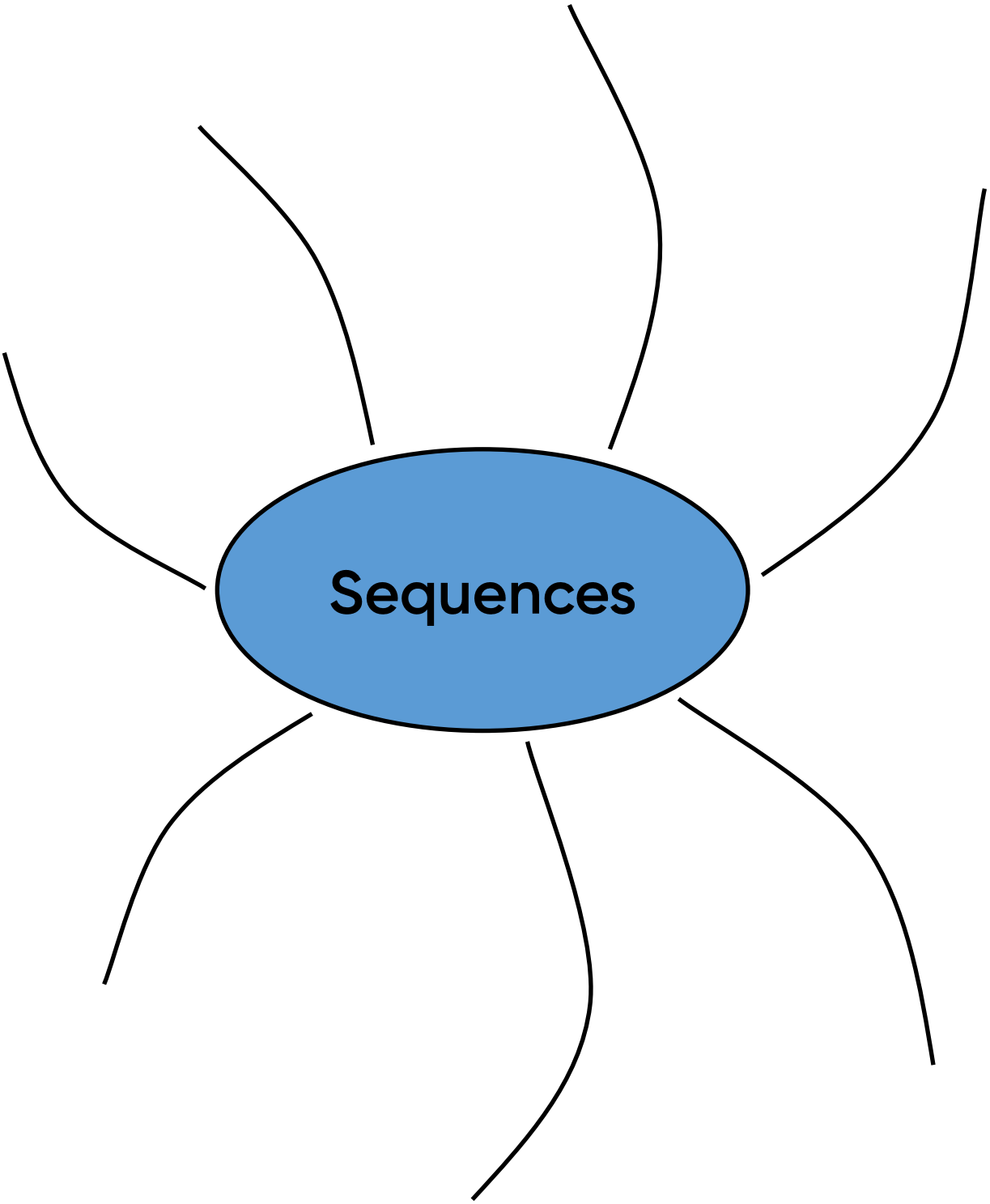
Finally, we will focus on exam technique to help you get that grade!

Glossary of keywords

Complete the table and add a few of your own that you think are relevant

Word	Definition	In a sentence
E.g. Sequence	An infinite ordered series of numerical quantities	A sequence of numbers can increase or decrease
Linear (arithmetic) sequence		
Quadratic sequence		
nth-term		
Multiple		
Geometric sequence		
Fibonacci sequence		
Expression		
Series		
Term		

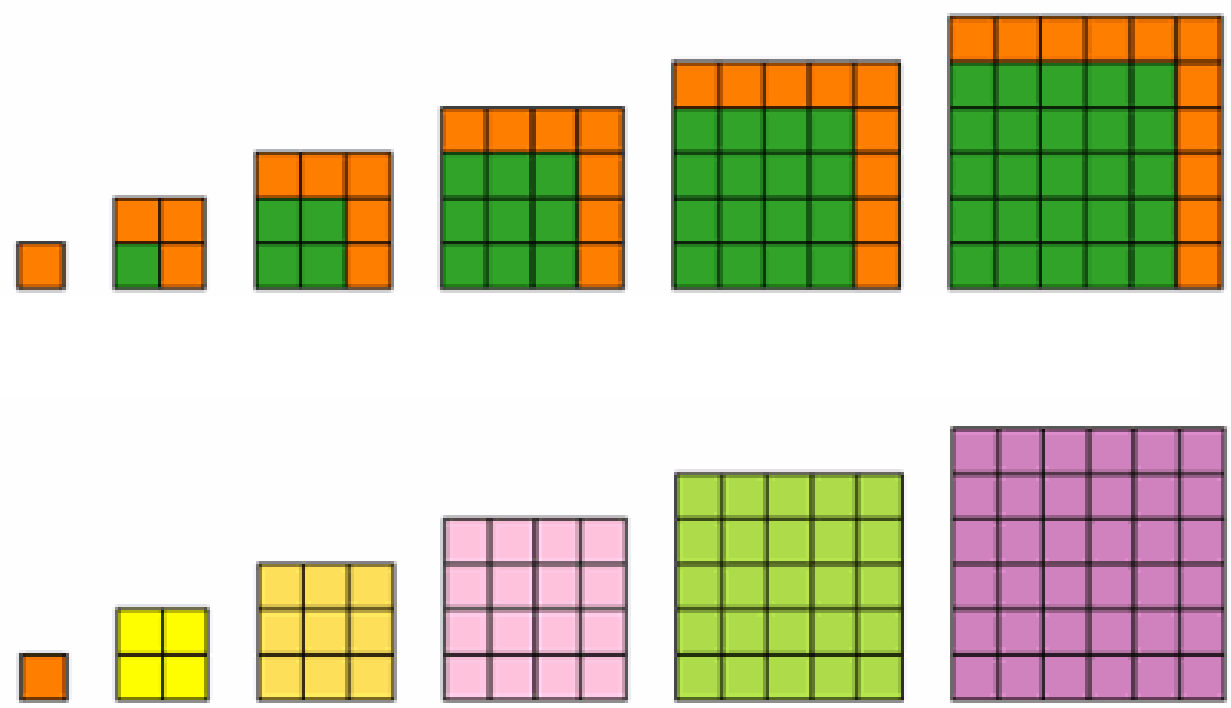
Knowledge Organiser



Tutorial 1 Baseline assessment

Calculations and notes

Tutorial 2 Generating sequences



What is the Purpose of Tutorial 2?

- What is a **sequence**?
- Generating terms of a sequence from **term-to-term**
- Generating terms of a sequence from **position-to-term**

Starter

Generate a sequence that increases by 7

Generate a sequence that increases by 7 and the first number is 10

Generate a sequence that increases by b and the first number is a

What is the **name** (or names) given to this type of sequence?

What is a sequence?

A mathematical sequence is "an infinite ordered series of numerical quantities"

Does a sequence have to **increase** or **decrease**?

If no, give an **example**.

Is this always by the **same** amount?

If no, give an **example**.

Practice

Are the following sequences or not? If not, can you change a number to make it a sequence?

a. 10, 20, 30, 35, 40,

b. 15, 25, 35, 45, 55,

c. -1, -5, -13, -19, -25,

d. 101, 132, 163, 192, 225,

e. -10, 1, 12, 23, 34, 45

f. 4, 0, -2, -5, -8,

g. 29, 21, 13, 5, -3,

h. -23, -2, 19, 40, 59,

Generating terms of a sequence from term-to-term

A **term** of a sequence is the name given to the numbers within a sequence, each of which has a **position**; e.g. the second term of the sequence 2, 4, 6, 8, is 4. Sequences can be produced by finding the next term/terms of an existing sequence:

E.g. For the sequence **5, 9, 13, 17,** the next term would be **17+4=21**

Practice

1. (multiple choice) The next **three** terms of the sequence **10, 21, 32, 43,** are:

a. 52, 61, 70

b. 54, 63, 74

c. 54, 65, 76

d. 54, 65, 74

2. Find the next **two** terms of the following sequences:

a. 1, 9, 17, 25,

b. 10, 25, 40, 55,

c. 18, 12, 6, 0,

d. 1, −10, −21, −32,

e. −34, −29, −24, −19,

f. −12, −3, 6, 15,

3. Make up your own sequence of **six** terms:

Generating terms of a sequence from position-to-term

Any term of a sequences can also be produced by using the **n^{th}-term** of the defined sequence:

(**e.g.** for the sequence 5, 9, 13, 17, the nth-term is 4n+1, so the 10th-term would be (4x10)+1=41)

Practice

1. (multiple choice) The **10th-term** of the sequence with **n^{th}-term** of 3 -n is:

a. 7

b. 13

c. 30

d. -7

2. Find the **12th-term** of the following sequences: if the **n^{th} -term** is:

a. n-1

b. n+12

c. 2n+3

d. 5n-1

e. 10n+33

f. 0.5n+2

3. Make up your own **n^{th}-term** and find the **100th-term** of your sequence

Plenary/Homework 1

Find the next three terms for the following sequences:

Q1) 6, 13, 20, 27

Q2) 7, 12, 17, 22

Q3) 3, 11, 19, 27

Q4) 8, 11, 14, 17

Q5) 10, 7, 4, 1,

Q6) 23, 12, 1, -10,

Q7) 111, 91, 71, 51,

Q8) -4.5, -7, -9.5, -12,

What are the first 5 terms of:

Q1) $9n - 5$

Q2) $n - 7$

Q3) $2n + 8$

Q4) $-5n$

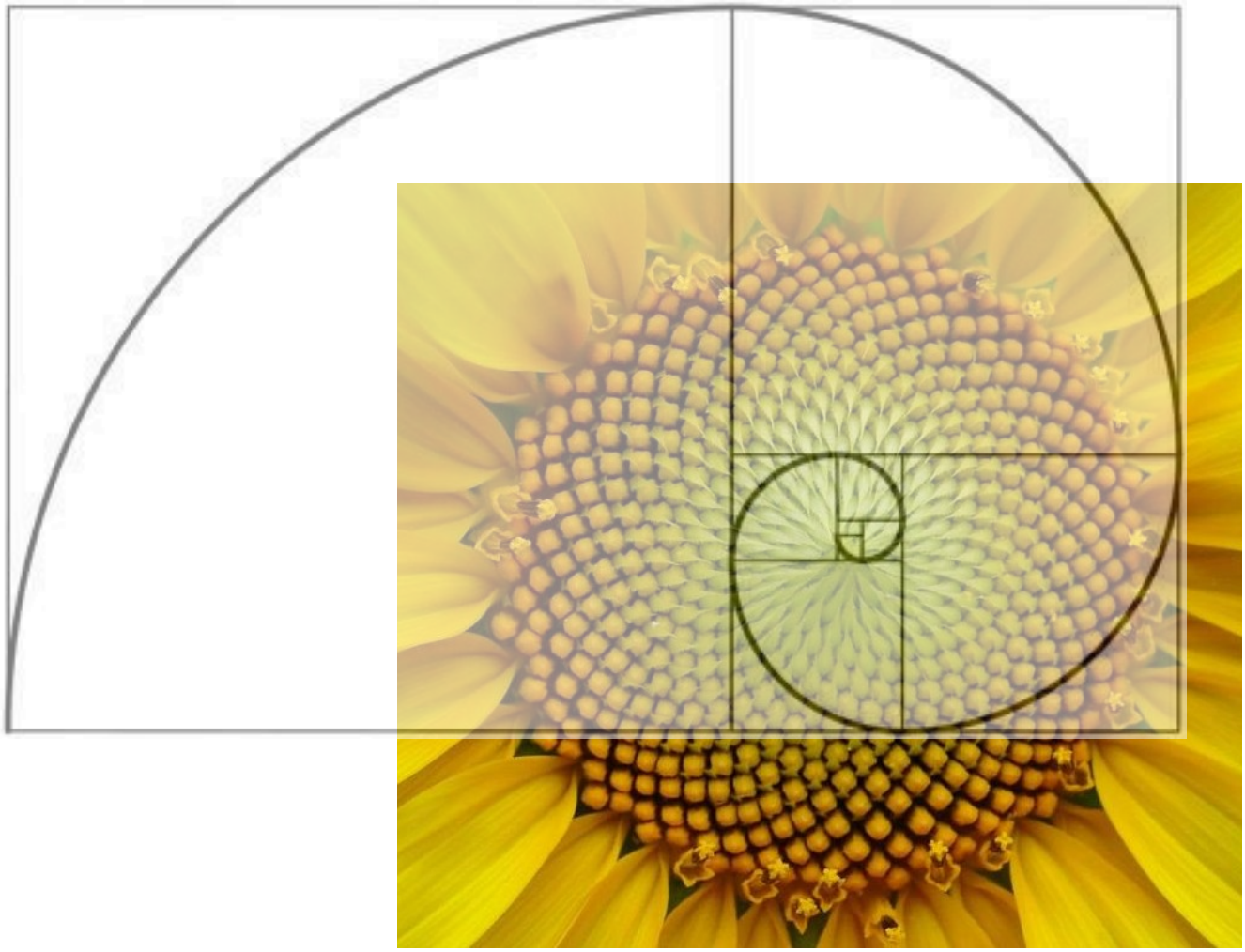
Q5) $-6n - 8$

Q6) $n - 6$

Q7) $5n - 6$

Q8) $6n - 2$

Tutorial 3 Recognising and using sequences



What is the Purpose of Tutorial 3?

- Recognise and use sequences of **triangular**, **square**, and **cube numbers**
- Recognise and use sequences of **Fibonacci**-type sequences and **geometric** progressions

Starter

How many **cube** numbers are there between **0** and **1000**?

What is a **quadratic** sequence?

A triangle of increasing size is represent by the sequence below. Can you **sketch** the first **three**. What is the **next** number in the sequence?

1, 3, 6, 10, 15, ...

Can you complete the following **quadratic** sequence:

3, 9, ..., ..., ..., ..., 99

Can you complete the following **cubic** sequence:

11, 18, 37, ..., ..., ..., ..., 522

Practice

1. The rule for the triangular number sequence is **n(n+1)/2**, where n is the term's position (**e.g.** the 5th term in the triangular sequence is 5 (5+1)/2 = 15).

a. Can you find the **7th term** of the triangular sequence?

b. Which is the **first term** to be greater than **100**?

c. Can you show whether the number **35** is a term in the triangular sequence?

d. Can you show whether the number **300** is a term in the triangular sequence? (you might need a calculator)

2. What are the first **15** square numbers?

3. What are the first **10** cube numbers?

Who or what is Fibonacci?

Leonardo Fibonacci was born c. 1170 in Italy and died c. 1250. He was a medieval Italian mathematician who wrote Liber abaci (1202; "Book of the Abacus"), the first European work on Indian and Arabian mathematics.

Little is known about Fibonacci's life beyond the few facts given in his mathematical writings. Except for his role in spreading the use of the Hindu-Arabic numerals, Fibonacci's contribution to mathematics has been largely overlooked. His name is known to modern mathematicians mainly because of the **Fibonacci sequence** (see below) derived from a problem in the Liber abaci.

The resulting number sequence, **1, 1, 2, 3, 5, 8, 13, 21, 34, 55** (Fibonacci himself omitted the first term), in which **each number is the sum of the two preceding numbers**, is the first recursive number sequence (in which the relation between two or more successive terms can be expressed by a formula) known in Europe.

Terms in the sequence were stated in a formula by the French-born mathematician **Albert Girard** in 1634: $u_{n+2} = u_{n+1} + u_n$, in which **u** represents the term and the subscript its rank in the sequence. The mathematician **Robert Simson** at the University of Glasgow in 1753 noted that, as the numbers increased in magnitude, the ratio between succeeding numbers approached the number α, the **golden ratio**, whose value is **1.6180...**, or $(1+\sqrt{5})/2$.

In the 19th century the term **Fibonacci sequence** was coined by the French mathematician **Edouard Lucas**, and scientists began to discover such sequences in nature; for example, in the spirals **of sunflower heads**, in **pine cones**, in the related logarithmic (equiangular) **spiral in snail shells**, in the arrangement of **leaf buds on a stem**, and in **animal horns**.

Fibonacci sequences

The first fifteen **Fibonacci** numbers are:

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610

What type of number is every **third** term? **Why**?

Find the next **four** terms of the following **Fibonacci** sequences.

(a) 2 3 5 8

(b) 1 3 4 7

(c) 2 7 9 16

(d) 3 8 11 19

A **Fibonacci** sequence has **second** term **20** and **fifth** term **80**. Find the **first** term.

The **sixth** and **seventh** terms of a **Fibonacci** sequence are **31** and **50**. What are the first **two** terms of the sequence?

If you take the first **ten** terms of any **Fibonacci** sequence, the sum of those **10** terms is **equal** to the **7th** term **multiplied** by **11**. **Show** that this is true for the following **Fibonacci** sequence:

4 5 9 14

When you find the **sum** of the first **six** numbers of a **Fibonacci** sequence the **sum** is always **four** times the **fifth** number in the sequence. **Show** this is true algebraically by using the following **Fibonacci** sequence

$a \quad b \quad a + b$

The **6th** and **7th** terms of a **Fibonacci** sequence are $12x + 25y$ and $20x + 40y$.

Write down the first **two** terms of the sequence.

Geometric sequences

A **geometric progression** (sequence) is one where each **term** is **multiplied** by a value known as the **common ratio** (**r**); the nth-term of a geometric progression takes the form r^n, where **r** is the **common ratio** (what each term is multiplied by) and **n** is the **term number**.

E.g. For the sequence **2, 4, 8, 16,** each term is **multiplied** by **2**, so the **common ratio** is **2** and so the **geometric progression** is 2^n.

1. Find the first **four** terms of the following sequences:

a. 4^n

b. $(3/4)^n$

c. 3×2^n

d. 2×2^n $(=2^{n+1})$

e. $(\sqrt{2})^n$

f. $(\sqrt{(3/4)})^n$

2. A colony of **ants** increases **exponentially** at a rate of **20%** per **week**. Initially there are **500** ants.

a. Show that the number of ants in the colony forms a **geometric** progression.

b. The number of ants in **week 8** is p times greater than in **week 6**. Find the value of p.

Plenary/Homework 2

What is the next term in the sequence?

Q1) $6, 24, 96, 384$

Q2) $9, 54, 324, 1944$

Q3) $2, 8, 32, 128$

Q4) $2, 6, 18, 54$

Q5) $3, 15, 75, 375$

Q6) $9, 45, 225, 1125$

Q7) $6, 18, 54, 162$

Q8) $4, 28, 196, 1372$

Write the first 4 terms of the sequence:

Q1) 3×2^n

Q2) 3×3^n

Q3) 4×5^n

Q4) 4^n

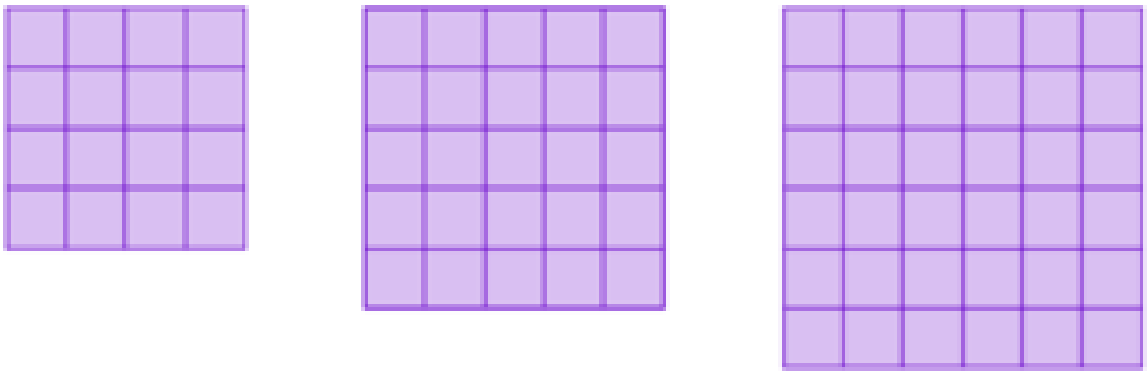
Q5) 2×3^n

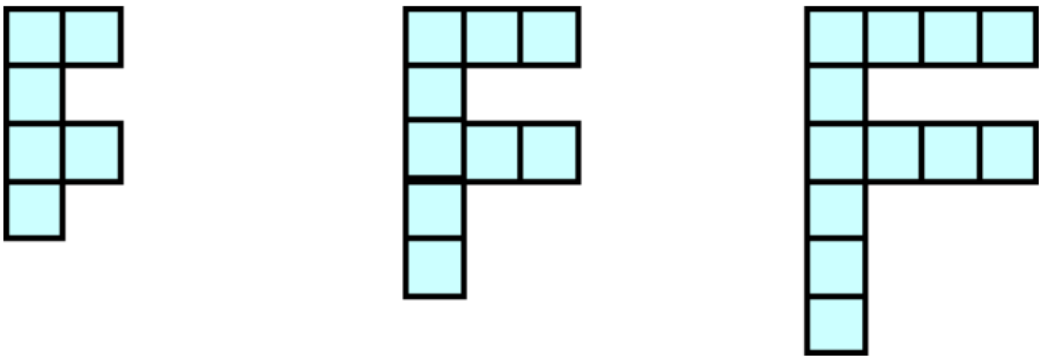
Q6) 3×5^n

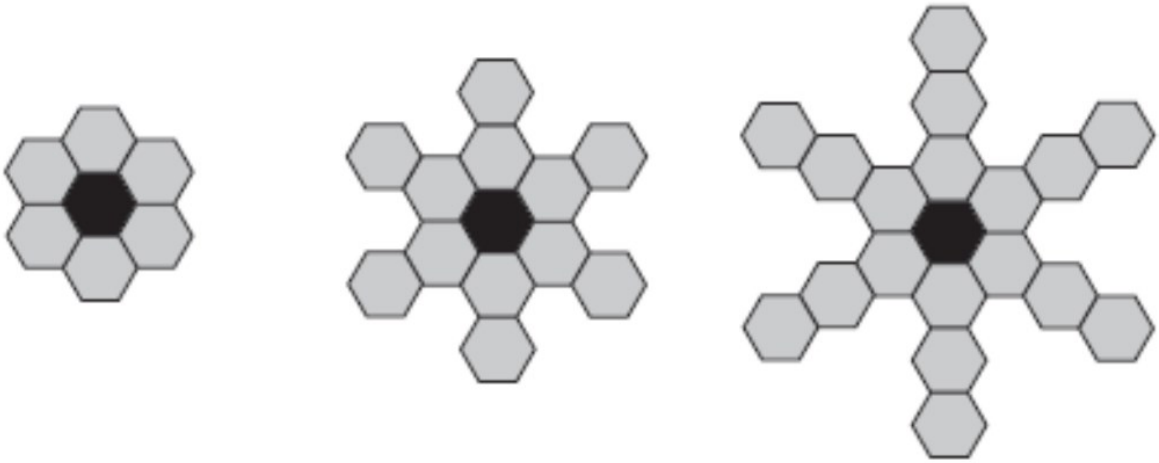
Q7) 2^n

Q8) 2×2^n

Tutorial 4 Finding the nth term of linear sequences







What is the Purpose of Tutorial 4?

- Recognise and use **linear** (arithmetic) sequences
- Deduce expressions to calculate the **n^{th}–term** of a liner (arithmetic) sequence

Starter

Find ***a*** and ***b*** for the following linear sequences:

a. n+*b*: 4, 5, 6, 7, 8,

b. *a*n+5: 7, 9, 11, 13, 15,

c. *a*n+*b*: 10, 18, 26, 34, 42,

d. *a*n+*b*: −12, −16, −20, −24, −28,

e. *a*n+*b*: 8, ..., −12, −22, ...

f. *a*n+*b*: −33, ..., ..., ..., 11,

Linear sequences

The difference between all the terms of a **linear sequence** is the same. It is essentially a list of multiples (timetable) with something consistently added or subtracted each time.

4, 11, 18, 25, 32, and **60, 51, 42, 33, 24,** are linear sequences; **why**?

Can you find the **missing terms** of the following linear sequences:

a. 1, 7, 13, ..., 25, ...,

b. ....., 6, ..., 20, ..., ...

c. 10, ..., ..., −35, ..., −65,

d. ....., 100, ..., ..., 220, ...

e. −63, ..., ..., ..., ..., −98

f. a, ..., ..., 4a−6b, ..., ...

Linear sequences

To find the **n^{th}–term** of a **linear** sequence, you need to:

- find the ..... between each term,
- write down the ..... of this difference,
- ..... or ..... whatever is required to obtain the sequence.

Find the **n^{th}–term** of the following **linear** sequences:

E.g. The sequence **14, 23, 32, 41, 50**, will have an **n^{th}–term** of **9n+5**

a. -3, 3, 9, 15, 21

b. -5, -1, 3, 7, 11

c. -2, -6, -10, -14, -18

d. 3, 1, -1, -3, -5

e. 3.4, 5.1, 6.8, 8.5, 10.2

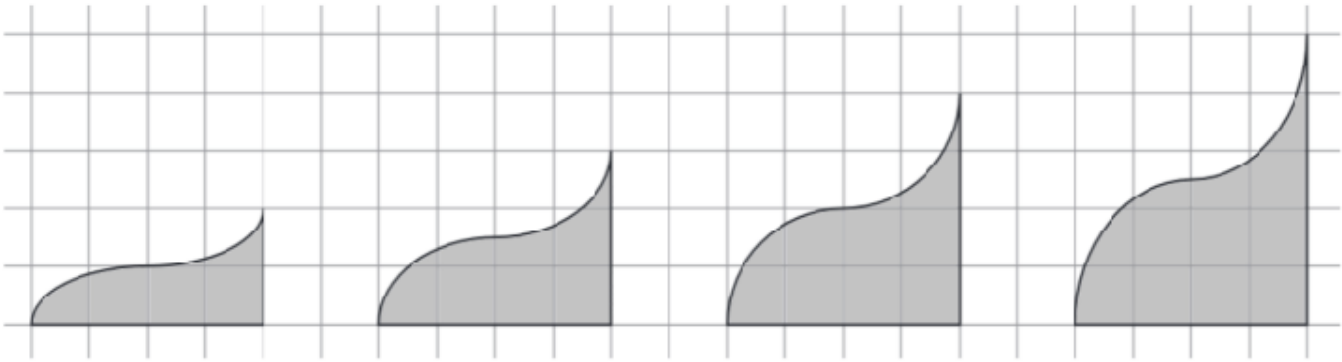
f. 1, -0.9, -2.8, -4.7, -6.6

Linear sequences

If a **linear** sequence has an **nth-term** of **5n-7**, which term in the sequence is equal to **48**? Is the number **92** in this sequence? **Why/why not**?

If a **linear** sequence has an **nth-term** of **80-9n**, which term in the sequence is equal to **-55**? Is the number **53** in this sequence? **Why/why not**?

A **linear** sequence can also involve **area**; **e.g.** What is the **nth-term** for the **areas** of the **shapes** below? **Hint**: the area for n=1 is 4 cm^2



n = 1 n = 2 n = 3 n = 4

Create your own **linear** sequence of **two-dimensional shapes** using the squares below; do this for **four** terms:

Plenary/Homework 3

Find the nth term for the following sequences:

Q1) 1, 6, 11, 16

Q2) 4, 5, 6, 7

Q3) 4, 10, 16, 22

Q4) 2, 8, 14, 20

Q5) 6, 10, 14, 18

Q6) 6, 7, 8, 9

Q7) 5, 9, 13, 17

Q8) 5, 13, 21, 29

Find the given term for the following sequences:

Q1) 36th term: 2, 10, 18, 26

Q2) 28th term: 7, 9, 11, 13

Q3) 42nd term: 5, 7, 9, 11

Q4) 50th term: 7, 13, 19, 25

Q5) 44th term: 2, 3, 4, 5

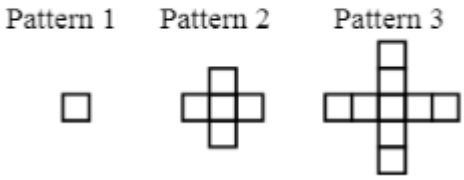
Q6) 23rd term: 5, 8, 11, 14

Q7) 26th term: 2, 9, 16, 23

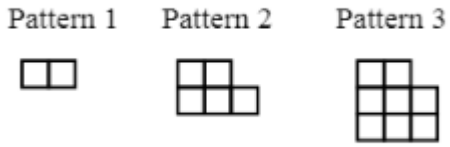
Q8) 26th term: 4, 13, 22, 31

How many squares in the given pattern?

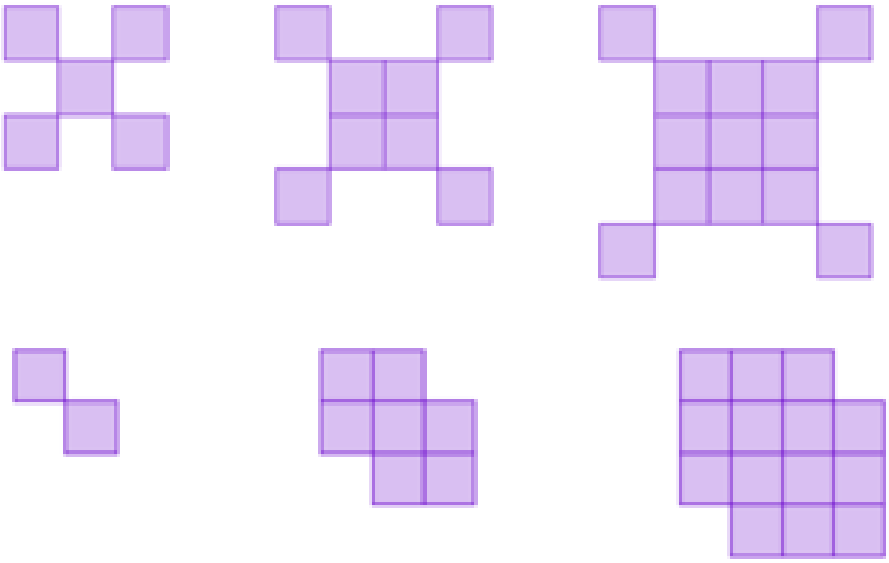
Q1) 21st pattern

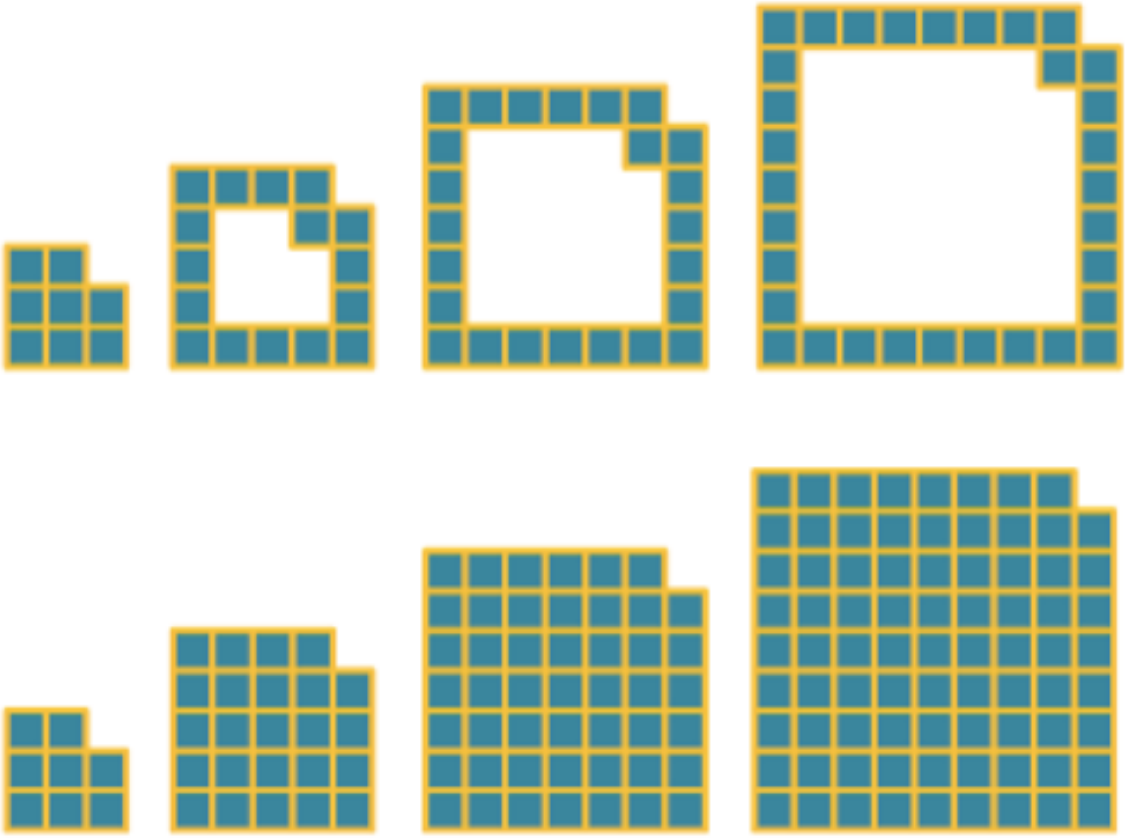


Q2) 28th pattern



Tutorial 5 Finding the nth term of quadratic sequences





What is the Purpose of Tutorial 5?

- Recognise and use **quadratic** sequences
- Deduce expressions to calculate the **n^{th}–term** of a quadratic

Starter

Can you find the **nth-terms** of the following **linear** sequences?

a)

4, 8, 12, 16, ...

b)

-3, -6, -9, -12, ...

c)

1, 3, 5, 7, ...

d)

3, 8, 13, 18, ...

e)

10, 8, 6, 4, ...

f)

45, 40, 35, 30, ...

g)

10, 25, 40, 55, ...

h)

35, 27, 19, 11, ...

i)

$a + b$, $a + 2b$,
$a + 3b$, $a + 4b$, ...

j)

$x + 2y$, $x + 3y$,
$x + 4y$, $x + 5y$, ...

k)

$2m + 3p$, $2m + p$,
$2m - p$, $2m - 3p$, ...

l)

$2c + 3d$, $3c + 5d$,
$4c + 7d$, $5c + 9d$, ...

Quadratic sequences

A quadratic sequence is a sequence of numbers in which the **second differences** between each consecutive term differ by the same amount (sometime called a common second difference).

1. U_2 of the sequence $U_n=n^2+9$ is $(2)^2+9=13$:

a. Find U_3 of the sequence $U_n=2n^2+1$

b. Find U_{10} of the sequence $U_n=7-3n^2$

c. Find U_5 of the sequence $U_n=7n^2-12$

d. Find U_4 of the sequence $U_n=n^2+4n+1$

e. Find U_1 of the sequence $U_n=13+5n-3n^2$

2. Find the first **five** terms of the following sequences:

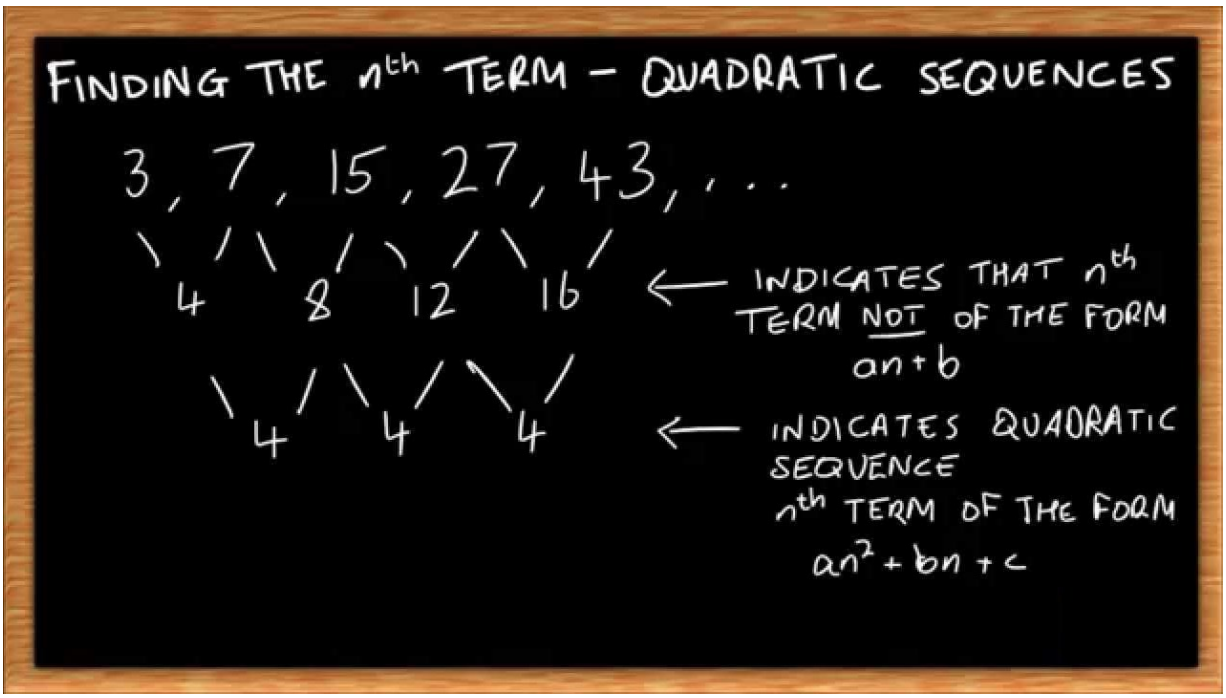
a. $U_n=3n^2-15$

b. $U_n=10-4n^2$

c. $U_n=2n^2-n+4$

d. $U_n=5n-1-n^2$

Finding the nth-term



To find the **n^{th}–term** of a **quadratic sequence**, you need to:

- find the ..... between each **term**,
- find the ..... between each of these **differences**
- divide this **second difference** by ..... and this becomes your **n^2** coefficient (a**n^2**)
- write down a**n^2** for the terms required
- ..... this from the **original** sequence
- You will now be left with a **linear** sequence (see previous tutorial) or a **constant**.
- Solve the **linear** sequence or use the **constant** to find the **n^{th}–term** of the **quadratic** sequence.

Find the **coefficient** of **n^2** for the following **quadratic** sequence:

14 23 32 41 50

Quadratic sequences

Fill the **gaps** the following **quadratic** sequences (**$U_n=an^2+c$**):

a. 5, 8, 13, 20, ..., ...

b. 2, ..., 18, 32, 50, ...

c. ....., ..., 8, 22, 40, 62

d. 97, 88, 73, 52, ..., ...

e. ....., -21, -41, -69, -105, ...

Let's find the **n^{th}–term** of the following **quadratic** sequence:

3 7 13 21 31

Quadratic sequences

1. If a sequence has an nth-term of $5n^2-7$, which term in the sequence is equal to 118? Is the number 238 in this sequence? Why/why not?

2. If a sequence has an nth-term of $80-9n^2$, which term in the sequence is equal to −1? Is the number 35 in this sequence? Why/why not?

3. If a sequence has an nth-term of $2n^2+4n-10$, which term in the sequence is equal to 38? Is the number 230 in this sequence? Why/why not?

4. Find the nth-term of the following sequences ($Un=an^2+bn+c$):

a. 9 14 21 30 41

b. −9 −14 −25 −42 −65

Plenary/Homework 4

Find the next term in the sequence:

Q1) $11, 19, 29, 41, 55$

Q2) $10, 14, 20, 28, 38$

Q3) $12, 19, 28, 39, 52$

Q4) $10, 20, 32, 46, 62$

Q5) $9, 10, 9, 6, 1$

Q6) $6, 1, -6, -15, -26$

Q7) $5, 10, 17, 26, 37$

Q8) $-2, 1, 10, 25, 46$

Find the nth term of the sequence:

Q1) $12, 20, 30, 42, 56$

Q2) $6, 11, 18, 27, 38$

Q3) $8, 14, 22, 32, 44$

Q4) $15, 22, 31, 42, 55$

Q5) $6, -1, -12, -27, -46$

Q6) $10, 10, 6, -2, -14$

Q7) $6, 12, 20, 30, 42$

Q8) $9, 7, -1, -15, -35$

What are the first 5 terms of:

Q1) $3n^2 - 2n + 2$

Q2) $n^2 - 2n - 3$

Q3) $4n^2 + 9$

Q4) $-2n^2 - 3n - 6$

Tutorial 6 Exam technique and final assessment

Exam technique

1. Things to remember

- In the exam you should check that you have been given the correct paper, that you know how many questions you have to answer on that paper and how long you have to do it. Try to spread your time equally between the questions. If you do this it will avoid the desire to rush the paper or spend too much time on some questions and not finish the paper.
- Many mathematics papers start with fairly straightforward questions which may be shorter than those that follow. If this is the case work through them in order to build up your confidence. Do not overlook any part of a question and double check that you have seen everything on each paper, look especially at the back page in case there is a question there!
- Take time to read through all the questions carefully and then start with the question(s) that you think you can do best.
- When there are about 15 minutes remaining in the examination then quickly check if you are running out of time. If you think that you will run out of time then try to score as many marks as possible by concentrating on the easier parts, the first parts, of any questions that you have not yet attempted.

2. Read each question carefully

- Make sure you understand what the question is asking. Some questions are structured and some are unstructured – called 'multi-step' questions – and for these you will have to decide how to tackle the question and it would be worthwhile spending a few seconds thinking the question through.
- Make sure you understand key words. The following glossary may help you:
    - Write down or state – no explanation is needed for an answer
    - Calculate, find, show, or solve – include enough working to make your method clear
    - Draw – plot accurately using the graph paper provided and selecting a suitable scale if one is not given. Such an instruction is usually followed by asking you to read one or more values from your graph.
- The number of marks is given in brackets [ ] at the end of each question or part question. This gives some indication of how many steps will be required to answer the question and therefore of what proportion of your time, you should spend on each part of the question.

Exam technique

3. Show your working and check your answers

- State units if required and give your final answer to an appropriate degree of accuracy.
- Write down the figures on your calculator and then make a suitable rounding. Don't round the numbers during the calculation. This will often result in an incorrect answer.
- Don't forget to check your answers, especially to see that they are reasonable. The mean height of a group of men will not be 187 metres!
- Lay out your working carefully and concisely. Write down the calculations you are going to make. You usually get marks for showing a correct method. (If you are untidy and disorganised, you might misread some of your own work and/or lose marks because the examiner cannot read your work or follow your method.)
- Remember that if all that is written down is an answer and that answer is wrong you gain no marks. Once you have finished the paper if you have any time left check the work you have done. The best way to do this is to work through the questions again.
- Remember that marks are given for the following:
    - using an appropriate method to answer a question
    - for facts found as you work through a question
    - for the final answer.

4. What examiners look for

The examiners look for the following:

- Work which is legible, clearly set out and easy to follow and understand. Use a pen, not pencil, except in drawings, and use the appropriate equipment.
- That drawings and graphs are neat, and graphs are labelled.
- That you always indicate how you obtain your answers.
- The right answer

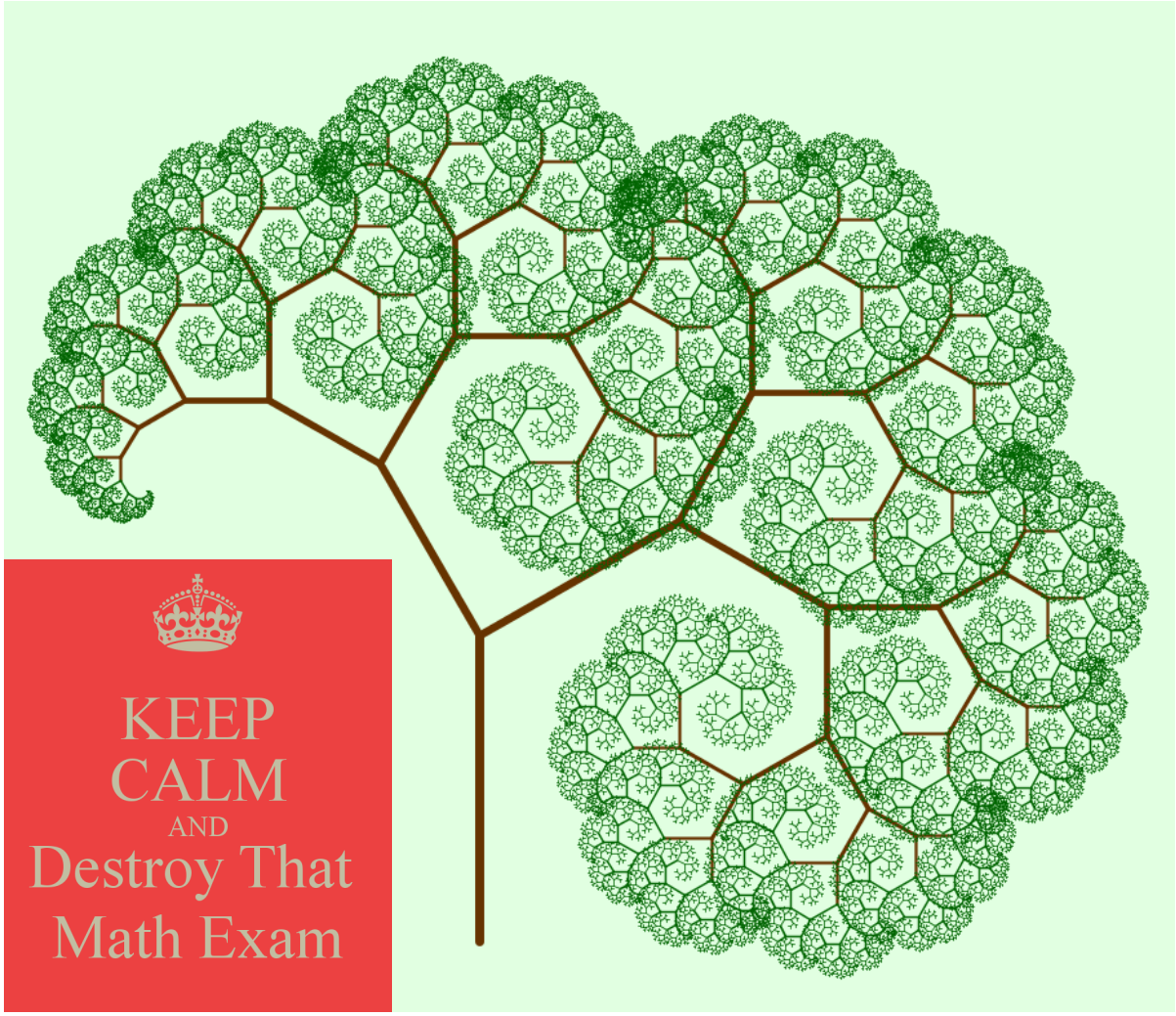
5. Practice makes perfect

- Practise all aspects of manipulative algebra, solving equations, rearranging formulas, expanding brackets, factorising, etc.
- Practise answering questions that ask for an explanation. Your answers should be concise and use mathematical terms where appropriate.
- Practise answering questions with more than one step to the answer, e.g. finding the radius of a sphere with the same volume as a given cone.
- Make sure you can use your calculator efficiently.
- To practice before the exam use past GCSE papers, revision books, or suitable online revision sites

Final assessment

Calculations and notes

Tutorial 7 Feedback and reflection



What is the Purpose of Tutorial 7?

- Provide **feedback** from the course and the final assessment
- **Reflecting** on the Uni Pathways course
- **Destroy** (do well in) that maths exam!

Feedback

We will do the following:

- Receive **feedback** on **final assignments**.
- **Reflect** on the course, including what you enjoyed and what you found more challenging.
- Write **targets** for improvement when in lessons/revising.
- **Share** examples of best practice/exam technique with the group.

Final assignment feedback:

What I did well...	What I could improve on...

Reflecting on Uni Pathways

What did you enjoy most about the Uni Pathways course this year?

What did you find most challenging about the programme?	How did you overcome these challenges?

My target(s) for future lessons/revision will be...

Notes

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