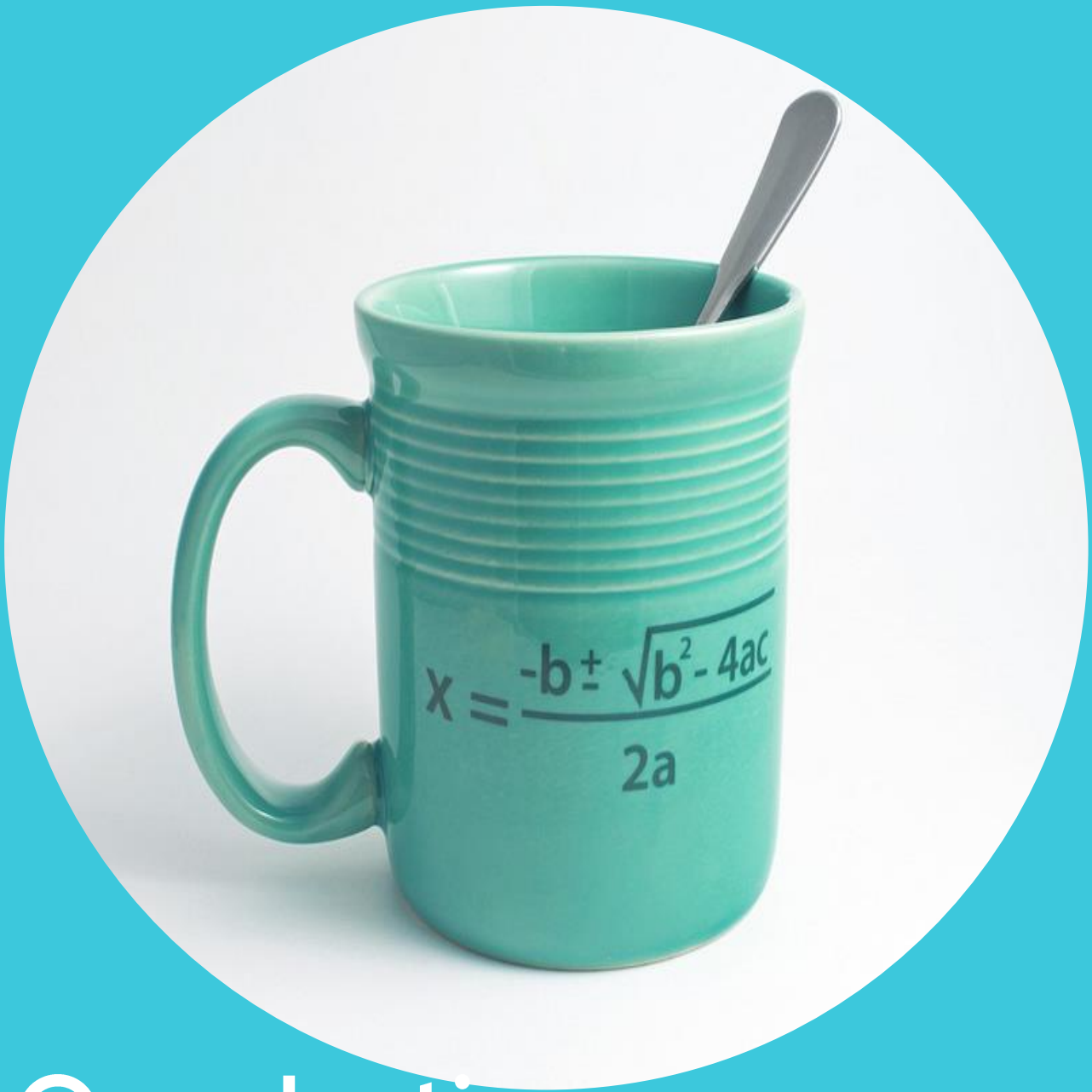


Quadratics

Key Stage 4 Revision Programme

Pupil Name

Handbook Designed by Dr Ripley

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5 (Final Assignment)			
6 (Feedback)			

Contents

Course Rationale	**P5**
Subject Vocabulary	**P6**
Tutorial 1 – Solving Quadratic Equations	**P7**
Tutorial 2 – Quadratic Graphs	**P10**
Tutorial 3 – Quadratic Simultaneous Equations	**P14**
Tutorial 4 – Quadratic Inequalities	**P18**
Tutorial 6	**P21**
Reflection on feedback	**P22**
Notes	**P23**

Course Rationale

Quadratic expressions and their associated graphs are a mainstay of maths and physics. Understanding how quadratics behave is a rich topic in and of itself, and has applications in almost every area of mathematics. Throughout this revision course we will cover how to solve quadratic equations through four different methods (factorising, using the quadratic formula, completing the square and using graphical methods) as well as understand their graphs and curves. We will also revisit how to deal with inequalities involving quadratic expressions, through algebraic and graphical methods.

This course will operate similar to a mathematics undergraduate tutorial at a university. In a university tutorial, students would be expected to look at and attempt a set of mathematical problems ahead of time. In the tutorial itself, students will have opportunities to ask questions about parts they struggled with, get feedback from their tutors and work on more challenging problems where appropriate.

Subject Vocabulary

Word	Definition	Example

Tutorial 1 – Solving Quadratic Equations

Objectives this tutorial:

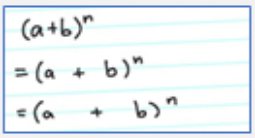
A) Solving quadratic equations by factorising

B) Solving quadratic equations by using the quadratic formula

Starter:

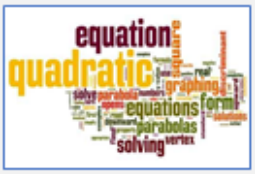
1) Expand the following brackets:

a) $(x+3)(x-2)$ b) $-(2x-4)$ c) $(2x+7)(4x+1)$



2) Explain why the following are **not** quadratic equations:

a) $2x^3+3x-1=0$ b) $x^2-\sqrt{x}+4=0$ c) $x^2-7x+3.5$



Objective A: Solving quadratic equations by factorising

A general quadratic equation has the form $ax^2+bx+c=0$.
Some quadratic equations can be solved by **factorising** the quadratic expression. The solutions to a quadratic equation are sometimes called the **roots**.

Worked Example 1: Solve $2x^2+7x+3=0$

a = 2, b=7, c=3 ac = 6

Factor pairs of 6? **1,6** and 2,3 Find a factor pair that adds to make b

$2x^2+7x+3$	$=0$	
$2x^2+x+6x+3$	$=0$	Split up b using the factor pair
$x(2x+1)+3(2x+1)$	$=0$	Factorise: the part in brackets should be the same for both!
$(x+3)(2x+1)$	$=0$	Collect like terms for (2x + 1)

Solutions are $x+3=0$ or $2x+1=0$ Find solutions by setting each bracket equal to zero

i.e. $x=-3$ or $x=-1/2$ Solve each **linear** equation for x

Worked example 2: Find the roots of $-2x^2+2x+12=0$

Key step: First look for any common factors or negative signs on the x^2 term

$-2x^2+2x+12 = -(2x^2-2x-12)$ Factor out the negative sign on the x^2

$= -2(x^2-x-6)$ Factor out the common factor of 2

Study tip
You can check your answer by substituting your roots back into the original equation!

Now focus on factorising x^2-x-6

a = 1, b = -1, c = -6 ac = -6

Factor pairs of ac? 1, -6 and **2, -3** and 3, -2 and 6, -1 2 + (-3) = -1

$-2(x^2-x-6)$	$=0$	Leave the factor of -2 from before
$-2(x^2+2x-3x-6)$	$=0$	Split up b using the factor pair
$-2(x(x+2)-3(x+2))$	$=0$	Factorise: the part in brackets should be the same for both!
$-2(x-3)(x+2)$	$=0$	Collect like terms for (2x + 1)

Solutions are $x-3=0$ or $x+2=0$ Find solutions by setting each bracket equal to zero

i.e. $x=3$ or $x=-2$ Solve each **linear** equation for x

Practice A: Solving quadratic equations by factorising

1) Solve the following quadratic equations by factorising:

a) $x^2 + 4x + 3 = 0$
b) $x^2 + 3x - 18 = 0$
c) $2x^2 - 17x + 30 = 0$
d) $x^2 + 4x + 5 = -2$
e) $8x^2 - 68x + 120 = 0$
f) $x^2 - 16 = 0$
g) $x^2 - 5x = 0$
h) $\frac{1}{2}x^2 + \frac{3}{2}x = 2$

Study tip
Part (f) is an example of something called the **difference of two squares.**

2) Are the given numbers roots of the following quadratics? If so what is the other root?

a) $(x+2)(x-7) = 0$ with $x = 2$
b) $(2x+5)(x+8) = 0$ with $x = -\frac{2}{5}$
c) $x^2 - 188x - 573 = 0$ with $x = 191$

3) A rectangle has an area of 45m^2 and sides of length $(2x-1)$ metres and $(x+4)$ metres. Form a quadratic equation and solve for x to find the length of the shortest side.

4) a) Is it possible to factorise $\sqrt{2}x^2 - \sqrt{8}x - 3\sqrt{2} = 0$? Explain why.
b) What about $\sqrt{2}x^2 - \sqrt{8}x - 4\sqrt{2} = 0$?

Assessment A: Solving quadratic equations by factorising

1) What are the roots of $(2x+1)(3x-4) = 0$?

A	-1 and 4	B	-2 and 3	C	$-\frac{1}{2}$ and $\frac{4}{3}$	D	-2 and $\frac{3}{4}$

2) Which of the following quadratics cannot be solved by factorising?

A	$x^2 + 16 = 0$	B	$x^2 + 7x - 12 = 0$	C	$\frac{1}{3}x^2 - \frac{1}{3}x - 2 = 0$	D	$(x-2)(x+1) = 1$

Objective B: Solving quadratic equations using the quadratic formula

Not all quadratic equations can be solved by factorising. We can always use the **quadratic formula** to try to solve quadratic equations.
If we want to solve $ax^2 + bx + c = 0$ then solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $b^2 - 4ac$ is called the **discriminant**. It can tell us how many solutions there are to a quadratic.

Worked Example1: Solve $1.7x^2 - 3x - 1 = 3.8$ giving your answer to 3 d.p.

$1.7x^2 - 3x - 1 = 3.8$
$1.7x^2 - 3x - 4.8 = 0$ First rearrange to make the right hand side zero
$a = 1.7$, $b = -3$, $c = -4.8$

Study tip
Asking for an answer to a number of decimal places is a strong hint to use the formula!

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times (1.7) \times (-4.8)}}{2 \times (1.7)}$ Substitute all values using brackets to avoid sign mistakes!

$x = \frac{-(-3) + \sqrt{(-3)^2 - 4 \times (1.7) \times (-4.8)}}{2 \times (1.7)} = 2.217$ or $x = \frac{-(-3) - \sqrt{(-3)^2 - 4 \times (1.7) \times (-4.8)}}{2 \times (1.7)} = -0.453$

Worked Example 2: How many solutions are there to the equation $x^2 + 5x + 30 = 0$?

a = 1, b = 5, c = 30

Discriminant = $b^2 - 4ac = (5)^2 - 4\times1\times30 = 25 - 120 = -95$

The discriminant is negative, so there are 0 solutions.
This is because you can't square-root a negative number.

Study tip
If the discriminant is 0 there is 1 unique solution.
If the discriminant is positive there are 2 unique solutions.

Practice B: Solving quadratic equations using the quadratic formula

1) Solve the following equations, giving your answers to 4 s.f.

a) $3x^2 + 2x - 13 = 0$
b) $3x^2 = 11x + 13$
c) $5x^2 = 3(2x + 1)$

2) Solve the following leaving your answer in the form $p \pm q\sqrt{r}$

a) $x^2 + 2x - 7 = 0$
b) $x^2 = 11 - 6x$
c) $x(x-4) - 1 = 0$

Study tip
Sometimes you will be required to write your answers exactly using surds.

3) I have the equation $x^2 - 3kx + 7 = 0$. What range of values for k will give:

a) No solutions
b) One unique solution
c) Two unique solutions

Assessment B: Solving quadratic equations using the quadratic formula

Work out the number of unique solutions for each equation below. Give the values of any solutions to 3 s.f.:

A: $x^2 + \sqrt{2}x + 18 = 0$

B: $-\frac{1}{3}x^2 - 5.2x + 4^{1.89} = 0$

C: $x + 3 = 2x(x - 5) + 8$

Plenary: Generate different quadratic equations that have

- **Two unique integer solutions**
- **One repeated irrational solution**
- **No solutions**

Tutorial 2 – Quadratic Graphs

Objectives this tutorial:

A) Solving quadratic equations by completing the square
B) Identifying turning points of quadratic graphs by completing the square
C) Sketching quadratic graphs including turning points and intercepts

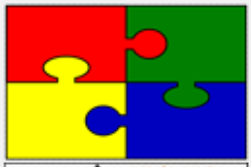
Starter:

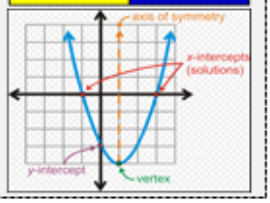
1) Expand the following:

a) $(x-2)^2$ b) $(x+3)^2+5$ c) $3(2x+1)^2$

2) What are the smallest values the following expressions could have?

a) x^2 b) $(x-5)^2$ c) $(x+2)^2-5$





Objective A: Solving quadratic equations completing the square

Another method for solving quadratic equations is by **completing the square**. This is where we re-write a quadratic expression as $(x+B)^2+C$.

Worked Example 1: Write x^2-4x-6 in the form $(x+B)^2+C$

x^2-4x-6

Try	$(x-2)^2$		The constant in the bracket is always **half** the coefficient of x
	$(x-2)^2$	$= x^2-4x+4$	Expand the bracket
	$(x-2)^2-4-6$	$= x^2-4x+4-4-6$	Add or subtract a constant to recreate the original quadratic
	$\mathbf{(x-2)^2-10}$	$\mathbf{= x^2-4x-6}$	

i.e. B = – 2 C = – 10

Study tip
You can check your answer by expanding your brackets.

Worked Example 2: Write $2x^2+12x+14$ in the form $A(x+B)^2+C$

There is one extra step to perform when the coefficient of x is not 1.

$2x^2+12x+14 = 2(x^2+6x+7)$ First take out a factor of the coefficient.
Then complete the square on the quadratic left in the bracket.

Try	$(x+3)^2$		The constant in the bracket is always **half** the coefficient of x
	$(x+3)^2$	$= x^2+6x+9$	Expand the bracket
	$(x+3)^2-9+7$	$= x^2+6x+9-9+7$	Add or subtract a constant to recreate the original quadratic
	$(x+3)^2-2$	$= x^2+6x+7$	

$\mathbf{2x^2+12x+14} = 2(x^2+6x+7) = \mathbf{2(x+3)^2-4}$

A = 2 B = 3 C = – 4

Worked Example 3: Solve $x^2-4x-6=0$

From completing the square we have

x^2-4x-6	$=0$	
$(x-2)^2-10$	$=0$	Complete the square
$(x-2)^2$	$=10$	Add 10 to both sides
$x-2$	$=\pm\sqrt{10}$	Square root each side
$\mathbf{x}$	$\mathbf{=2\pm\sqrt{10}}$	Add 2 to both sides

Study tip
Remember the "±" when square-rooting in equations.

Practice A: Solving quadratic equations by completing the square

1) Complete the square for the following quadratic expressions:

a) $x^2 + 6x + 1$
b) $x^2 + 2x - 7$
c) $x^2 - 8x + 20$
d) $x^2 - 7x - 3$
e) $3x^2 - 18x + 9$
f) $2x^2 + 9x$
g) $-x^2 - 2x + 9$
h) $\frac{1}{2}x^2 + 3x - \frac{1}{4}$

2) Find the solutions to the following quadratic equations, or explain why no such solutions exist:

a) $(x-4)^2 = 3$
b) $(x+5)^2 + 6 = 0$
c) $2(2x-3)^2 + 3 = (2x-3)^2 - 7$
d) $-(x+\sqrt{2})^2 = 6$

3) Solve $x^2 + 2bx + c = 0$ by completing the square. How does this link to the previous tutorial?

Assessment A: Solving quadratic equations by completing the square

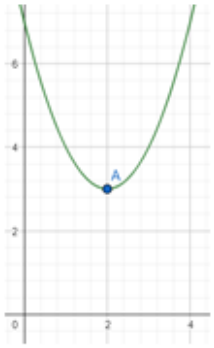
1) Which of these quadratic equations has no real solutions?

A	$x^2 - 4x + 7 = 0$	B	$x^2 - 4x + 1 = 0$	C	$11 - (x+1)^2 = 3$	D	$(x+7)^2 = 0$

Objective B: Identifying turning points of quadratic graphs by completing the square

By completing the square we can easily find the maximum or minimum values (turning points) of a quadratic function.

Worked Example 1: Find the minimum value of $y = x^2 - 4x + 7$ and the value of x that achieves the minimum



$y = x^2 - 4x + 7$
$= (x-2)^2 + 3$ First complete the square

We want the minimum value of y i.e. the smallest value it can take.
The square of any number is positive, so the smallest value $(x-2)^2$ can take is 0.
The square bracket takes the value of 0 when $x = 2$.
So by completing the square we can see that the minimum is $y = 3$ when $x = 2$.

Worked Example 2: Find the maximum value of $y = -x^2 - 8x - 9$ and the value if x that achieves the maximum

$y = -x^2 - 8x - 9$
$= -(x^2 + 8x + 9)$ First factor out the -1 to help us complete the square
$= -((x+4)^2 - 7)$ Complete the square inside the bracket
$= -(x+4)^2 + 7$ Expand the outside bracket

The minimum value of $(\text{something})^2$ is 0, so the *maximum* value of $-(\text{something})^2$ is 0.
The square bracket takes the value 0 when $x = -4$.
The maximum is $y = 7$ when $x = -4$.

Practice B: Identifying turning points of quadratic graphs by completing the square

1) Identify the value of x that minimises or maximises the following quadratics:

a) $y = (x - 5)^2 + 3$
b) $y = (x - 5)^2 - 100$
c) $y = (x + 5)^2 - 100$
d) $y = -(x + 5)^2 + 13$
e) $y = -(x - 5)^2 + 24$
f) $y = -(x - A)^2 - B$

2) Identify the coordinate of the turning point for these quadratics. State whether the turning point is a maximum or a minimum:

a) $y = (x - 6)^2 + 11$
b) $y = -(x + 13)^2 - 7$
c) $y = -((x - 1)^2 + 3)$
d) $y = 2(x - 5)^2 + 1$
e) $y = 13 - (x + 3)^2$
f) $y = x^2 + 3$
g) $y = x^2 + 6x + 1$
h) $y = -x^2 + 4x - 3$

Study tip
If the coefficient of x^2 is positive, the turning point will be a minimum.
If the coefficient is negative then the turning point will be a maximum.

3) I have the graph of $y = x^2 + 2Bx + C = 0$. What values of B and C will make sure:

a) The turning point has a positive x coordinate?
b) The turning point has a positive y coordinate

Assessment B: Identifying turning points of quadratic graphs by completing the square

Which of the following graphs has a turning point that is a minimum with a negative x coordinate?

A	$y = -x^2 - 6x - 7$	B	$y = 4 - (x - 4)^2$	C	$y = x^2 + 10x + 15$	D	$y = x^2 - 10x + 15$

Objective C: Sketching quadratic graphs including turning points and intercepts

To sketch a quadratic graph we need to know:

1. The coordinates of the y-intercept (the point where the curve crosses the y-axis)
2. The coordinates of the x-intercept(s) (the point(s) where the curve crosses the x-axis)
3. The coordinates of the turning point
4. Whether the curve is a U shape or a ∩ shape

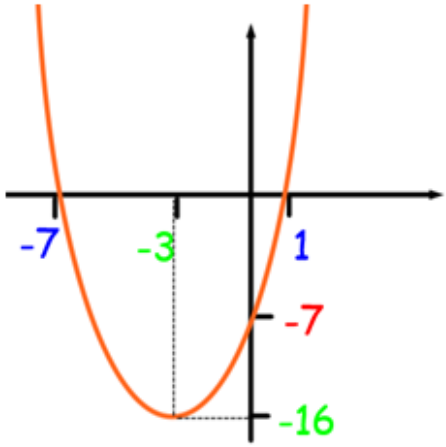
Worked Example 1: Sketch the curve $y = x^2 + 6x - 7$ highlighting the key features

1. y-intercept is found by substituting x = 0:
   $y = 0^2 + 6\times0 - 7 = -7$

2. x-intercepts are the roots of the equation, found by setting y = 0:
   $0 = x^2 + 6x - 7$
   $0 = (x + 3)^2 - 16$ Completing the square
   $x = -3 \pm \sqrt{16}$ Rearranging
   $x = 1$ or -7

3. Turning point:
   $y = (x + 3)^2 - 16$ Completing the square
   Turning point at $x = -3$, $y = -16$

4. Shape determined by coefficient of x^2:
   coefficient is positive so we have a U shape.



Worked Example 2: Sketch the curve $y = -x^2 + 6x - 5$ highlighting the key features

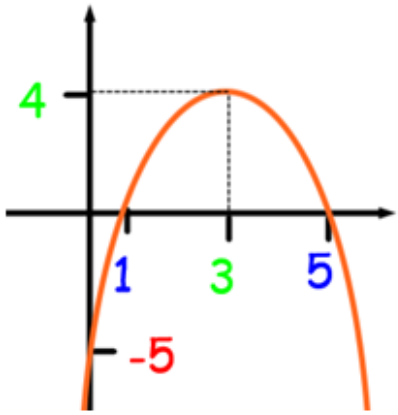
1. y-intercept is found by substituting x = 0:
   $y = -0^2 + 6\times0 - 5 = -5$

2. x-intercepts are the roots of the equation, found by setting y = 0:

0	$= -x^2 + 6x - 5$	
0	$= -(x^2 - 6x + 5)$	Factor out -1
0	$= -(x-1)(x-5)$	Factorise
x	= 1 or 5	

3. Turning point:
   $y = -(x-3)^2 + 4$ Completing the square
   Turning point at x = 3, y = 4

4. Shape determined by coefficient of x^2:
   coefficient is negative so we have a ∩ shape.



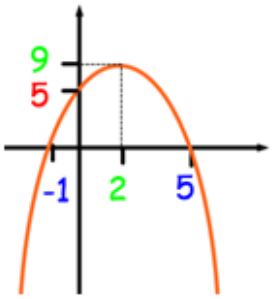
Practice C: Sketching quadratic graphs including turning points and intercepts

1) Sketch curves for the following quadratics:
a) $y = (x-5)^2 - 1$
b) $y = (x+5)^2 + 6$
c) $y = -(x+1)^2 + 16$
d) $y = -(x-4)^2 - 9$

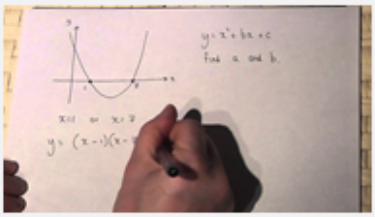
2) Sketch curves for the following:
a) $y = (x+2)(x+3)$
b) $y = (x+4)(x-9)$
c) $y = x(x-5)$
d) $y = (x-3)^2$
e) $y = -(x+3)^2$
f) $y = -(x+4)(x-6)$
g) $y = 2(x+1)(x-1)$
h) $y = (2x+3)(x-4)$

Assessment C: Sketching quadratic graphs including turning points and intercepts

Which quadratic does this sketch correspond to?
A: $y = (x+1)(x-5)$
B: $y = -(x+2)^2 + 9$
C: $y = -x^2 + 4x + 5$



Plenary: How many different quadratics can you write down with a turning point at (1, 4)? Are any of these extra properties impossible?
- **Only positive roots**
- **Only negative roots**
- **No roots at all**



Tutorial 3 – Quadratic Simultaneous Equations

Objectives this tutorial:

A) Solve simultaneous equations involving quadratics using algebra
B) Solve and estimate simultaneous equations involving quadratics using graphical methods
C) Solve and estimate quadratic equations using graphical methods

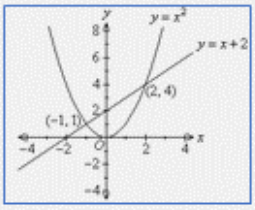
Starter:

1) Substitute into the expression $a^2 + 2a - b$:

a) a = 1, b = 2 b) a = – 3, b = 4 c) a = x + 1, b = – 1

2) Sketch the straight lines:

a) $y = 0$ b) $y = 4$ c) $y = 2x + 1$



Objective A: Solve simultaneous equations involving quadratics using algebra

We might be asked to solve a pair of simultaneous equations where one is a quadratic equation. We can solve these algebraically by using **substitution**.

Worked Example: Solve simultaneously the pair

$$\boldsymbol{y = 3x^2 - 4}$$
$$\boldsymbol{y = 2x + 3}$$

First substitute the second equation into the first to get an equation only involving x:

y	$= 3x^2 - 4$	
2x + 3	$= 3x^2 - 4$	
0	$= 3x^2 - 4 - 2x - 3$	Collect all terms to one side
0	$= 3x^2 - 2x - 7$	Simplify

Using quadratic formula: $\boldsymbol{x} = \frac{-(-2)\pm\sqrt{(-2)^2-4\times3\times(-7)}}{2\times3}$

x = 1.896805253… or x = – 1.230138587…

Study tip
You can use any method you like to solve the quadratic once you have written everything in terms of just one variable.

Use each (unrounded) value of x to find a corresponding value of y by substituting into the second equation:

when x = 1.896805253: y = 2x + 3
= 2 × 1.896805253 + 3
= 6.79 (2 d.p.)

when x = – 1.230138587: y = 2x + 3
= 2 × (– 1.230138587) + 3
= 0.54 (2 d.p.)

Solutions **are x = 1.90, y = 6.79** and **x = – 1.23, y = 0.54** rounded to 2 d.p.

Practice A: Solve simultaneous equations involving quadratics using algebra

1) Solve the following simultaneous equations using substitution

a) $y = x^2 - 4$, $y = 3x$ *b)* $y = x^2 + 5x$, $y = 2x + 10$ *c)* $y = 2x^2 + x - 3$, $y - 1 = 3x$ *d)* $x - y^2 = 8$, $6y + x = 5$

2) Solve the following simultaneous equations using substitution

a) $x^2 + y^2 = 5$, $y = x - 1$

b) $x^2 + y^2 = 58$, $y - x = 4$

c) $x^2 + y^2 = 25$, $-x + y = 1$

d) $x^2 + y^2 = 2$, $x + 3y = 2$

3) Solve the following simultaneous equations using substitution

a) $4x^2 + y^2 = 4$, $2x + 3y = 2$

b) $2x^2 + xy + 1 = 0$, $y - x = 4$

c) $4x^2 + y^2 = 37$, $2y + x = 1$

d) $2x^2 + y^2 = 6$, $x + 2y = 5$

Assessment A: Solve simultaneous equations involving quadratics using algebra

1) One of a pair of simultaneous equations is $y = x^2 - 1$. If there are no solutions to the pair of simultaneous equations, what could the second equation be? There may be more than one answer.

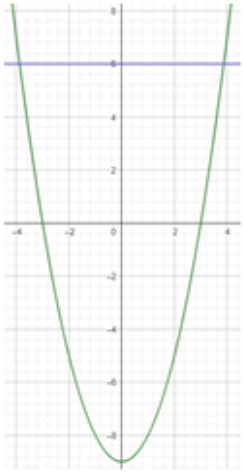
A	$y = x - 1$	B	$y = 1 - x$	C	$y = 1 + x$	D	$y = -1 - x$

Objective B: Solve and estimate simultaneous equations involving quadratics using graphical methods

When we solve simultaneous equations we can find solutions graphically by plotting each function and looking for the points where they intersect. Sometimes we can only estimate solutions in this way.

Worked Example 1: Use the graph to estimate the solution to the pair

$$y = x^2 - 9$$
$$y = 6$$



First plot graphs of $y = x^2 - 9$ and $y = 6$

From the graph we can see intersections at approximately $x = 3.9$ and $x = -3.9$

Our approximate solutions are $x = 3.9$, $y = 6$ and $x = -3.9$, $y = 6$

We can check the approximations by substituting back into the original:

$(3.9)^2 - 9 = 6.21 \approx 6$

$(-3.9)^2 - 9 = 6.21 \approx 6$

Worked Example 2: Use the graph to estimate the solution to the pair

$$y = x^2 - 9$$
$$y = 3x - 6$$

First plot graphs of $y = x^2 - 9$ and $y = 3x - 16$

From the graph we can see intersections at approximately $(-0.8, -8.4)$ and $(3.9, 5.7)$

Our approximate solutions are $x = -0.4$, $y = -8.2$ and $x = 3.8$, $y = 5.4$

We can check these approximations by substituting back into the original equations:

$x^2 - 9 = (-0.8)^2 - 9 = -8.36 \approx -8.4 = y$

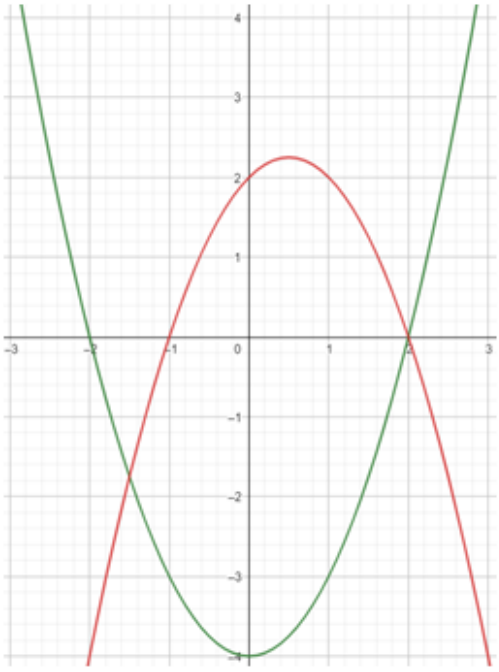
or

$x^2 - 9 = (3.8)^2 - 9 = 5.44 \approx 5.4 = y$

Practice B: Solve and estimate simultaneous equations involving quadratics using graphical methods

1) The graph shows the curves $y = x^2 - 4$ and $y = -x^2 + x + 2$.
   By drawing suitable lines, estimate the solutions to:

a) $y = x^2 - 4, y = 2$
b) $y = -x^2 + x + 2, y = x$
c) $y = x^2 - 4, y = x + 1$
d) $y = -x^2 + x + 2, y = 3$



2) Verify your estimations to Q1 by solving the simultaneous equations exactly using substitution.

3) Using the graph, estimate the smallest value of k such that $y = -x^2 + x + 2$ and $y = x + k$ has only one simultaneous solution.

Assessment B: Solve and estimate simultaneous equations involving quadratics using graphical methods

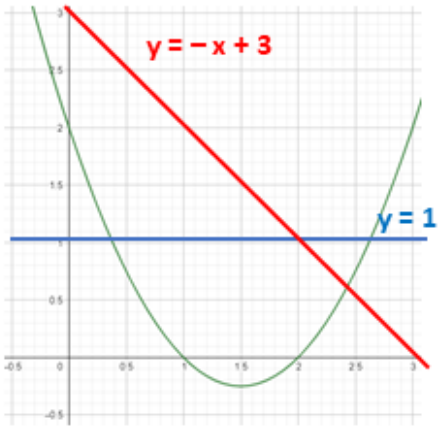
Which of the following is the best estimate to a simultaneous solution of $y = x^2 - 4$ and $y = -x^2 + x + 2$:

A	$(-1.75, -1.5)$	B	$(1.75, 1.5)$	C	$(-1.5, -1.8)$	D	$(-1.5, -1.7)$

Objective C: Solve and estimate quadratic equations using graphical methods

We can approximate solutions to quadratic equations using known graphs by converting the problem into a pair of simultaneous equations.

Worked Example 1: The graph below shows $y = x^2 - 3x + 2$. Using the graph, estimate positive solutions to (a) $x^2 - 3x + 1 = 0$ and (b) $x^2 - 2x - 1 = 0$



(a) First rearrange in terms of y.

$x^2 - 3x + 1 = 0$
$x^2 - 3x + 2 = 1$ Add 1 to both sides
$y = 1$

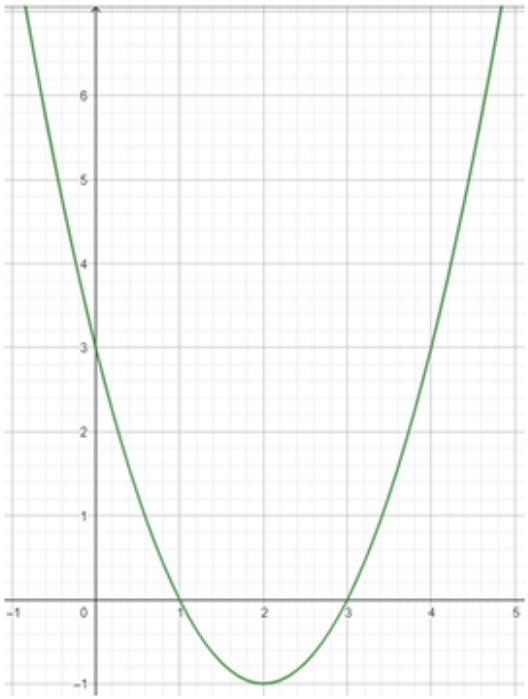
We need to find points on the original graph where y = 1.
These are approximately at **x = 0.4 and x = 2.6** (blue line)

(b) For second solution:

$x^2 - 2x - 1 = 0$
$x^2 - 3x - 1 = -x$ Subtract 2x from both sides
$x^2 - 3x + 2 = -x + 3$ Add 3 to both sides
$y = -x + 3$

We need to find points on the original graph where y = − x + 3 for positive x
Positive solution can be estimated graphically as **x = 2.4** (red line)

Practice C: Solve and estimate quadratic equations using graphical methods



1) The graph shows $y = x^2 - 4x + 3$. Using the graph and by drawing suitable lines, estimate the solutions to:

a) $x^2 - 4x + 1 = 0$
b) $x^2 - 4x = -4$
c) $x^2 = 5x - 3$
d) $x(x + 2) = 2 - 4x$
e) $(x + 1)^3 = x^2(x + 2) + 7x - 2$

2) Using the graph, estimate the range of k such that $x^2 + (k - 4)x + 3 = 0$ has no solutions.

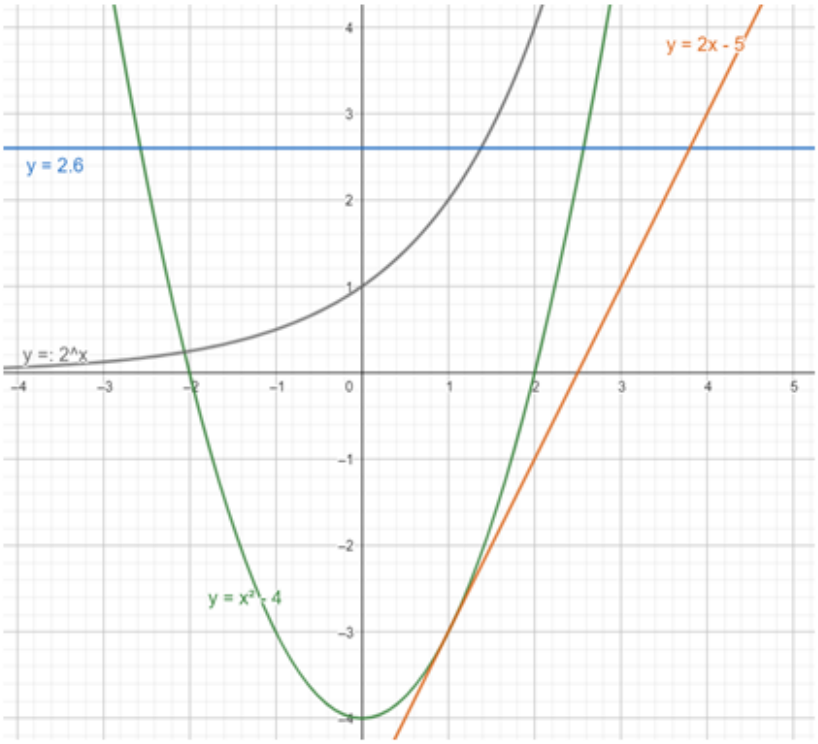
3) Verify your answer to Q2 using the quadratic formula.

4) By sketching a suitable curve on the graph, estimate solutions to $2x^2 - 2x + 11 = 0$.

Assessment C: Solve and estimate quadratic equations using graphical methods

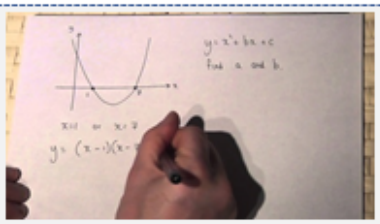
Given a graph of $y = 2x^2 + x + 7$, which curve would **not** help to solve $x^2 - x - 1 = 0$:

A	$y = 3x + 9$	B	$y = 6x - 9x^2$	C	$y = x^2 + 2x + 8$	D	$y = 3x^2 + 6$



Plenary: Given the curves above, write down four different equations that you could estimate solutions to.

Can you verify any estimations by finding the exact solutions?



Tutorial 4 – Quadratic Inequalities

Objectives this tutorial:

A) Solve quadratic inequalities by sketching curves

B) Solve quadratic inequalities using graphical methods

Starter:

1) Sketch the following:

a) $y = (x+1)(x-2)$ b) $y = x^2 + 5x + 6$ c) $y = -x^2 + 4x - 3$

2) Solve the following inequalities:

a) $x + 5 < 11$ b) $2x - 5 < 8$ c) $4 - 3x \geq 10$

Objective A: Solving quadratic inequalities by sketching curves

When we solve quadratic equations we find specific values for our variable x that make some quadratic expression equal to zero. When we solve a **quadratic inequality** we find an **interval** that makes some quadratic expressions always greater than or less than some value. Techniques for solving quadratic equations and quadratic inequalities are very similar.

Worked Example 1: Solve $x^2 + 7x + 12 \leq 0$

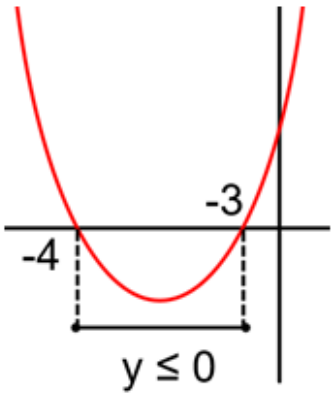
$x^2 + 7x + 12 \leq 0$

$(x + 3)(x + 4) \leq 0$

We can solve this by sketching a graph of $y = x^2 + 7x + 12$. From the factorised form we can see that the graph has roots at $x = -3$ and $x = -4$

From the sketch we can see that $y = x^2 + 7x + 12$ is less than or equal to 0 when x is between -4 and -3.

Solution: $-3 \leq x \leq -4$



Worked Example 2: Solve $x^2 - x > 6$

If the right-hand-side of the inequality is not zero, rearrange to make it so

$x^2 - x > 6$

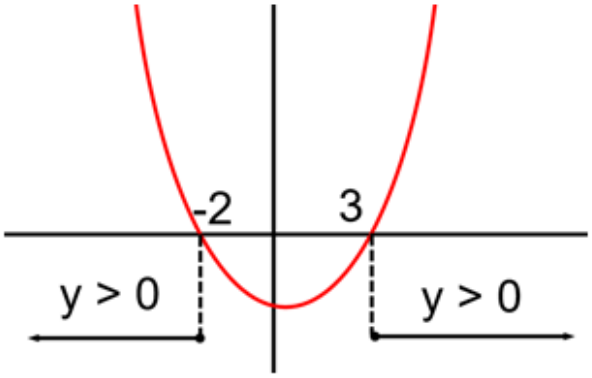
$x^2 - x - 6 > 0$

$(x - 3)(x + 2) > 0$

The graph of $y = x^2 - x - 6$ has roots are $x = -2$ and $x = 3$

From the sketch we can see that $y = x^2 - x - 6$ is greater than 0 when x is greater than 3 or less than -2

Solution: $x > 3$ or $x < -2$



Practice A: Solve quadratic inequalities by sketching curves

1) Solve the following quadratic inequalities:

a) $x^2 + 4x + 3 < 0$
b) $x^2 - x - 4 < 16$
c) $x(x-3) > 2(12+x)$
d) $x^2 - 49 \geq 0$
e) $2x^2 - 11x + 9 \leq 0$
f) $6x + 27 - x^2 > 0$
g) $\frac{1}{2}x^2 - 4x - \frac{9}{2} < 0$
h) $\frac{x-12}{x} < \frac{-35}{x^2}$

2) Find all the integers that satisfy the following inequalities:

a) $x^2 - 7x + 11 < 0$
b) $2x^2 + 10 < 10x$

3) Show that the curve $y = x^2 + x + 1$ has no real roots. Use this to solve $x^2 + x + 1 > 0$.

4) For what values of k does $x^2 + kx + 1 < 0$ have no solutions?

Assessment A: Solve quadratic inequalities by sketching curves

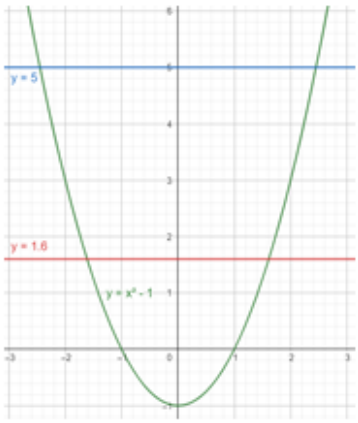
1) Which of these inequalities has a solution involving only one interval?

A	$(x-3)(x+7) > 0$	B	$21 - 4x \geq x^2$	C	$2 - 3x - 2x^2 > 0$	D	$2x^2 + 3x > 2$

Objective B: Solve quadratic inequalities using graphical methods

We can use graphs to solve or estimate solutions to more complicated quadratic inequalities.

Worked Example 1: The graph shows the curve $y = x^2 - 1$. Use the graph to estimate solutions to $x^2 - 1 < 5$ and $x^2 > 2.6$



For the first we can draw the line y = 5 onto the graph.
Graphically we can see that $y = x^2 - 1$ is below (less than) $y = 5$ when $-2.4 < x < 2.4$ (approximately).

For the second, we can rearrange:
$x^2 > 2.6$
$x^2 - 1 > 1.6$
If we compare $y = x^2 - 1$ and $y = 1.6$ we can see that $y = x^2 - 1$ is above (greater than) $y = 1.6$ when $x > 1.6$ or $x < -1.6$

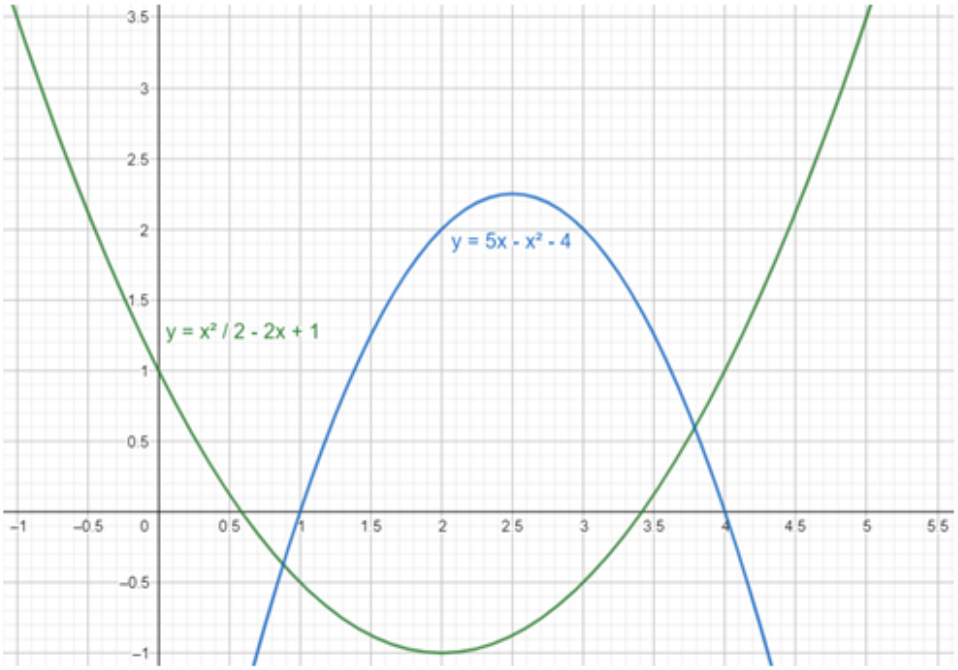
Worked Example 2: The graph shows the curves $y = 2 - 3x - x^2$ and $y = 2x^2 + x - 1$. Use the graph to estimate a solution to $3x^2 + 4x - 3 \leq 0$.

$3x^2 + 4x - 3 \leq 0$
$2x^2 + x - 1 \leq -x^2 - 3x + 2$
$2x^2 + x - 1 \leq 2 - 3x - x^2$

Look for interval where the green curve is below (less than or equal to) the blue curve.

From the graph we can see that this is when $-1.9 \leq x \leq 0.5$ (approximately).

Practice B: Solve quadratic inequalities using graphical methods



1) Using the graph, estimate solutions to the following inequalities:

a) $5x - x^2 - 4 \geq 0$

b) $\frac{x^2}{2} - 2x + 1 > \frac{1}{2}$

c) $x^2 - 4x < 2$

d) $5(x-1) < x^2$

e) $7x - \frac{3}{2}x^2 - 5 > 0$

2) What are the integer solutions to the inequalities in Q1?

3) Using the graph above, estimate the smallest value of M such that $5x - x^2 - 4 > Mx$ has no solutions.

Assessment B: Solve quadratic inequalities using graphical methods

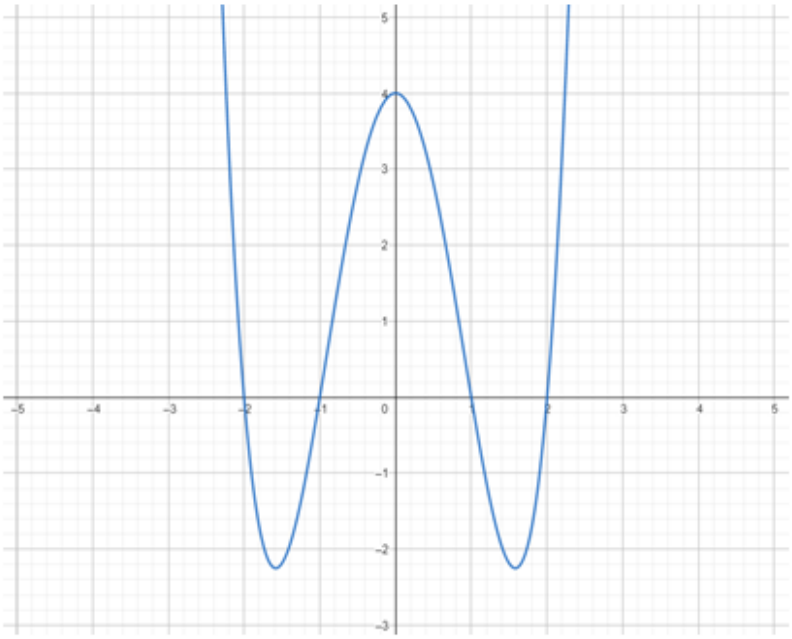
Using the graph above, which of these inequalities has no solutions?

A	$5x - x^2 - 6 > 0$	B	$5x - x^2 - 6 < 0$	C	$5x - x^2 - 7 > 0$	D	$5x - x^2 - 7 < 0$

Plenary: The graph shows $y = x^4 - 5x^2 + 4$.
Estimate the solutions to:

- **$y < 0$**
- **$y \geq 3$**
- **$y > 5$**

Is it possible to make an inequality involving this graph with only one interval as the solutions?



Tutorial 6 – Feedback

What is the Purpose of Tutorial 6?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
•   •   •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Notes

Notes

Notes

Quadratics

Key Stage 4 Programme

researchersinschools.org