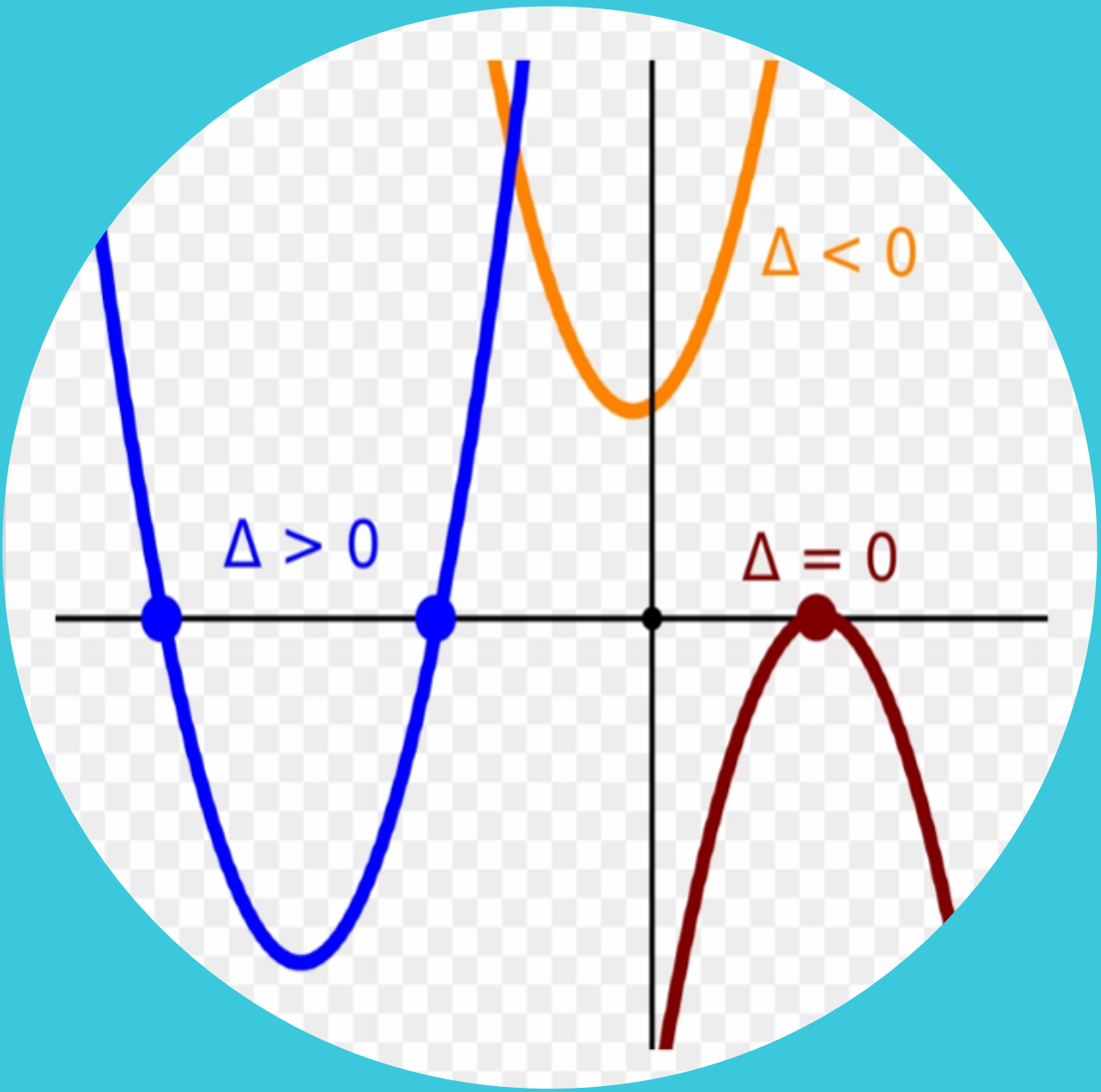
Researchers in Schools



Quadratics

Key Stage 4 Programme

Pupil Name

Handbook Designed by Dr Makwana

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

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Course Rationale

The name **Quadratic** comes from "quad" meaning square, because the equations have the squared variable ($ax^2 + bx + c$). Quadratic equations allow us to model situations that happen in real life, such as the rise and fall of profits from selling goods, the decrease and increase in the amount of time it takes to run a mile based on your age etc.

The vertex and x-intercepts are especially useful. Intercepts tell you where numbers change from positive to negative or negative to positive, so you know, for instance, where the ground is located in a physics problem or when you'd start making a profit or losing money in a business venture. The vertex tells you where you can find the absolute maximum or minimum cost, profit, speed, height, time, or whatever you're modelling.

In this course four different methods of solving quadratic equations will be taught with regards to graphs. This course will operate similar to a mathematics undergraduate tutorial at a university. In a university tutorial, you would be expected to look at and attempt a set of mathematical problems ahead of time. In the tutorial itself, you will have opportunities to ask questions about parts you struggle with, get feedback from your tutor and work on more challenging problems where appropriate.

For each tutorial, in order to meet the objectives, practice will be done and an assessment will be completed to check if the objectives are met. This will allow you to retain knowledge and skills developed in each tutorial and also improve your revision skills.

Tutorial 1 Quadratics

Objectives

Baseline and Pre-SEF

Tutorial 2 – Graphs of Quadratics

Objectives

A. To demonstrate understanding of quadratic equations.
B. To identify the key characteristics of quadratics graphs.
C. To be able to plot the graph of a quadratic equation.

Objective A: To demonstrate understanding of quadratic equations.

The name Quadratic comes from "quad" meaning square, because the variable gets squared (like x^2). This language derives from the area of a square being its side length multiplied by itself. The word "quadratic" comes from quadratum, the Latin word for square.

An example of a **Quadratic Equation**:

this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

Quadratic equations characterize a great number of phenomena in the real world, such as where a rocket ship will land, how much to charge for a product or how long it will take a person to row up and down a river.

Standard Form

The Standard Form of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0$$

a, b and c are known values. a can't be 0. "x" is the variable or unknown.

<u>Example 1</u>: For quadratic equation $3x^2 + 5x + 1 = 0$, define a, b, and c.

Comparing with the standard form $ax^2 + bx + c = 0$, a= 3, b = 5, and c = 1.

<u>Example 2</u>: For quadratic equation $3x^2 - 5x - 1 = 0$, define a, b, and c.

Comparing with the standard form $ax^2 + bx + c = 0$, a= 3, b = −5, and c = −1.

<u>Example 3</u>: For quadratic equation $3x^2 = 5x + 1$, define a, b, and c.

Comparing with the standard form $ax^2 + bx + c = 0$,
Equation can be written as $3x^2 - 5x - 1 = 0$,
Therefore: a= 3, b = −5, and c = −1.

Practice A: For the following quadratic equations, identity a, b and c.

1. $x^2 - 3x = 0$

2. $x^2 = 3x - 1$

3. $2(w^2 - 2w) = 5$

4. $z(z-1) = 3$

Assessment A: Attempt multiple choice questions.

1. $x^2 + 2x + 1$ is a

   A. polynomial of degree 2
   B. polynomial of degree 0
   C. polynomial of degree 3
   D. polynomial of degree n

2. If $-x^2 - 3x = 1$, then a =,

   A. 1
   B. -1
   C. 0
   D. 2

3. If $z(z-1) = 3z + 2$, then b = ,

   a. 4
   b. -1
   c. -3
   d. -4

Objective B: To identify the key characteristics of quadratics.

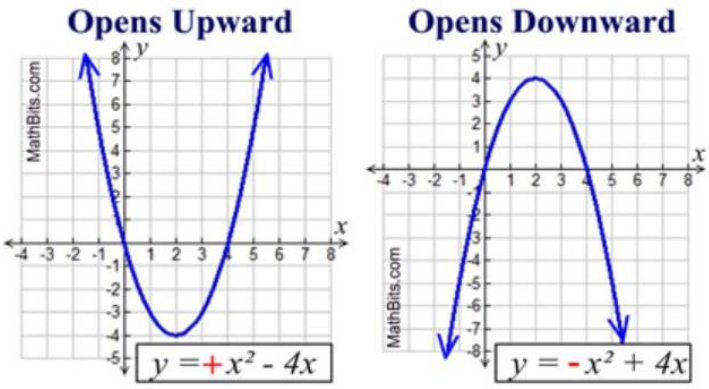
All quadratic equations: $y = x^2$, $y = x^2 + 5$, $y = x^2 + 5x = 6$, $y = 3x^2 - 5x + 4$, produces quadratic graphs, which is a smooth curve called parabola.

Parabolas have a shape that resembles (but is not the same as) the letter U.

Parabolas may open upward or downward.

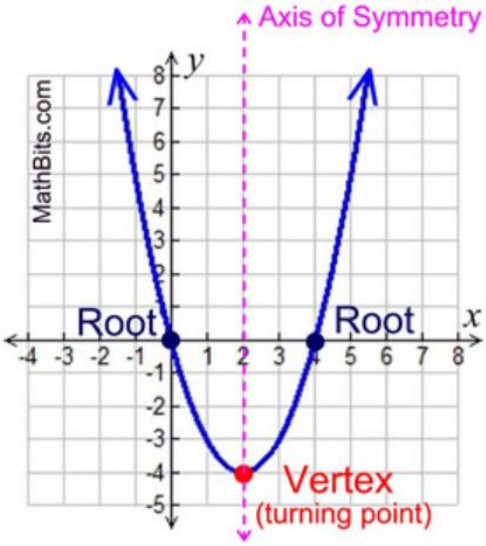
If the sign of the leading coefficient, *a*, is positive (*a>0*), the parabola opens upward.

If the sign of the leading coefficient, *a*, is negative (*a<0*), the parabola opens downward.



The bottom (or top) of the U is called the vertex, or the turning point. The vertex of a parabola opening upward is also called the minimum point. The vertex of a parabola opening downward is also called the maximum point.

The *x*-intercepts are called the roots, or the zeros. To find the *x*-intercepts, set $ax^2 + bx + c = 0$.



Quadratic equations with graphs

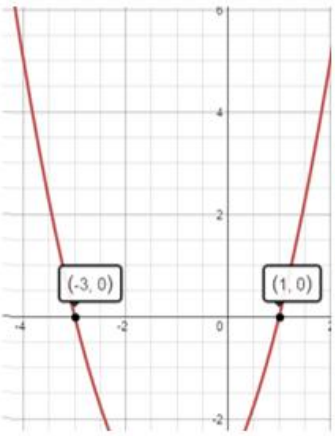
1. Roots

$x^2 + 2x - 3 = 0$

Factorised:
$(x + 3)(x - 1) = 0$

Therefore:
$x = -3$ and $x = 1$

Look at the graph →



The **Roots** of a quadratic are where the graph intersects with the x-axis.

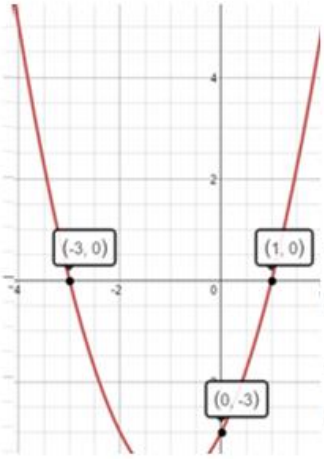
They can be found by solving the quadratic equation when y = 0.

This can be done by factorising, completing the square or using the quadratic formula.

2. Intercept

$y = x^2 + 2x - 3$

We can also see the intercept →

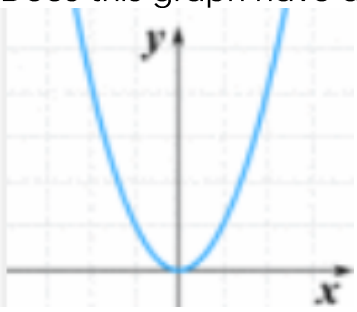


The **Intercept** is where the graph cuts the y-axis.

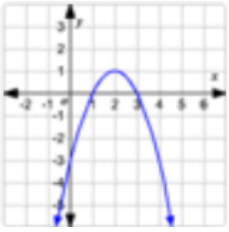
This is always the (+c) (end number) of the quadratic equation

Practice B

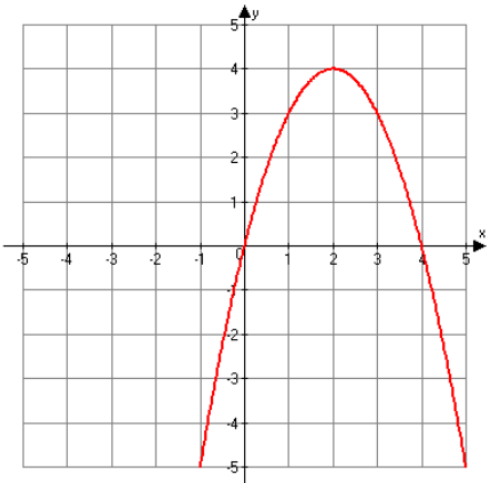
1. Does this graph have a maximum of minimum value?



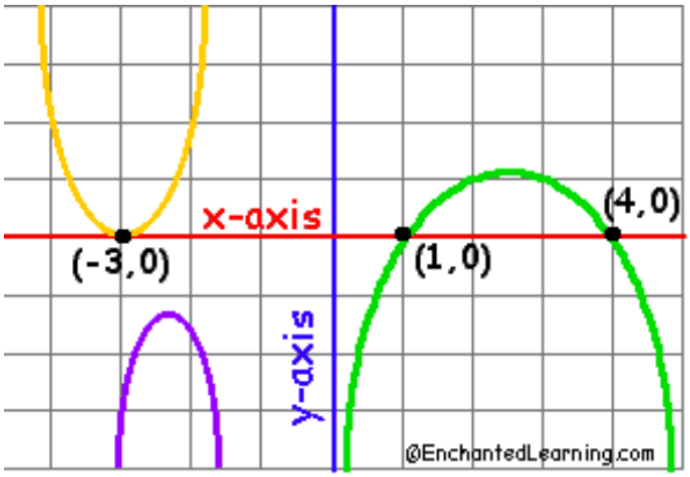
2. Use the graph to determine the solutions.
   *Hint: Solutions = roots = zeros = x-intercepts



3. What are the *x*-intercepts?



4. The point in which a graph hits the x-axis is known as the _____



Objective C To be able to plot the graph of a quadratic equation.

<u>Example</u>

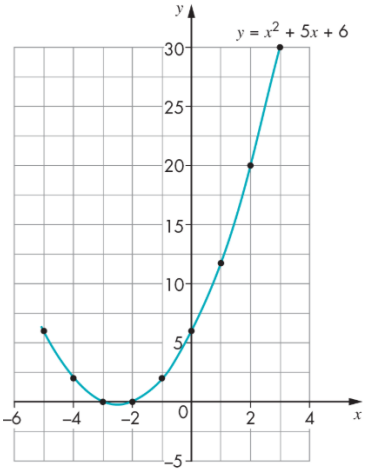
Draw the graph of $y = x^2 + 5x + 6$ for $-5 \leq x \leq 3$

X	−5	−4	−3	−2	−1	0	1	2	3
Y	6	2	0	0	2	6	12	20	30

When $x = -5$, then $y = (-5)^2 + 5(-5) + 6$ (Substitute in, the value of x)

$y = 25-25+6$

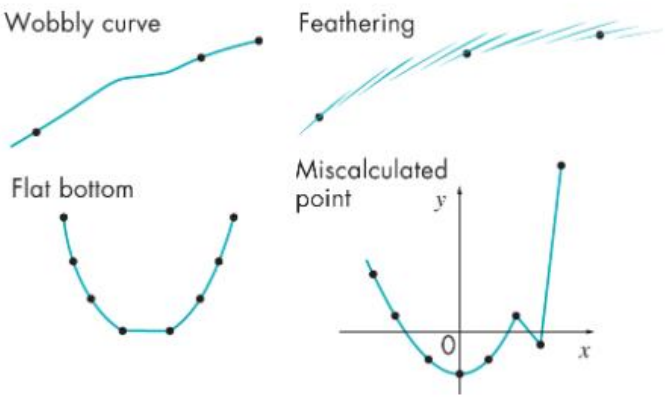
The graph crosses at −3 and −2.



Drawing Accurate Graphs

Although it is difficult to draw accurate curves, you need to make sure that whatever you draw is the right shape and accurate enough to read off values. Try to avoid the following common errors:

- When the points are too far apart, a curve tends to 'wobble'
- Drawing curves in small section leads to 'feathering'
- The place where a curve should turn smoothly is drawn 'flat'
- A line is drawn through a point that, clearly, has been drawn incorrectly plotted



Here are some tips that will make it easier for you to draw smooth, curved lines:

- If you are right-handed, turn your paper and exercise book round so that you draw from left to right. Your hand is steadier this way than when you are trying to draw from right to left or away from your body. If you are left-handed, you should find drawing from right to left the more accurate way
- Move your pencil over the points as a practice run without drawing the curve
- Do one continuous curve and only stop at a plotted point
- Use a sharp pencil and do not press too heavily, so that you may easily rub out mistakes

Practice C

1 **a.** Copy and complete the table or use a calculator to work out values for the graph of $y = x^2 - 2x - 8$ for values of x from -5 to 5. Plot the graph using $-5 \leqslant x \leqslant 5$ and $-10 \leqslant y \leqslant 30$ for your axes.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	27							-8			

b. Use your graph to find the value of y when $x = 0.5$.

c. Use your graph to find the values of x that give a y-value of -3.

2 **a.** Copy and complete the table or use a calculator to work out the values for the graph of $y = x^2 + 2x - 1$ for values of x from -3 to 3. Plot the graph using $-3 \leqslant x \leqslant 3$ and $-2 \leqslant y \leqslant 14$ for your axes.

x	-3	-2	-1	0	1	2	3
y	2					7	

b. Use your graph to find the y-value when $x = -2.5$.

c. Use your graph to find the values of x that give a y-value of 1.

d. On the same axes, draw the graph of $y = \frac{x}{2} + 2$.

e. Where do the graphs $y = x^2 + 2x - 1$ and $y = \frac{x}{2} + 2$ cross?

3 **a.** Copy and complete the table or use a calculator to work out values for the graph of $y = 2x^2 - 5x - 3$ for values of x from -2 to 4.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4
y	15	9			-3	-5				-3			9

b. Where does the graph cross the x-axis?

MR 4 Shayla is writing out a table of values for the graph of $y = x^2 - 4x + 7$.

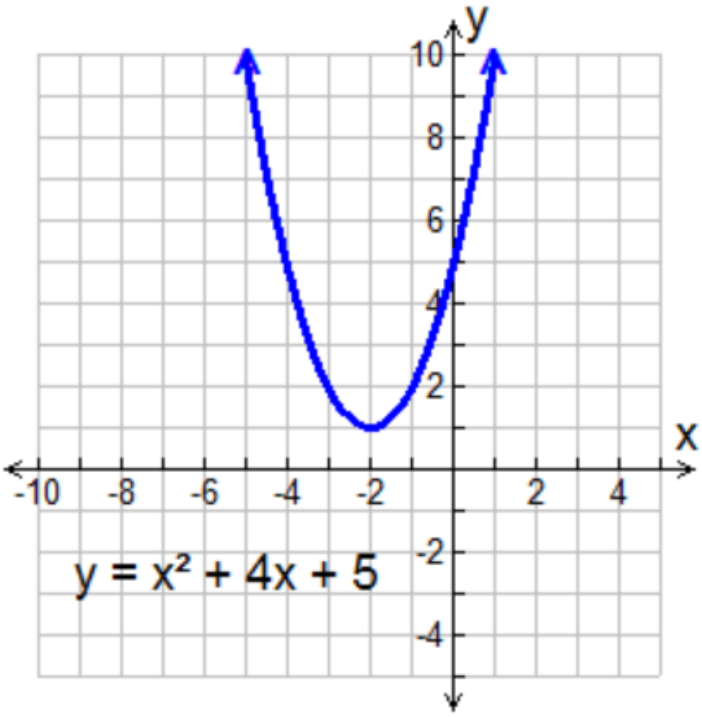
x	-3	-2	-1	0	1	2	3	4	5	6	7
y					4	3	4	7	12	19	28

a. What do you notice about the value of y when $x = 1$ and when $x = 3$?

b. Complete the table without substituting any values, explaining how it is possible to do so.

Assessment A

How many solutions does this graph have?

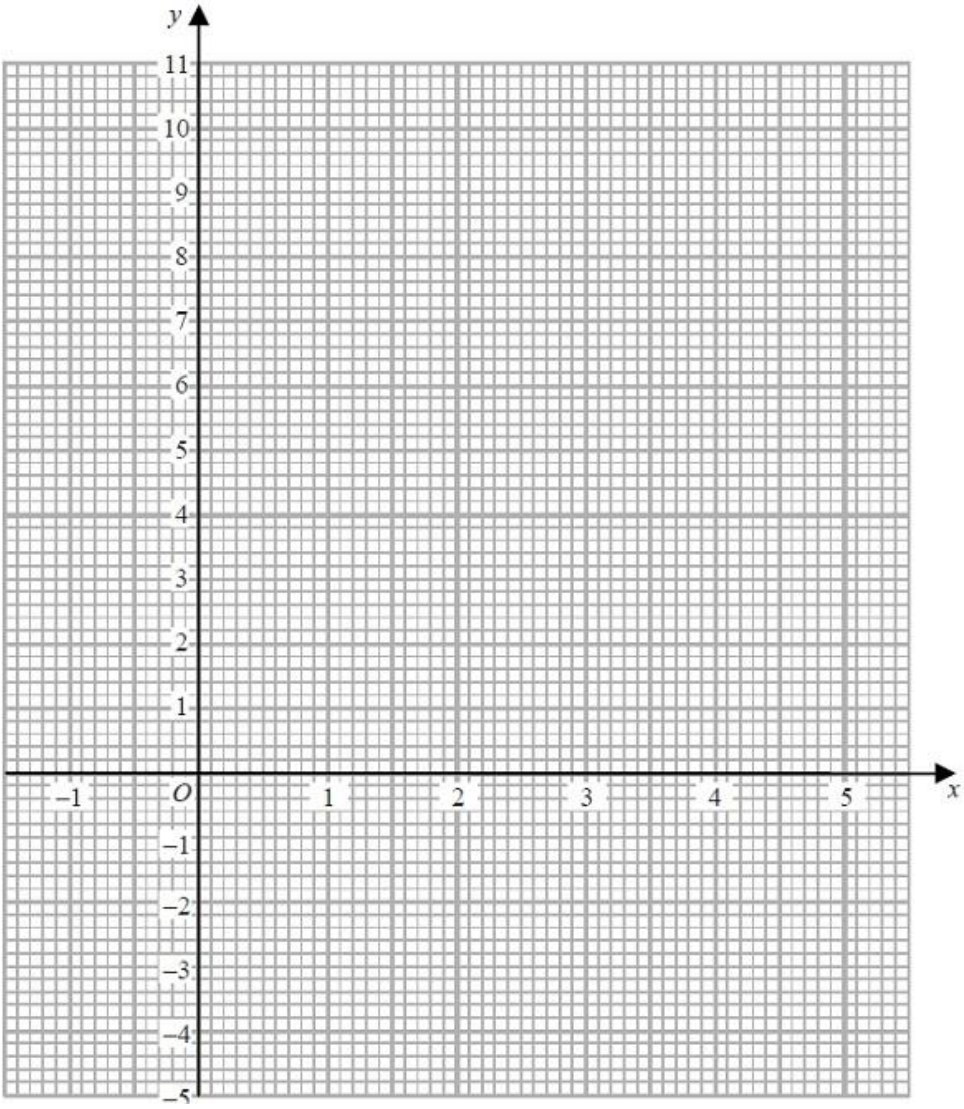


Assessment B

(a) Complete the table of values for $y = x^2 - 5x + 3$

x	−1	0	1	2	3	4	5
y		3	−1		−3		3

(b) On the grid below, draw the graph of $y = x^2 - 5x + 3$ for values of x from x = −1 to x = 5



Plenary

If the roots of a quadratic equation are 2 and 3, then the equation is

(a) $x^2 + 5x + 6 = 0$ (b) $x^2 + 5x - 6 = 0$

(c) $x^2 - 5x + 6 = 0$ (d) $x^2 - 5x + 6 = 0$

Tutorial 3 Solving a Quadratic Equation by Factorisation

Recall: Standard form of a quadratic equation: $ax^2 + bx + c = 0$
a, b and c are known values. a can't be 0. "x" is the variable or unknown.

Objective A: Solve a quadratic equation by factorisation

Some quadratic equations can be solved by factorisation. The solutions to a quadratic equation are sometimes called the root, zeros and x-intercept.

Worked Example 1: Solve $2x^2 + 7x + 3 = 0$

a = 2, b=7, c=3 ac = 6

Factor pairs of 6? **1,6** and 2,3 Find a factor pair that adds to make b

$2x^2 + 7x + 3 = 0$		
$2x^2 + x + 6x + 3 = 0$		Split up b using the factor pair
$x(2x + 1) + 3(2x + 1) = 0$		Factorise: the part in brackets should be the same for both!
$(x + 3)(2x + 1) = 0$		Collect like terms for (2x + 1)

Solutions are	$x + 3 = 0$	or	$2x + 1 = 0$	Find solutions by setting each bracket equal to zero
i.e.	$x = -3$	or	$x = -1/2$	Solve each **linear** equation for x

Worked example 2: Find the roots of $-2x^2 + 2x + 12 = 0$

Key step: First look for any common factors or negative signs on the x^2 term

$-2x^2 + 2x + 12 = -(2x^2 - 2x - 12)$	Factor out the negative sign on the x^2
$= -2(x^2 - x - 6)$	Factor out the common factor of 2

Study tip
You can check your answer by substituting your roots back into the original equation!

Now focus on factorising $x^2 - x - 6$

a = 1, b = -1, c = -6 ac = -6

Factor pairs of ac? 1, -6 and **2, -3** and 3, -2 and 6, -1 2 + (-3) = -1

$-2(x^2 - x - 6) = 0$	Leave the factor of -2 from before
$-2(x^2 + 2x - 3x - 6) = 0$	Split up b using the factor pair
$-2(x(x+2) - 3(x+2)) = 0$	Factorise: the part in brackets should be the same for both!
$-2(x - 3)(x + 2) = 0$	Collect like terms for (2x + 1)

Solutions are	$x - 3 = 0$	or	$x + 2 = 0$	Find solutions by setting each bracket equal to zero
i.e.	$x = 3$	or	$x = -2$	Solve each **linear** equation for x

Practice A

1 Solve the following equations.

HINTS AND TIPS
Look out for the special cases where b or c is zero.

a $3x^2 + 8x - 3 = 0$ **b** $6x^2 - 5x - 4 = 0$

c $5x^2 - 9x - 2 = 0$ **d** $4t^2 - 4t - 35 = 0$

e $18t^2 + 9t + 1 = 0$ **f** $3t^2 - 14t + 8 = 0$

g $6x^2 + 15x - 9 = 0$ **h** $12x^2 - 16x - 35 = 0$ **i** $15t^2 + 4t - 35 = 0$

j $28x^2 - 85x + 63 = 0$ **k** $24x^2 - 19x + 2 = 0$ **l** $16t^2 - 1 = 0$

m $4x^2 + 9x = 0$ **n** $25t^2 - 49 = 0$ **p** $9m^2 - 24m - 9 = 0$

Assessment A

Recall: Expanding brackets and Rearranging equations (If you need any help with either of this please see me before attempting assessments questions)

$$(x-2)(x-3)=0$$

What is the solution to the equation above?

A $x=-2,\ x=-3$ B $x=6$

C $x=2,\ x=3$ D x^2-5x+6

Correct Answer: A B C D

Explanation:

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$$(x+5)^2=0$$

What is the solution to the equation above?

A $x=-5$ B $x=5$

C $x=25$ D $x=-5\ or\ 5$

Correct Answer: A B C D

Explanation:

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$$3(x-4)(x+1)=0$$

What is the solution to the equation above?

A $x=-4,\ x=1$ B $x=-8,\ x=2$

C $x=4,\ x=-1$ D $x=8,\ x=-2$

Correct Answer: A B C D

Explanation:

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The sum of the roots of the quadratic $5x^2 - 6x + 1 = 0$ is

(a) $\frac{6}{5}$

(b) $\frac{1}{5}$

(c) $-\frac{5}{6}$

(d) $-\frac{1}{5}$

Plenary

Write the steps to factorise quadratic equations.

Tutorial 4 Solving equations using quadratic method

Objectives:

A. To be able to solve quadratic equations using quadratic method.
B. To be able to use discriminant to identity number of possible solutions.

Objective A: To be able to solve quadratic equations using quadratic method.

Many quadratic equations cannot be solved by factorisation because they do not have simple factors. For example, try to factorise $x^2 - 4x - 3 = 0$ or $3x^2 - 6x + 2 = 0$.

One way to solve this type of equation is to use the **quadratic formula**. You can use this formula to solve *any* quadratic equation that is **soluble**. (Some are not, which the quadratic formula would immediately show. You will learn about this later in this section.)

The solution of the equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a and b are the coefficients of x^2 and x respectively and c is the constant term.

This is the quadratic formula.

The symbol $\pm$ states that the square root has a positive and a negative value, and you must use *both* of them in solving for x.

Example A

Solve $5x^2 - 11x - 4 = 0$, giving solutions correct to 2 decimal places.

Substitute $a = 5$, $b = -11$ and $c = -4$ into the formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So $x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(-4)}}{2(5)}$

Note: Using brackets can help you to avoid arithmetic errors. A common error is to write -11^2 is -121.

$$x = \frac{11 \pm \sqrt{121 + 80}}{10} = \frac{11 \pm \sqrt{201}}{10}$$

$x = 2.52$ or -0.32

Practice A

Solve the following equations using the quadratic formula.
Give your answers to 2 decimal places.

1. $2x^2 + x - 8 = 0$
2. $3x^2 + 5x + 1 = 0$
3. $x^2 - x - 10 = 0$
4. $5x^2 + 2x - 1 = 0$
5. $7x^2 + 12x + 2 = 0$
6. $3x^2 + 11x + 9 = 0$

HINTS AND TIPS

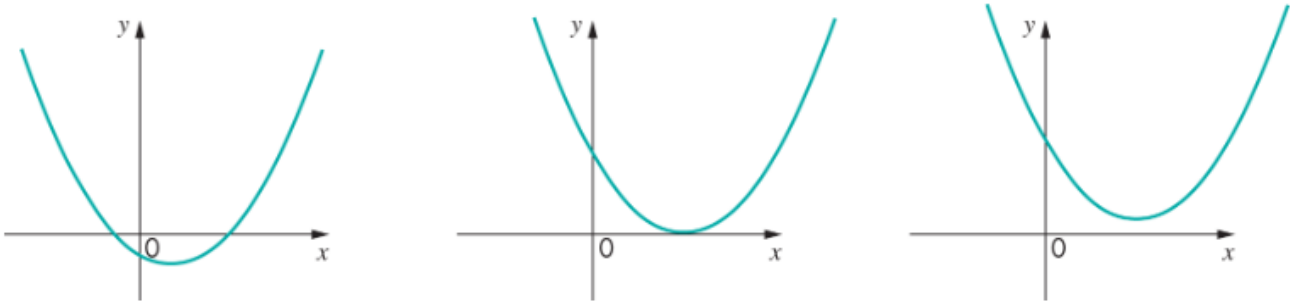
Use brackets when substituting and do not try to work two things out at the same time.

Objective B: To be able to use discriminant to identity number of possible solutions.

The quantity $(b^2 - 4ac)$ in the quadratic formula is known as the **discriminant**.

All quadratic equations can be shown as parabolas.

Here are the graphs of the three types of quadratic equations:



When the discriminant, $b^2 - 4ac$, is positive it means there are two solutions and the graph crosses the x-axis twice	When $b^2 - 4ac$ equals zero, it means there is only one solution and the graph just touches the x-axis	When $b^2 - 4ac$ is negative, you would have to find the square root of a negative number, so there are no solutions and the graph does not cross the x-axis

These rules are also true for negative parabolas.

Although it is possible to take the square root of a negative number and get something called an imaginary num you won't learn about them at this level. So there are no solutions if the discriminant is negative.

Example 11

How many times does the graph of $y = 13 - 2x - 3x^2$ meet the x-axis?

$$\begin{aligned} b^2 - 4ac &= (-2)^2 - 4(13)(-3) \\ &= 4 + 156 \\ &= 160 \end{aligned}$$

Since $b^2 - 4ac$ is positive, the graph meets the x-axis at two points.

Practice B

1 Work out the discriminant $b^2 - 4ac$ of the equations. In each case say how many solutions the equation has.

a. $3x^2 + 2x - 4 = 0$ **b.** $25x^2 - 30x + 9 = 0$ **c.** $4x^2 + 3x + 2 = 0$

d. $2x^2 + x + 1 = 0$ **e.** $x^2 - 2x - 16 = 0$ **f.** $5x^2 + 5x + 3 = 0$

g. $6x - x^2 - 10 = 0$ **h.** $48x - 9x^2 - 64 = 0$ **i.** $45 - 7x - 2x^2 = 0$

Assessment

1. If the roots of the quadratic $2x^2 + kx + 2 = 0$ are equal then the value of 'k' is

(a) 4 (b) –4

(c) ± 4 (d) ± 16

2. The ten quadratic equations below have been sorted into two categories, ones which can be factorised and ones which cannot be factorised. Evaluate $b^2 - 4ac$ for each one and determine what is special about the value of $b^2 - 4ac$ for equations that can be factorised.

Can be factorised	Cannot be factorised
$14x^2 + 27x - 20 = 0$	$8x^2 - 19x + 10 = 0$
$15x^2 + 11x + 2 = 0$	$3x^2 + 7x - 18 = 0$
$12x^2 - 25x + 12 = 0$	$30x^2 - 13x - 28 = 0$
$63 - 4x - 4x^2 = 0$	$9 - 6x - 5x^2 = 0$
$9x^2 + 18x - 16 = 0$	$10x^2 + 26x + 15 = 0$

Plenary

A. Explain to your partner how and when will you use quadratic method to find solutions

B. What is discriminant and what is the use of calcating it?

Tutorial 5 Solving Quadratic Equations using Completing Square Method

Objective: Solve a quadratic equation by completing the square.

You will remember that:

$$(x + a)^2 = x^2 + 2ax + a^2$$

which gives:

$$x^2 + 2ax = (x + a)^2 - a^2$$

This is the basic principle behind completing the square.

Example

Consider the quadratic expression

$$x^2 + 6x - 4$$

We compare this with the complete square

$$x^2 + 2ax + a^2$$

Clearly the coefficients of x^2 in both expressions are the same.

We would like to match up the term $2ax$ with the term $6x$. To do this note that $2a$ must be 6, so that $a = 3$.
Recall that

$$(x + a)^2 = x^2 + 2ax + a^2$$

Then with $a = 3$

$$(x + 3)^2 = x^2 + 6x + 9$$

This means that when trying to complete the square for $x^2 + 6x - 4$ we can replace the first two terms, $x^2 + 6x$, by $(x + 3)^2 - 9$. So

$$\begin{aligned} x^2 + 6x - 4 &= (x + 3)^2 - 9 - 4 \\ &= (x + 3)^2 - 13 \end{aligned}$$

We have now written the expression $x^2 + 6x - 4$ as a complete square plus or minus a constant. We have **completed the square**. It is important to note that the constant term, 3, in brackets is half the coefficient of x in the original expression.

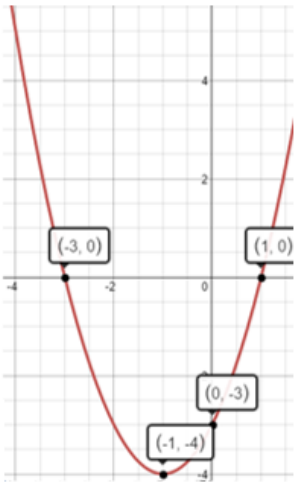
Deducing graphical information from competing the square.

3. Turning Point

$y = x^2 + 2x - 3$

Completed Square:

$y = (x + 1)^2 - 4$



The **turning point** of a quadratic is where the gradient changes from negative to positive or vice versa.

This can be found by completing the square in the form $y = a(x + b)^2 + c$

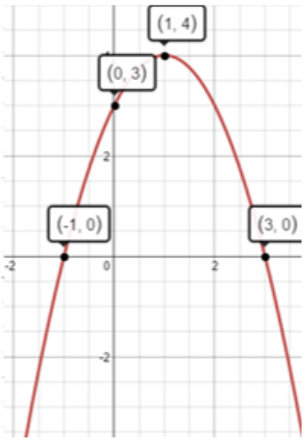
The coordinate of the turning point is (−b, c)

4. Max or Min

$y = x^2 + 2x - 3$

If a quadratic is positive (U Shape) the turning point will be a Minimum.

If it is a negative (n Shape) it will be a maximum



The Turning Point is either a **Maximum** value or a **Minimum** value.

If a quadratic is positive (U Shape) the turning point will be a **Minimum.**

If it is a negative (n Shape) it will be a **Maximum**

Example 1: Solve $x^2 + 4x + 1 = 0$

Step 1 can be skipped in this example since the coefficient of x^2 is 1

Step 2 Move the number term to the right side of the equation:

$x^2 + 4x = -1$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation.

$(b/2)^2 = (4/2)^2 = 2^2 = 4$

$x^2 + 4x + 4 = -1 + 4$

$(x + 2)^2 = 3$

Step 4 Take the square root on both sides of the equation:

$x + 2 = \pm\sqrt{3} = \pm 1.73$ (to 2 decimals)

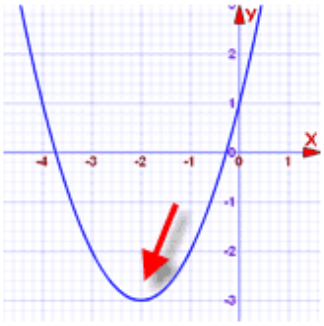
Step 5 Subtract 2 from both sides:

$x = \pm 1.73 - 2 = -3.73$ or -0.27

And here is an interesting and useful thing.

At the end of step 3 we had the equation:

$(x + 2)^2 = 3$



It gives us the **vertex** (turning point) of $x^2 + 4x + 1$: **(-2, -3)**

Practice A: For the following quadratic equations using completing square method find:

a the point of intersection of the graph with the y-axis

b the coordinates of the minimum point (vertex) of each graph

c Explain the connection between these points and the original equation.

a Write the equation $y = x^2 - 4x + 4$ in the form $y = (x - p)^2 + q$.

b Write down the minimum value of the equation $y = x^2 - 4x + 4$.

a Write the equation $y = x^2 - 6x + 3$ in the form $y = (x - p)^2 + q$.

b Write down the minimum value of the equation $y = x^2 - 6x + 3$.

a Write the equation $y = x^2 - 8x + 2$ in the form $y = (x - p)^2 + q$.

b Write down the minimum value of the equation $y = x^2 - 8x + 2$.

a Write the equation $y = -x^2 + 2x - 6$ in the form $y = -(x - p)^2 + q$.

b Write down the minimum value of the equation $y = -x^2 + 2x - 6$.

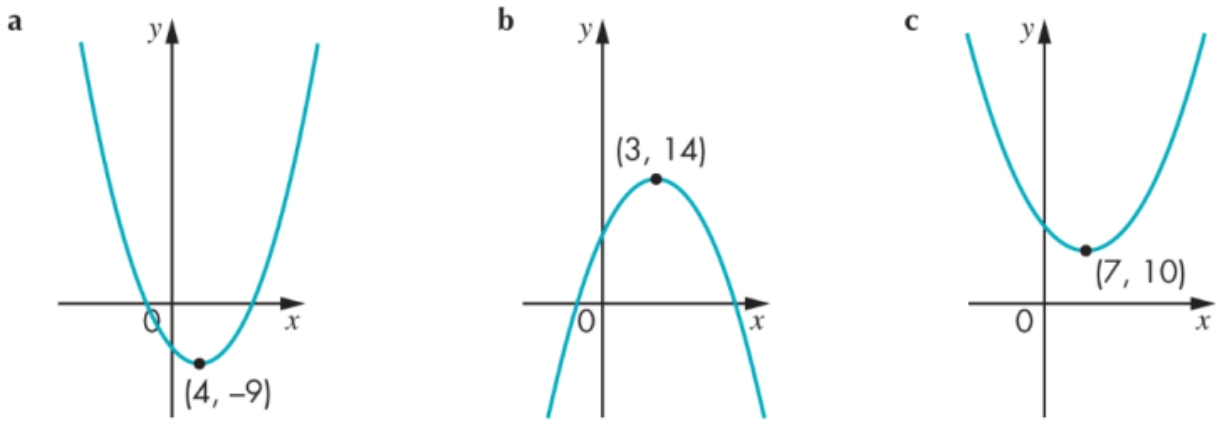
Assessment A

1. When completing the square, what do you do if your x squared term has a coefficient?

- Divide both sides by this coefficient
- Nothing
- Multiplying both sides by this coefficient
- Move it to the other side

2. (i) Express $2x^2 - 20x + 49$ in the form of $p(x-q)^2 + r$
(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 20x + 49$

3. Each of these graphs can be written in the form $y = \pm x^2 \pm ax \pm b$
State the equation of each graph.



Plenary

Discuss the following questions with your partner and make notes of the responses.

1. What is a quadratic equation?
2. What is a discriminant? And What information about the roots does it gave?
3. How many types of methods are there to solve quadratic equations? Discuss advantage and disadvantage of each of this method.

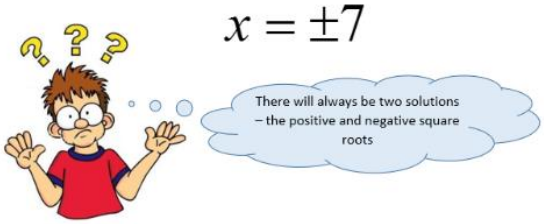
Clearing Up Misconceptions in Quadratics

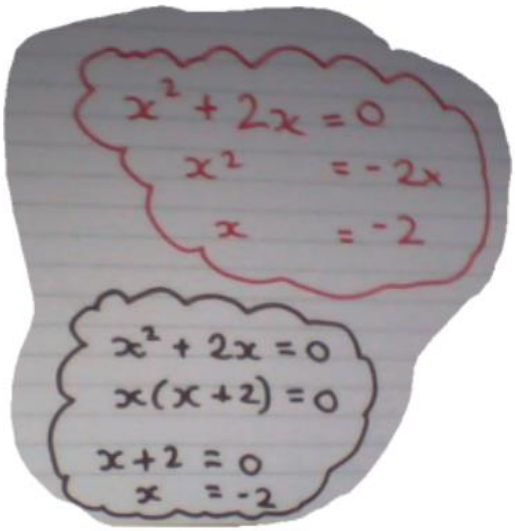
7 x 7 = 49

(-7) x (-7) = 49

$$x^2 = 49$$

$$x = \pm 7$$





Zero Product Property

If a × b = 0 then a = 0 or b = 0

Solve $x^2 + 7x + 10 = 0$

$x^2 + 7x + 10 = 0$

$(x + 2)(x + 5) = 0$

$x + 2 = 0$ or $x + 5 = 0$

$x = -2$ or $x = -5$

Solve $x^2 + 7x = 0$

$x^2 + 7x = 0$

$x(x + 7) = 0$

$x = 0$ or $x + 7 = 0$

$x = 0$ or $x = -7$

Tutorial 6 – Final Assessment

Tutorial 7 – Feedback Tutorial

What is the Purpose of Tutorial 7?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
• • •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Notes

Notes

Notes

Notes

Quadratics

Key Stage 4 Programme

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