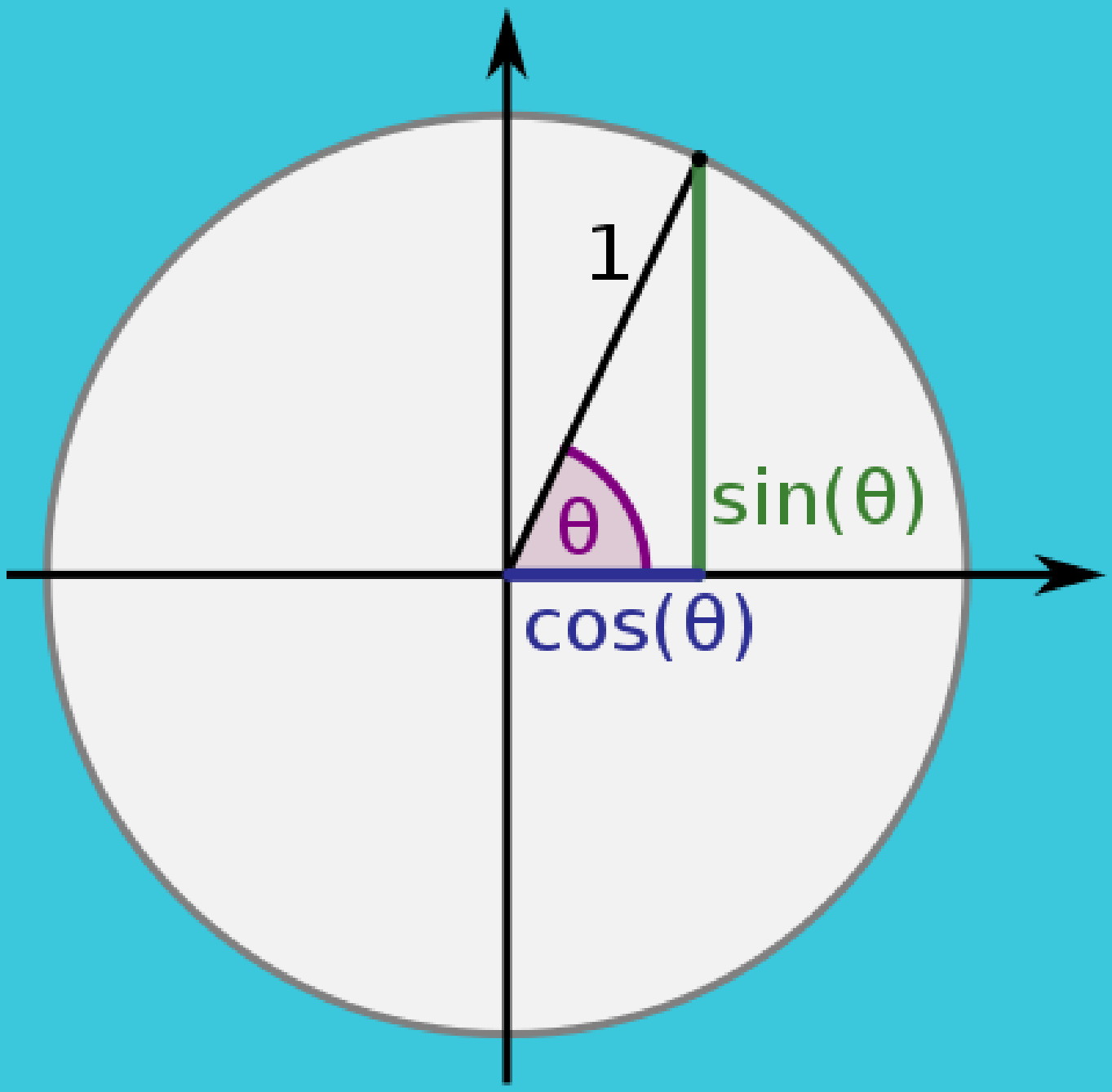

Trigonometry

Pupil Name

Handbook Designed by Dr Stone

Timetable

Timetable – Tutorials

Tutorial	Date	Time	Location
1 (Baseline)			
2			
3			
4			
5			
6 (Final Assignment)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

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Glossary of Keywords	P5
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Trig Ratios	P7
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Course Rationale

The aim of this course over the next few weeks is to take our knowledge of Trigonometry from a good level to a mastery level. We will be looking at a range of topics and questions within Trigonometry. By the end of the course the intention is that every possible GCSE question involve Trigonometry should be approachable to you.

Trigonometry is mentioned in the Edexcel GCSE mathematics specification in the following points:

R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

G20 know the formulae for: Pythagoras' theorem and the trigonometric ratios; apply them to find angles and lengths in right-angled triangles in two-dimensional figures and, where possible, general triangles in two and three dimensional figures

G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

G22 know and apply the sine rule and cosine rule to find unknown lengths and angles

G23 know and apply Area = ab sinC to calculate the area, sides or angles of any triangle

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function, exponential functions y = kx for positive values of k, and the trigonometric functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size.

A13 sketch translations and reflections of a given function

Subject Vocabulary

Word	Definition	In a sentence

Tutorial 1 – What are the Trigonometric Ratios?

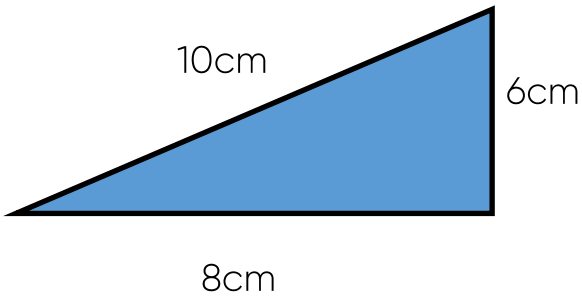
What is the Purpose of Tutorial 1?

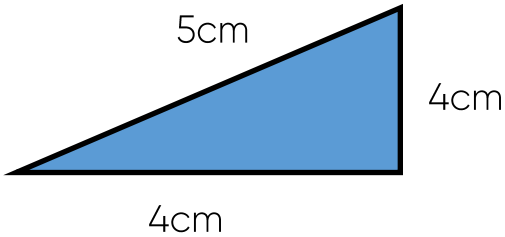
- To understand the definitions and origins of the three basic trig ratios
- To be able to apply the three trig ratios to 2D and 3D shapes

Hipparchus (c 190BC - 120 BC) was a Greek mathematician, astronomer and geographer. He is regarded as the founder of trigonometry and was the first to write down tables of trig values - he was doing this in the context of chord lengths within a circle

1.1 Similar Triangles

Two triangles are **similar** if they are exact scales of each other





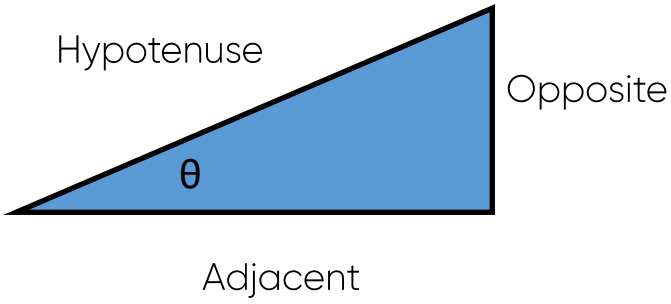
The **angles** within similar triangles must be equal.

On the other hand if you have two triangles with equal angles they must be similar.

For any given angle within a right angle triangle there is a **common ratio** between any two of the side lengths. These ratios are known as trigonometric ratios.

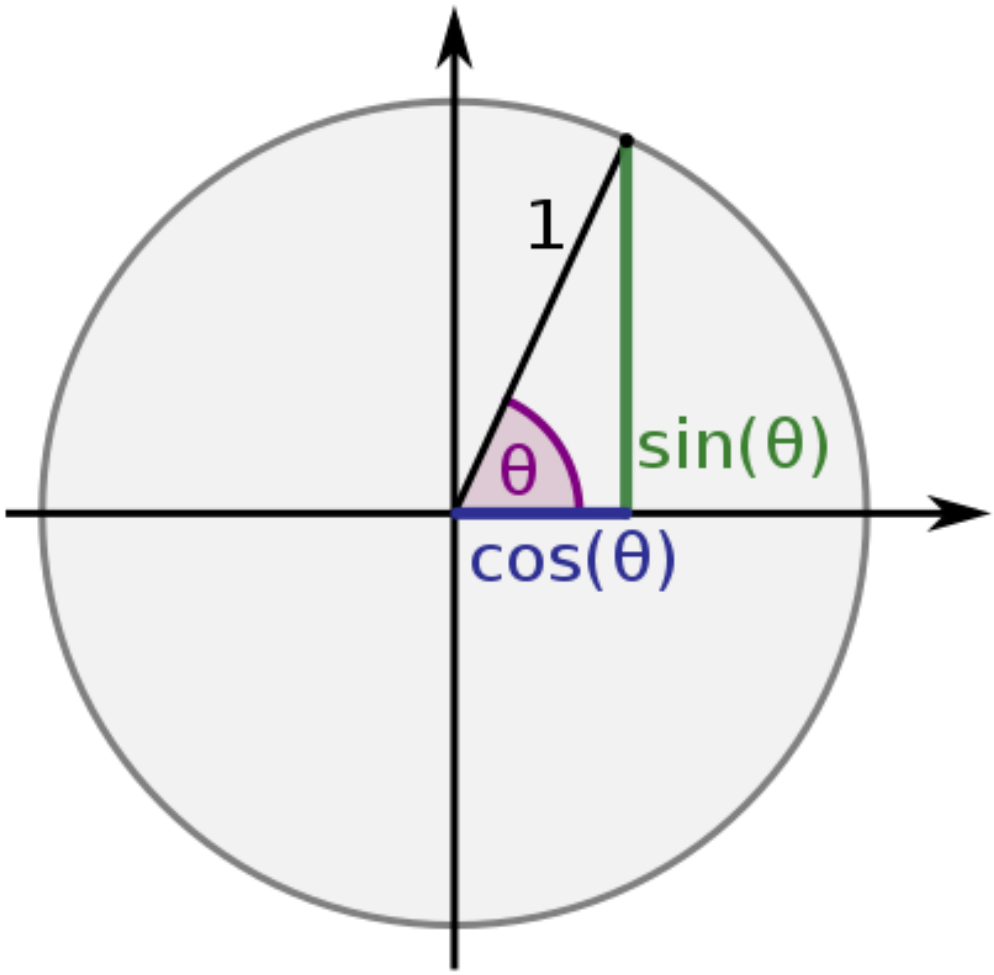
1.2 The Trigonometric Ratios

Identify one of the angles within a right angle triangle that's not the right angle. We label the sides of a right angle triangle

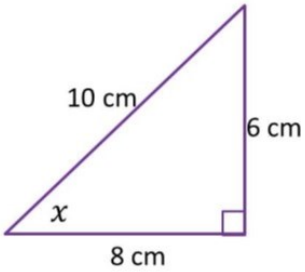


Since we know that the ratios between sides for similar triangles are the same, we give special names to these ratios (that depend on the angle).

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$



Write down the value of $\sin x$



A 0.6

B 0.75

C 0.8

D 1.7

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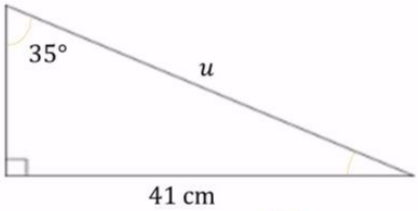
Which is the correct method to work out side u?



A $\sin(35) \times 41$

B $\dfrac{41}{\sin(35)}$

C $\dfrac{41}{\cos\ (35)}$

D $\cos(35) \times 41$

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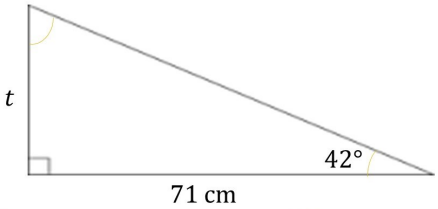
edexcel

Which is the correct method to work out side t?



A $\dfrac{\cos\ (42)}{71}$

B $\sin(42) \times 71$

C $\dfrac{\tan(42)}{71}$

D $\tan(42) \times 71$

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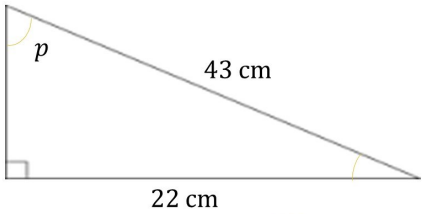
Which is the correct method to work out angle p?



A $\sin^{-1}(\frac{22}{43})$

B $\cos^{-1}(\frac{22}{43})$

C $\cos(\frac{22}{43})$

D $\sin(\frac{22}{43})$

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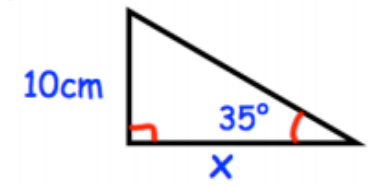
(a)



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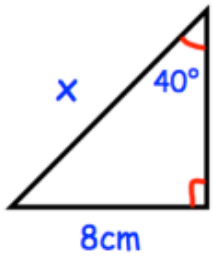

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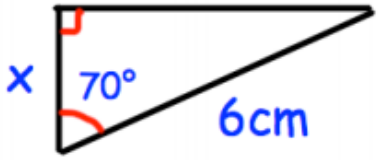

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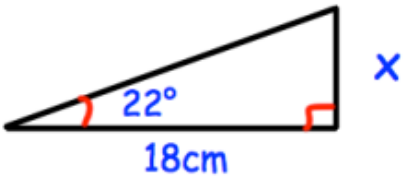

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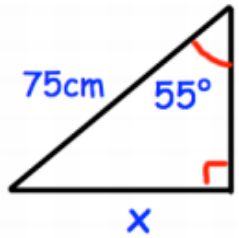

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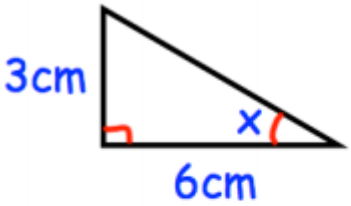

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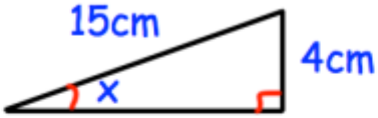

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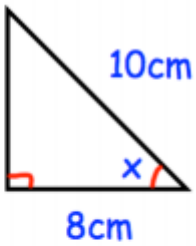

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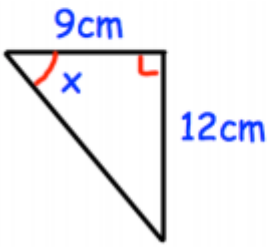

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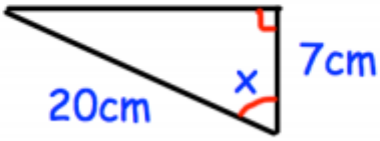

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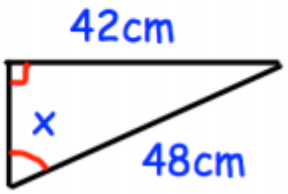
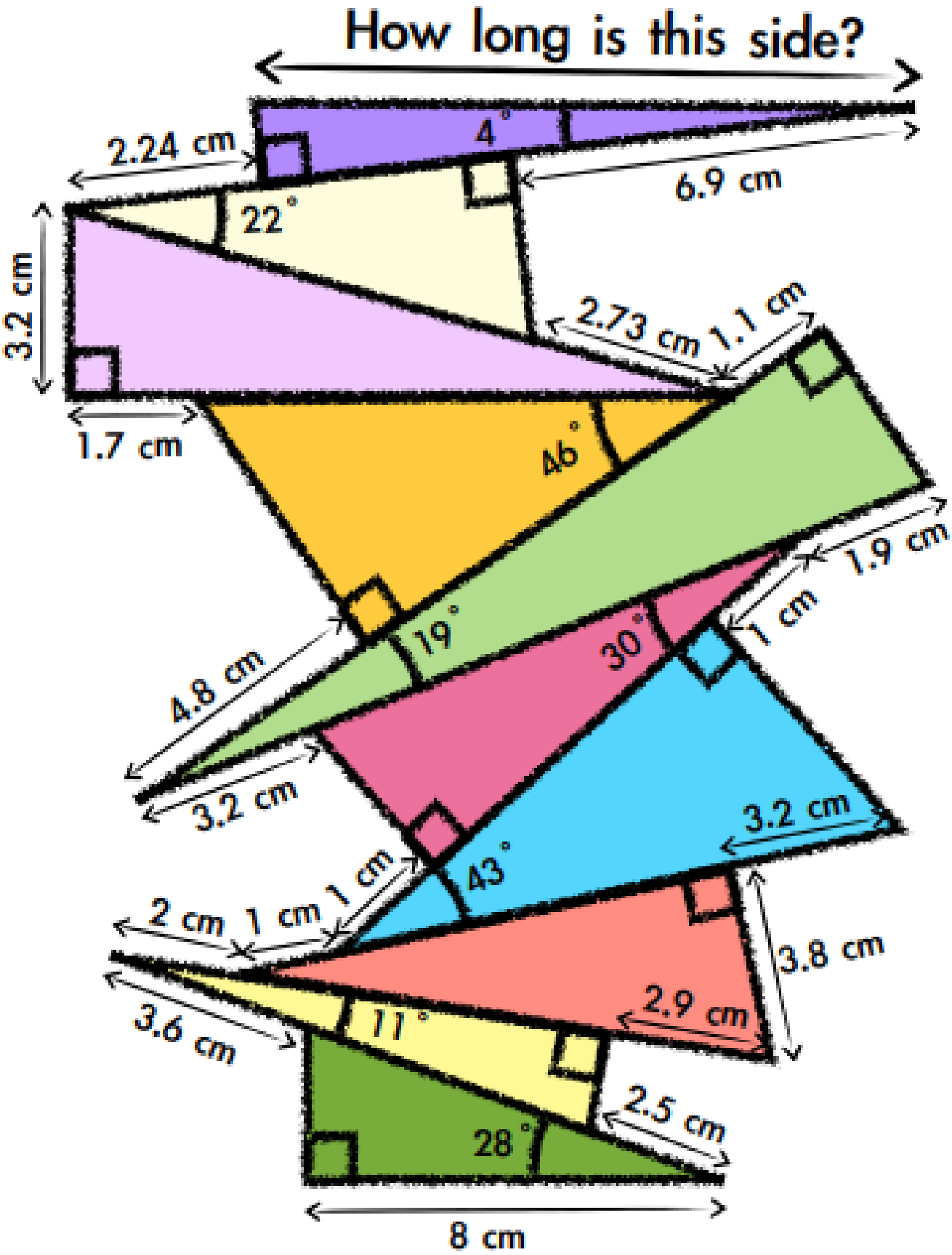


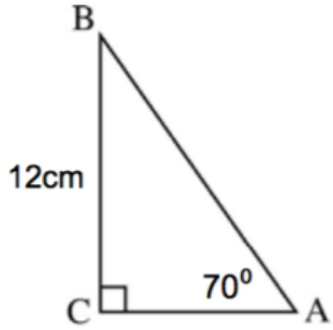

How long is this side?
2.24 cm
4°
6.9 cm
22°
3.2 cm
2.73 cm
1.1 cm
1.7 cm
46°
1.9 cm
1 cm
19°
30°
4.8 cm
3.2 cm
3.2 cm
43°
1 cm
2 cm
1 cm
3.8 cm
2.9 cm
3.6 cm
11°
2.5 cm
28°
8 cm

Question 5: (a) Find the length of AC.

(b) Find the length of AB.

(c) Find the perimeter of triangle ABC.

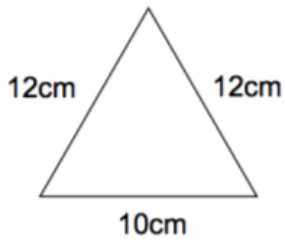
(d) Find the area of triangle ABC.



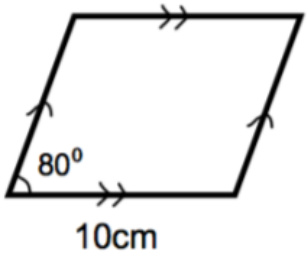
Question 6: A helicopter leaves A and flies 40 miles due east. Then the helicopter flies 10 miles due south and arrives at B. Work out the bearing of B from A.

Question 7: A boat leaves a port and sails 55km due west and then 30km due north and arrives at an oil rig. What is the bearing of the oil rig from the port?

Question 8: Shown is an isosceles triangle. Calculate its area.



Question 9: Shown is a rhombus of side length 10cm. Calculate its area.



Question 10: Can you spot any mistakes in the question below?

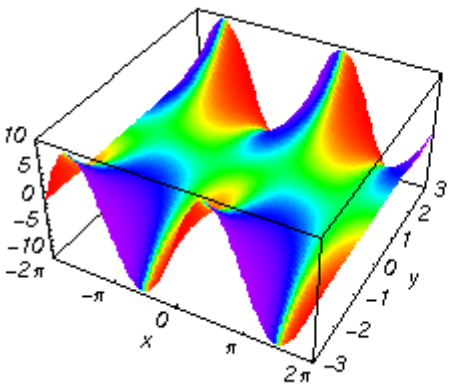
Notes

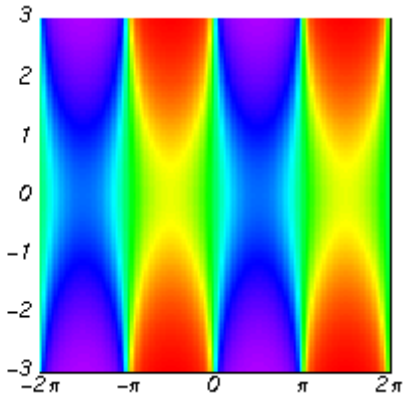
Notes

Tutorial 2 – Exact Values and Graphs

What is the Purpose of Tutorial 1?

- To know the common exact values of trig functions
- To be able to memorise these and to know how to compute them
- To be able to plot the graphs of trig functions
- To be able to perform graph transformations on trig graphs





Trig functions can be extended and defined on complex numbers - giving these interesting looking graphs.

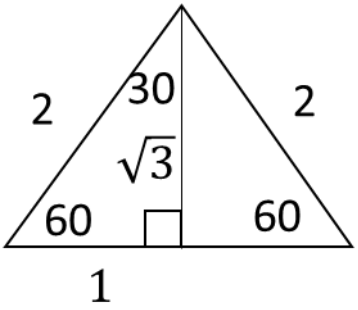
1.1 Exact Values

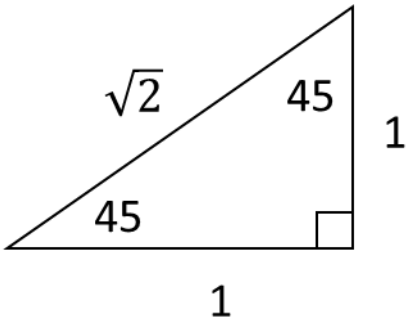
Certain values of sin, cos and tan give exact answers. You need to memorise these.

	0	30	45	60	90
Sin					
Cos					
Tan					

	0	30	45	60	90
Sin					
Cos					
Tan					

These exact values of the trig ratios come from two specific triangles - an equilateral triangle with side length 2 and an isosceles triangle with the two equal sides length 1 .





You also need to use the fact that

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Using the above triangles complete the table

	0	30	45	60	90
Sin					
Cos					
Tan					

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What is the exact value of $\tan(30)$?

A $\frac{\sqrt{3}}{3}$

B $\sqrt{3}$

C $\frac{\sqrt{3}}{2}$

D $\frac{1}{2}$

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Which of the following does **not** have a value of 1?

A $\sin(90)$

B $\tan(90)$

C $\cos(0)$

D $\cos(360)$

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Which of the following has a value of 1?

A $\sin(0)$

B $\tan(0)$

C $\cos(0)$

D $\cos(90)$

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I= Eedi

Which of the following has a value of 0?

A $\sin(90)$

B $\tan(90)$

C $\sin(270)$

D $\cos(90)$

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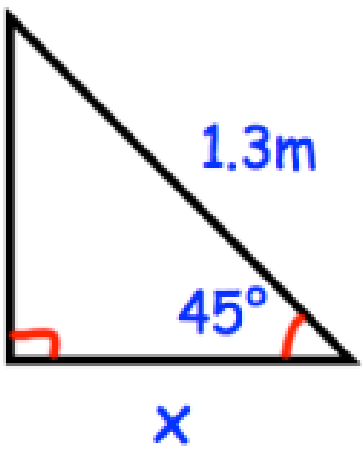
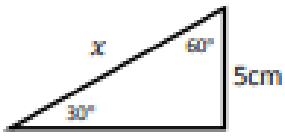
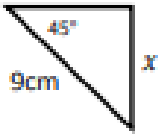
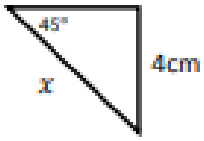
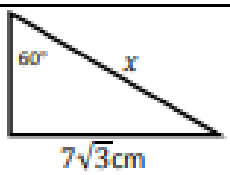
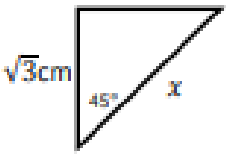
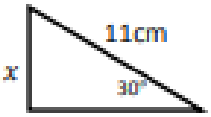
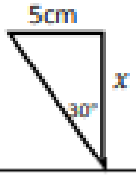
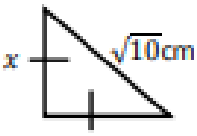
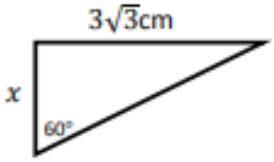
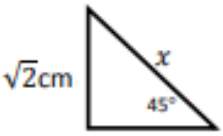
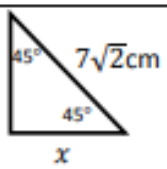
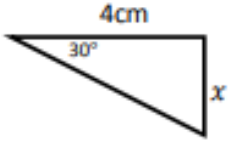
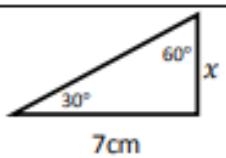
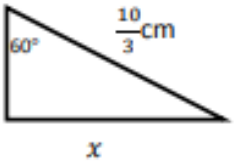


Trigonometry – Using Exact Values

In each triangle below find the exact value of x then put the letter of the question below its corresponding answer to discover a message.

A		R	
G		S	
I		T	
K		U	
M		W	
N		Y	
O		!	

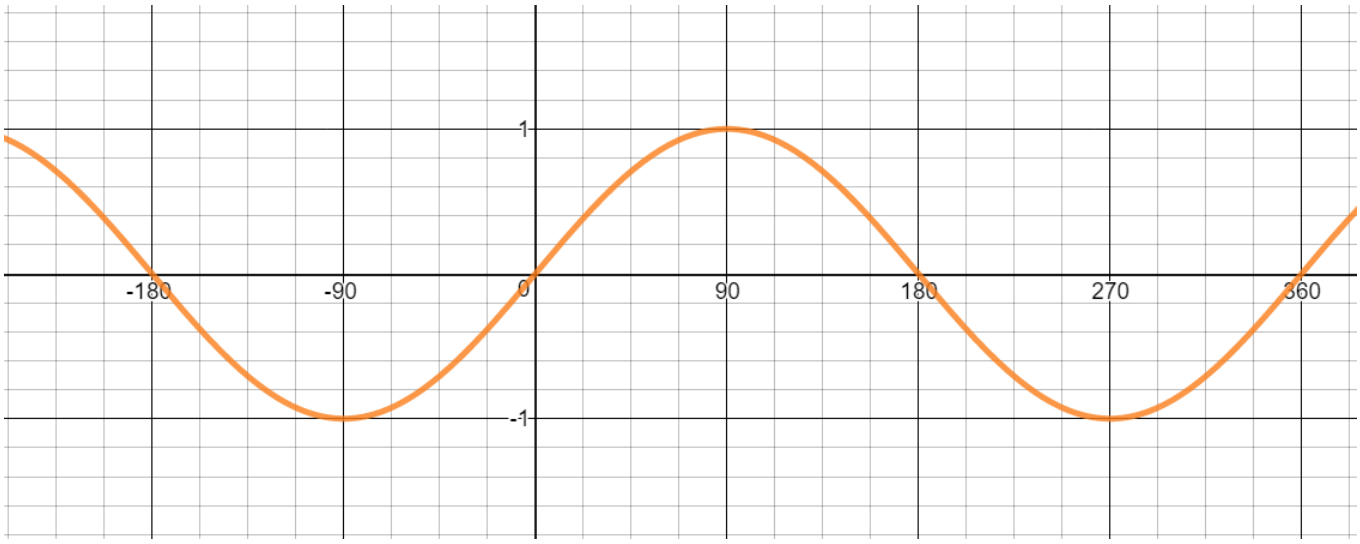
$\frac{4\sqrt{3}}{3}$	$\frac{7\sqrt{3}}{3}$	$\sqrt{5}$		3	$\sqrt{5}$	14	$\frac{11}{2}$		$5\sqrt{3}$	7	$\frac{7\sqrt{3}}{3}$	2		$\frac{4\sqrt{3}}{3}$	$\frac{7\sqrt{3}}{3}$	$\sqrt{5}$	$\frac{9\sqrt{2}}{2}$

$\frac{11}{2}$	$\frac{9\sqrt{2}}{2}$	$\sqrt{6}$	$4\sqrt{2}$		$\frac{9\sqrt{2}}{2}$	10	$\frac{11}{2}$	$\sqrt{6}$	$\frac{7\sqrt{3}}{3}$	14	$\frac{5\sqrt{3}}{3}$

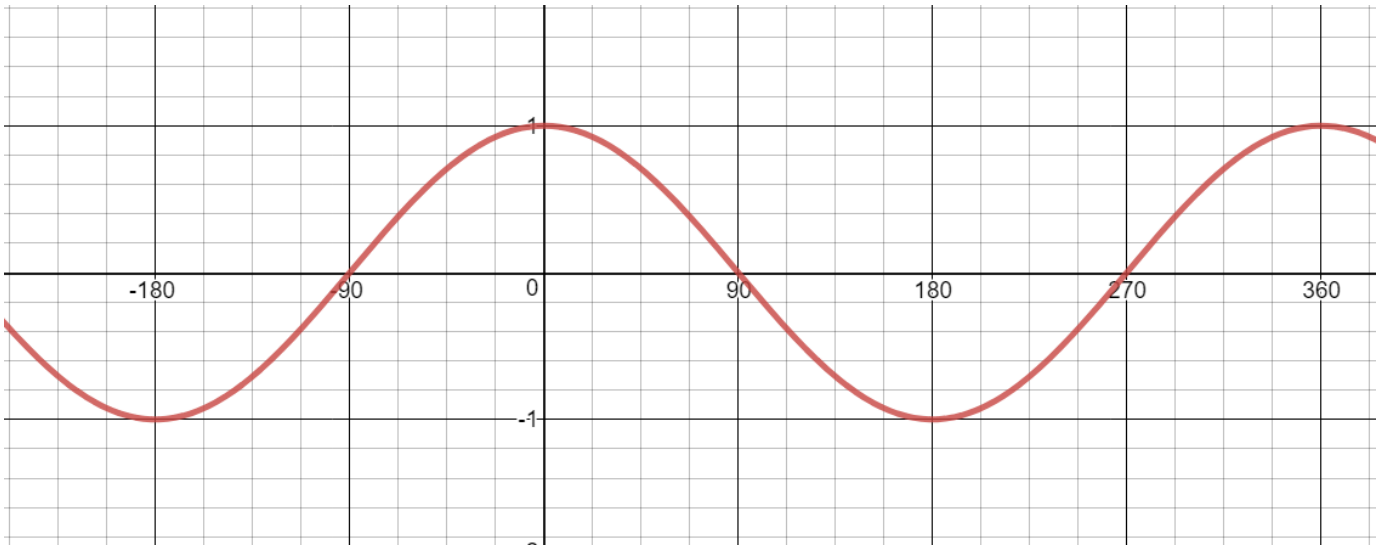
2.2 Graphs of Trig Functions

You need to be able to plot, recognise the graphs of trig functions.

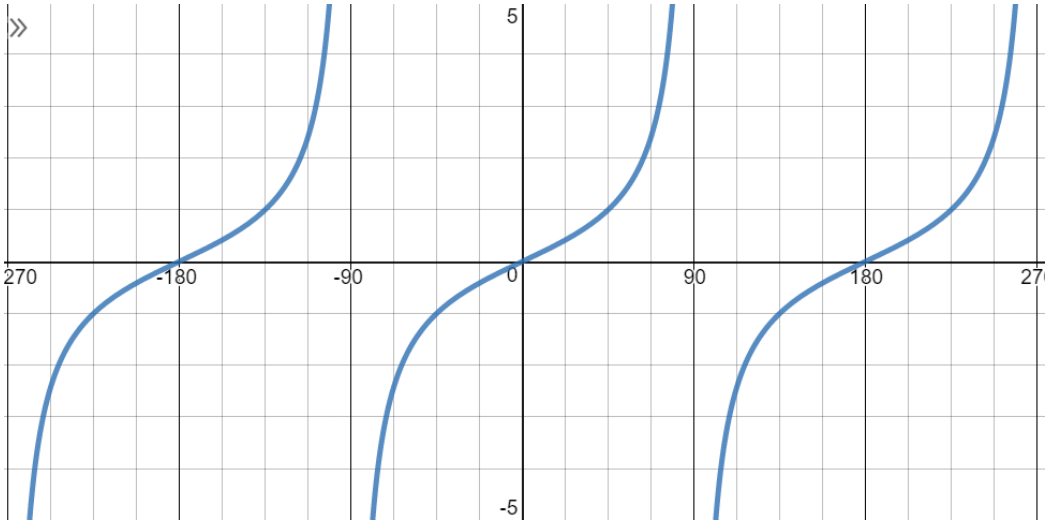
Sin



Cos



Tan



Complete the table and plot the graphs

Sine

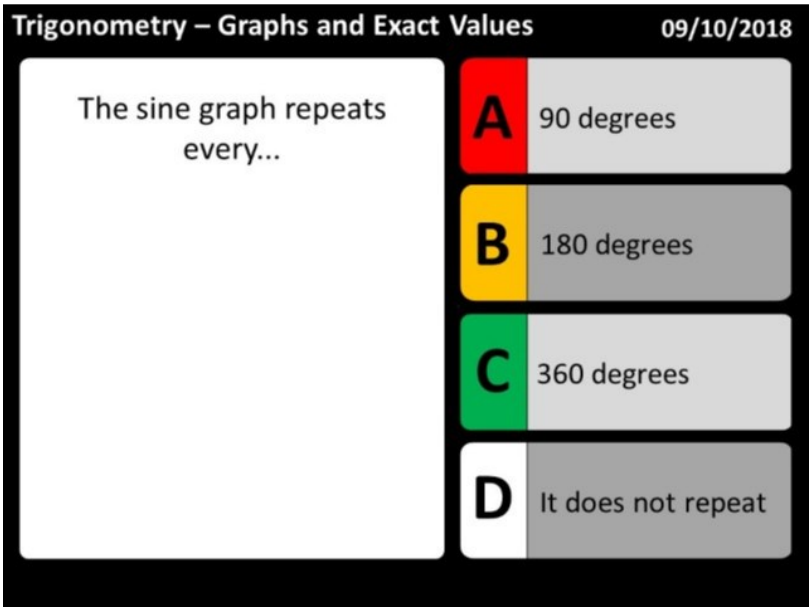
Θ(^{0})	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420
Sin(Θ)	0	0.5		1			0			-1				0.5	

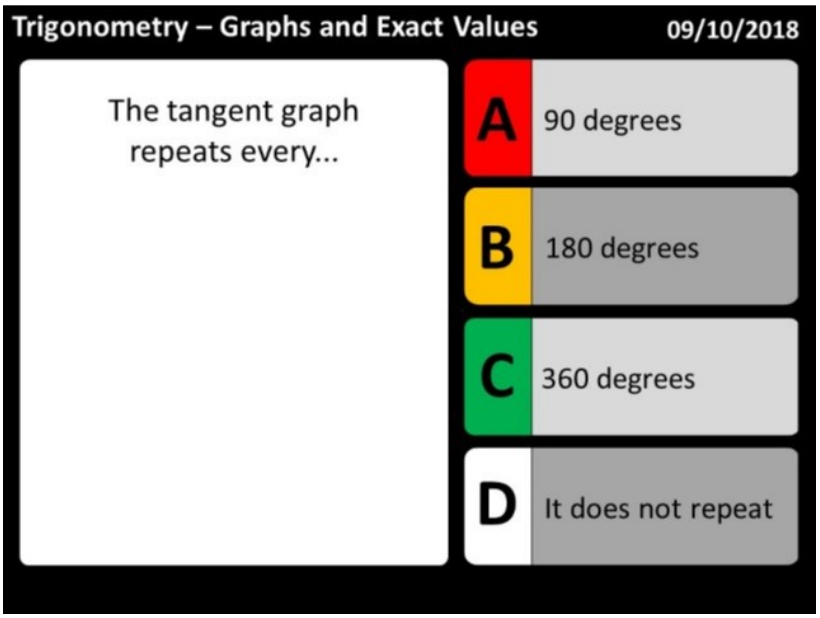
Cosine

Θ	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420
Cos(Θ)	1									0	0.5				

Tangent

Θ	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420
Tan(Θ)															


Trigonometry – Graphs and Exact Values
09/10/2018
The sine graph repeats every...
A
90 degrees
B
180 degrees
C
360 degrees
D
It does not repeat


Trigonometry – Graphs and Exact Values
09/10/2018
The tangent graph repeats every...
A
90 degrees
B
180 degrees
C
360 degrees
D
It does not repeat

2.3 Transformations

You need to be able to perform transformations of graphs, and recognise graph transformations. Including multiple transformations. You need to understand the following transformations,

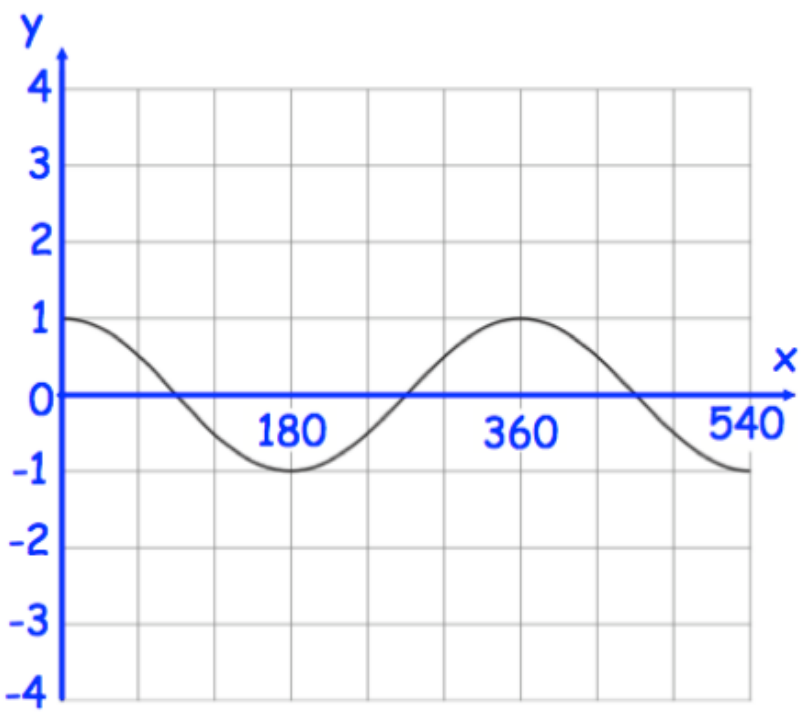
$$y = f(x)$$

$$y = f(x) + a$$

$$y = bf(x)$$

$$y = f(x + c)$$

Shown below is the graph of y = cos x



On the grid, sketch the graph of y = 3 + cos x for values of x from 0° to 540°

edexcel

Let $f(x) = \sin(x)$

Point P lies at $(180, 0)$ on the graph of $y = \sin(x)$

If the graph is transformed to $y = f(x + 30)$, what are the new coordinates of point P?

A $(210, 30)$

B $(150, 0)$

C $(210, 0)$

D $(180, 30)$

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edexcel

Let $g(x) = \cos(x)$

Point Q lies at $(180, -1)$ on the graph of $y = \cos(x)$

If the graph is transformed to $y = g(-x) + 1$ what are the new coordinates of point Q?

A $(-180, 0)$

B $(180, 2)$

C $(180, -2)$

D $(-180, -2)$

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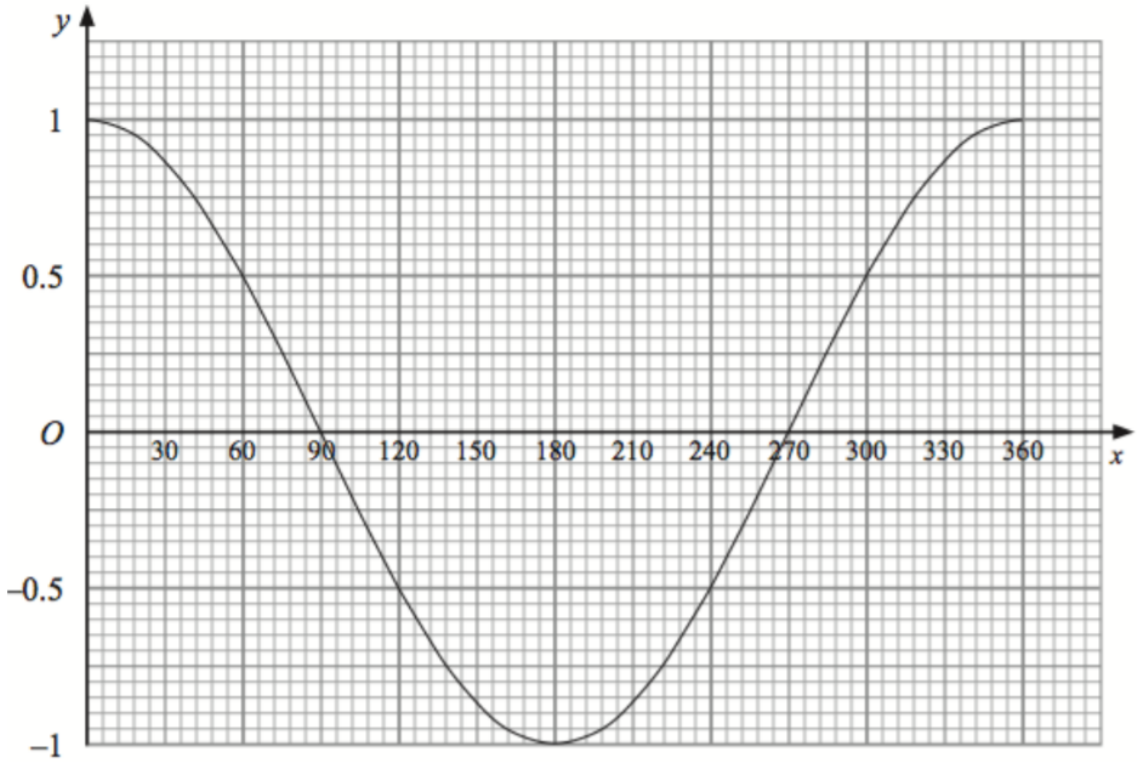
13. Shown below is the graph of y = cos x



On the grid, sketch the graph of y = cos(x – 90°) for values of x from 0° to 360°

Describe the transformation that maps the curve with equation y = sin(x) onto the curve with equation

(a) y = –sin(x)

.....

.....

(2)

(b) y = 1 + sin(x)

.....

.....

(2)

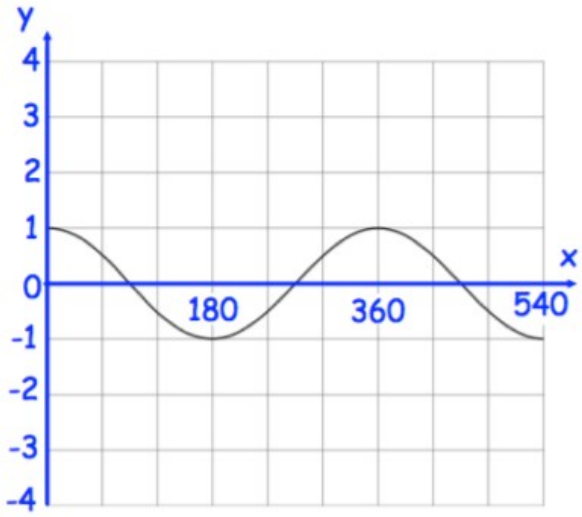
(c) y = sin(x – 30°)

.....

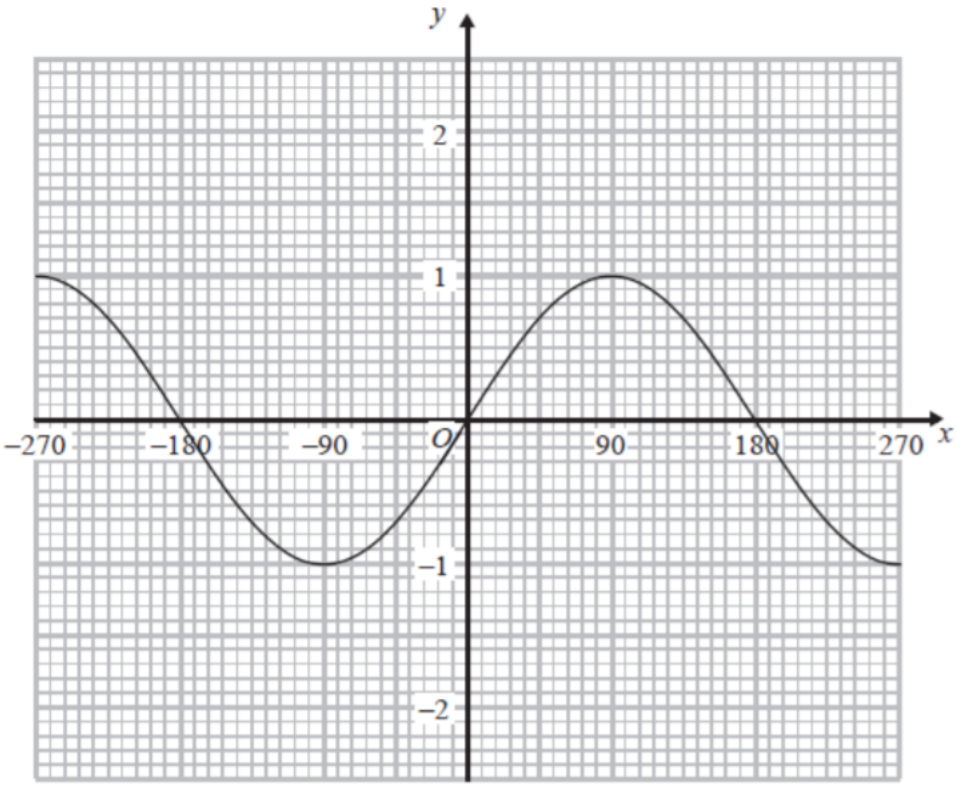
.....

(2)

Shown below is the graph of y = cos x



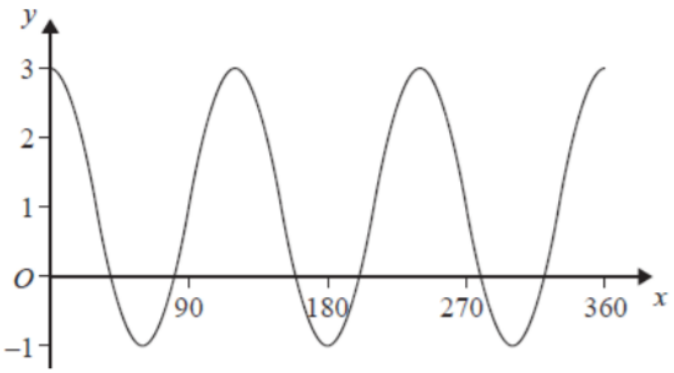
On the grid, sketch the graph of y = 2 – cos(x) for values of x from 0° to 540°



The graph of $y = \sin x°$ for values of x from –270 to +270 is shown above.

(b) On the same axes, draw the graph of $y = 1 - \sin x°$ for values of x from –270 to +270

Here is a sketch of the curve $y = a \cos bx° + c$, $0 \leq x \leq 360$



(c) Find the values of a, b and c.

$a =$ .....

$b =$ .....

$c =$ .....

(3)

Notes

Notes

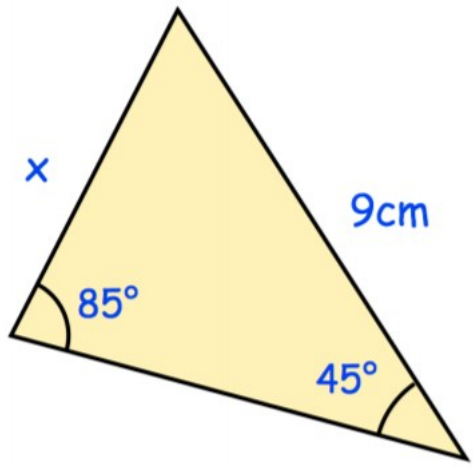
Tutorial 3 Sin and Cos Rule

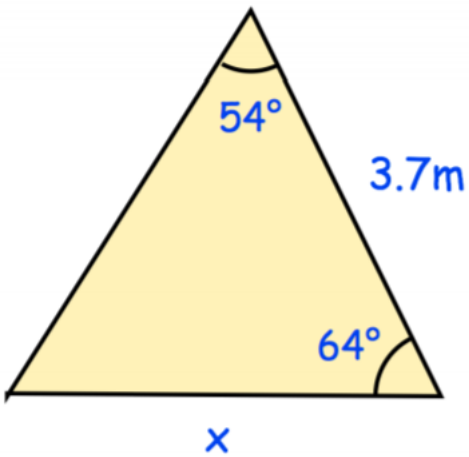
What is the Purpose of Tutorial 1?

- To know the sin and cos rule, and the sin formula for the area of a triangle
- To be able to apply these in a variety of 2d shapes

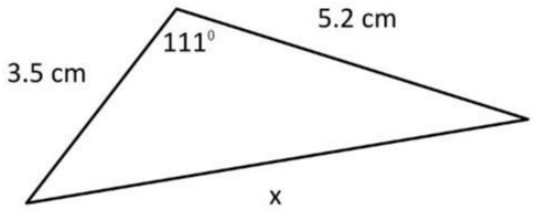
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$





Find the length of the side x

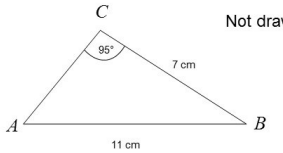


A	B	C	D
52.3 cm	7.23 cm	6.77 cm	5.12 cm

AQA
Realising potential

Which of these is correct for this triangle?

Not drawn accurately



A	B	C	D
$11 \sin 95° = 7 \sin A$	$\frac{\sin 95°}{11} = \frac{A}{7}$	$\frac{11}{\sin 95°} = \frac{7}{A}$	$\frac{\sin 95°}{11} = \frac{\sin A}{7}$

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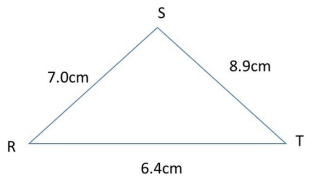
!! NOT TO SCALE !!



What is the correct value of angle RST to two decimal places?

A	B	C	D
51.34°	44.45°	45.55°	0.70°

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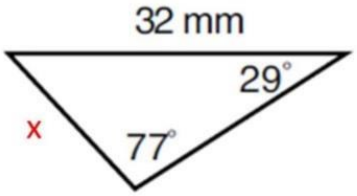
What is the most appropriate rule to use to find the missing value?



A Sine Rule

B Cosine Rule

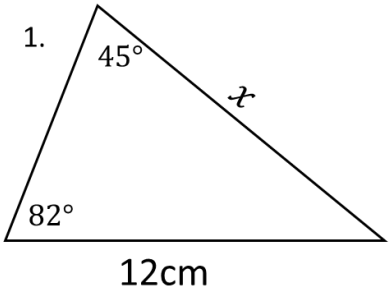
C SOHCAHTOA

D Pythagoras

Practice using the Sine Rule

Sine Rule: Missing Sides

1.



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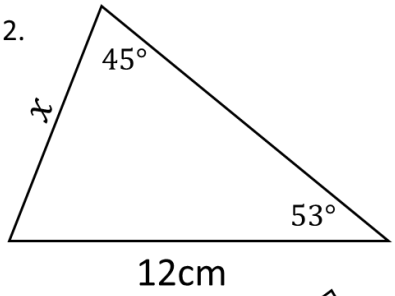

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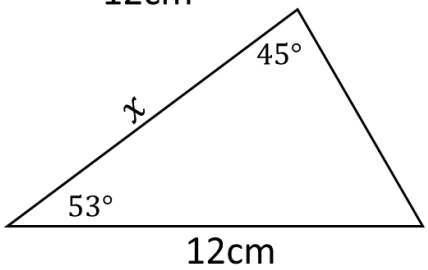

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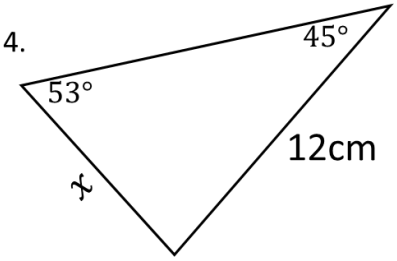

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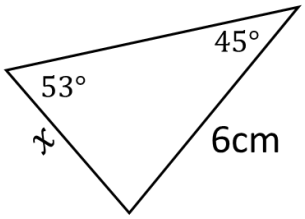

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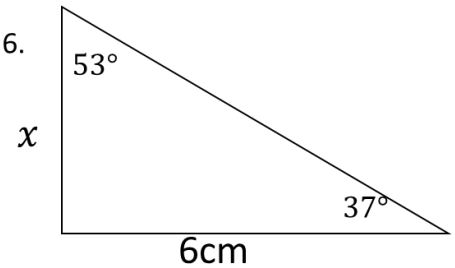

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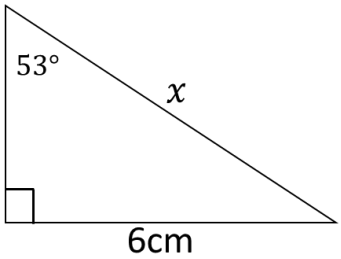

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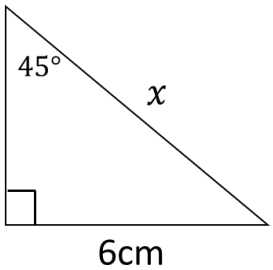
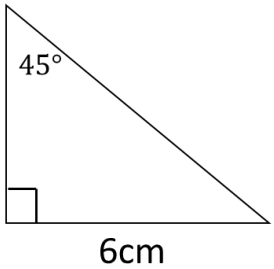
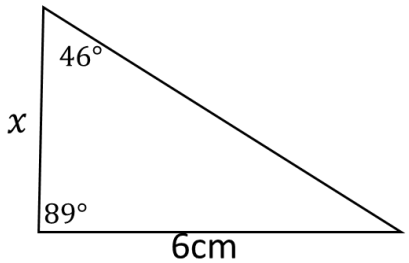


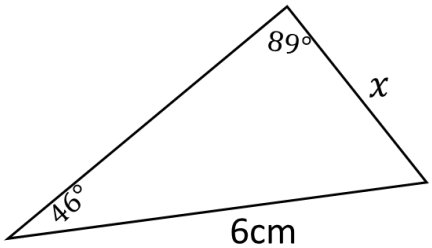
9.



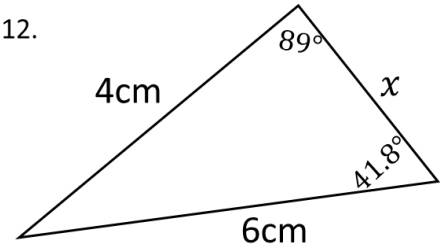
10.



11.

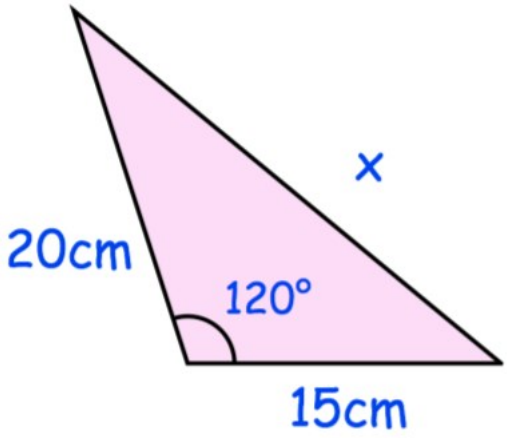


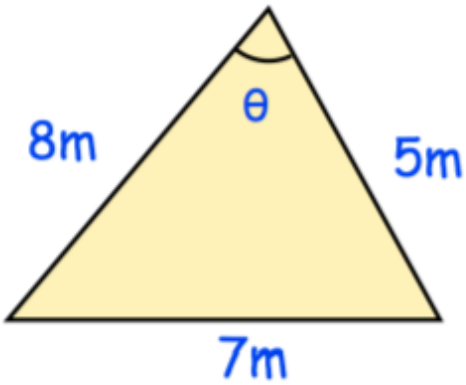
12.



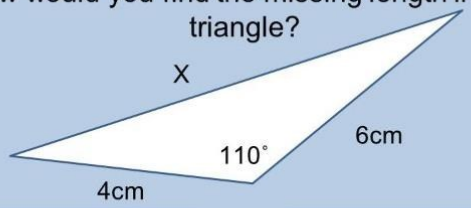
The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$





How would you find the missing length in this triangle?



A) Sine Rule

B) SOHCAHTOA

C) Pythagoras' Theorem

D) Cosine Rule

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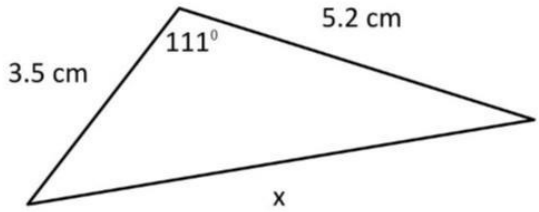
Find the length of the side x



A 52.3 cm

B 7.23 cm

C 6.77 cm

D 5.12 cm

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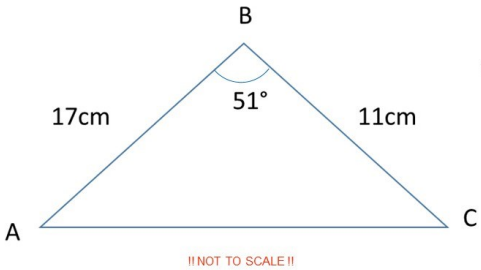

What is the correct value of the length AC to three significant figures?

A $13.215cm$

B $10.9cm$

C $13.2cm$

D $175cm$

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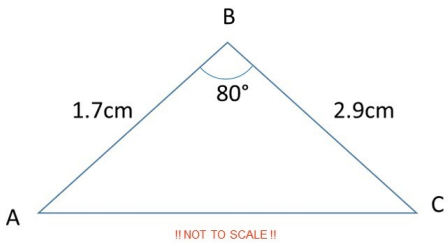

What is the correct value of the length AC to three significant figures?

A $9.59cm$

B $3.10cm$

C $3.096cm$

D $1.26cm$

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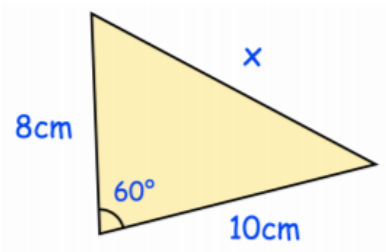
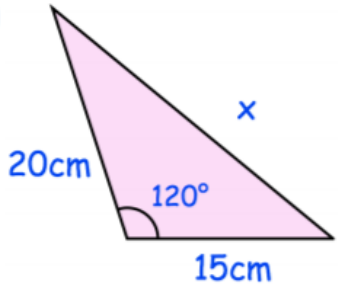
Question 1: Find x for each of the triangles below.

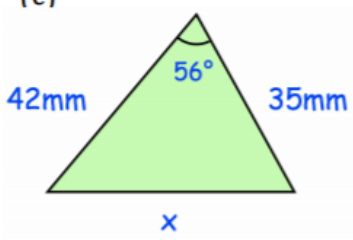
(a)



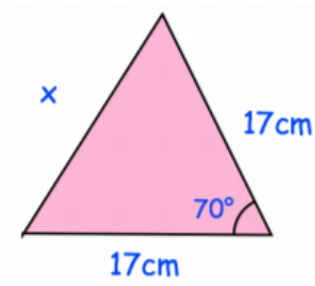
(b)



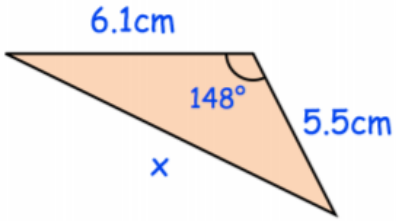
(c)



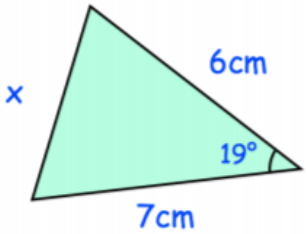
(d)



(e)

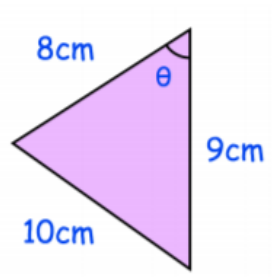


(f)

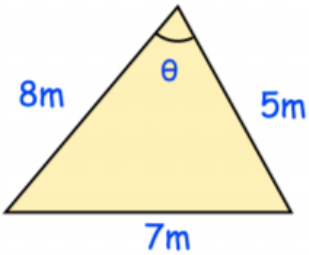


Question 2: Find the size of θ for each of these triangles.

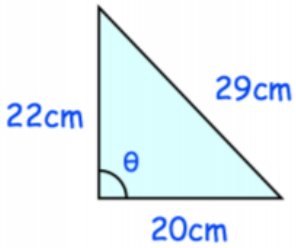
(a)



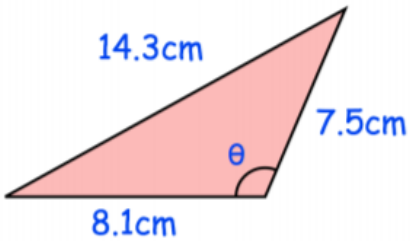
(b)



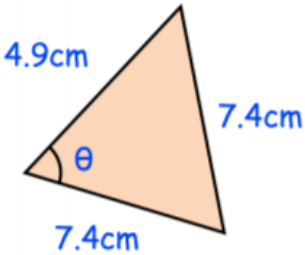
(c)



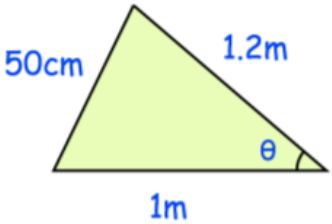
(d)



(e)



(f)

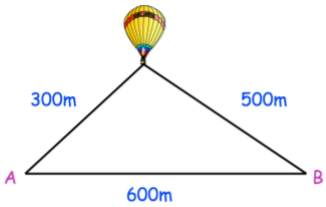


A hot air balloon is flying above two point, standing on the ground at points A and B, 600m apart.

The hot air balloon is 300m from A and 500m from B.

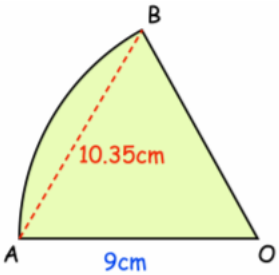
(a) Work out the angle of elevation from point B

(b) How high is the hot air balloon from the ground?



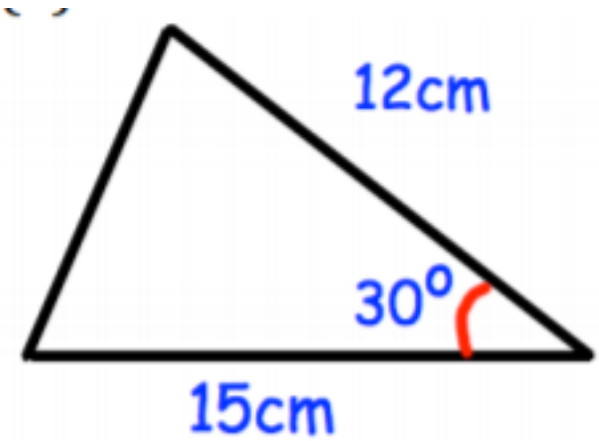
Shown is sector OAB.
O is the centre of the circle with radius 9cm
A and B are points on the circle.
The length of the chord AB is 10.35cm

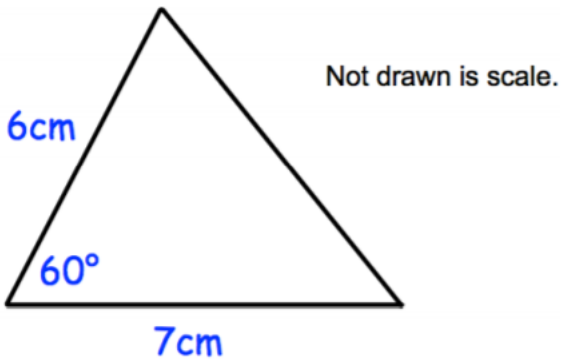
Find the area of sector OAB



The Sine Rule for area:

$$Area = \frac{1}{2}ab\sin C$$





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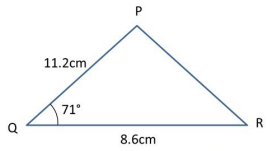
What is the correct value for the area of triangle PQR to two decimal places?



!! NOT TO SCALE !!

A 45.54cm^2

B 48.16cm^2

C 45.80cm^2

D 91.07cm^2

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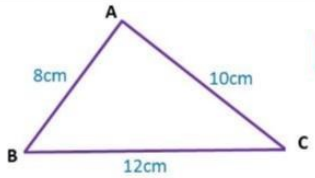
Follow up Questions for Edexcel GCSE – June 2014 – Higher – Paper 2 (Calc) – Question 27

Find the area of the triangle ABC



Step 1

What formula is the best to use to get us started:

A) Sine Rule

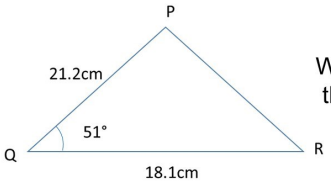
B) Cosine Rule

C) Area of a Triangle = ½ ab sin(C)

D) Area of a Triangle = ½ base x height

edexcel

!! NOT TO SCALE !!



What is the correct value for the area of triangle PQR to one decimal place?

A $149.1cm^2$

B $191.9cm^2$

C $128.6cm^2$

D $298.2cm^2$

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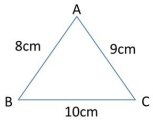
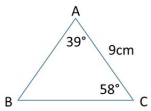
edexcel

The formula for the area of a non right-angled triangle is $\frac{1}{2}ab\sin C$. Which of these triangles can directly use this formula to find the area of the triangle?

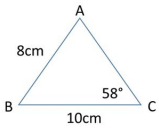
A



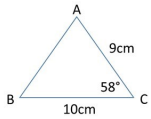
B



C



D



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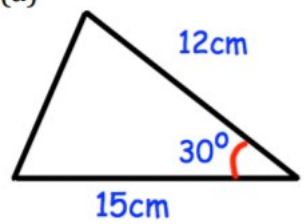
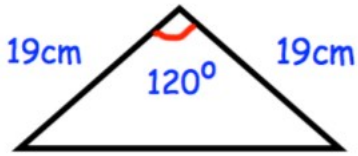
Question 1: Find the area of each of these triangles.

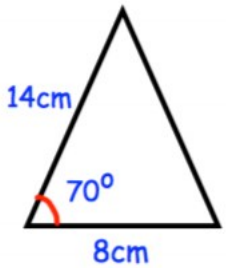
(a)



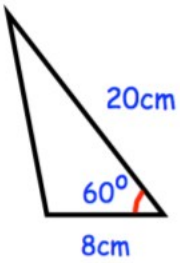
(b)



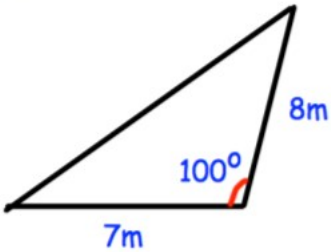
(c)



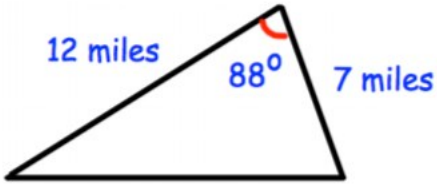
(d)



(e)

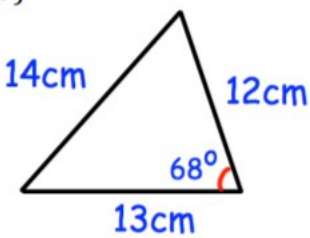


(f)

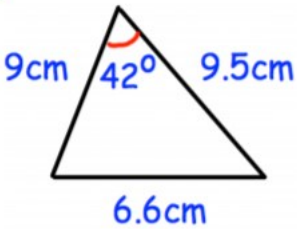


Question 2: Find the area of each of these triangles.

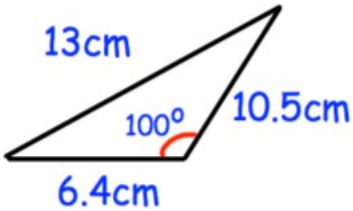
(a)



(b)



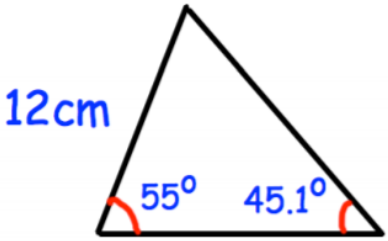
(c)



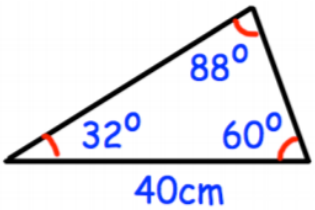
Question 1: Find the area of these triangles.

(a)

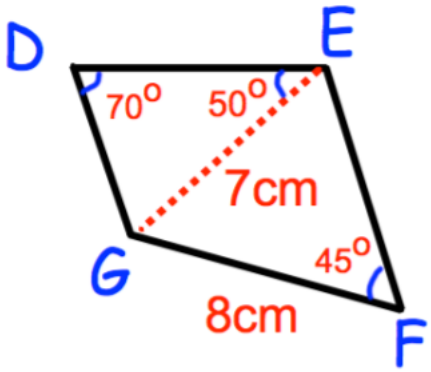
(b)



(c)



Question 4: Find the area of quadrilateral DEFG.



Notes

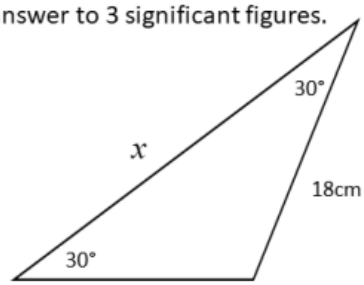
Notes

Tutorial 4 More Involved Trig

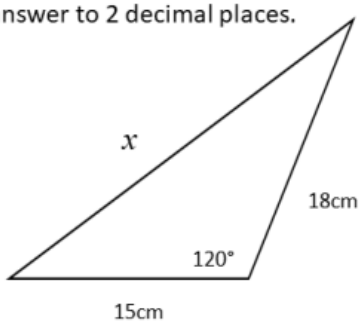
What is the Purpose of Tutorial 4?

- To be able to identify the correct approach to trig problems
- To be able to approach 3D trig problems

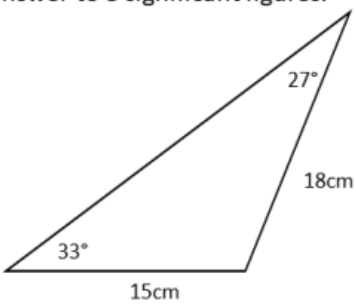
Find the length of the marked side.
Give your answer to 3 significant figures.



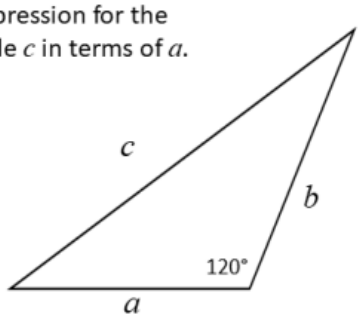
Find the length of the marked side.
Give your answer to 2 decimal places.

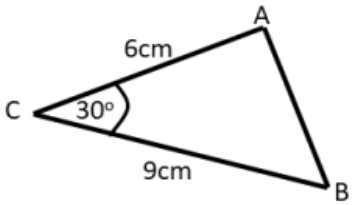


Find the area of the shape below.
Give your answer to 3 significant figures.

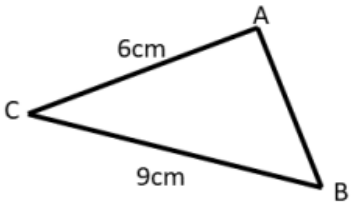


$a:b = 3:5$
Write an expression for the length of side c in terms of a.

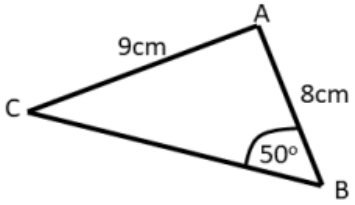




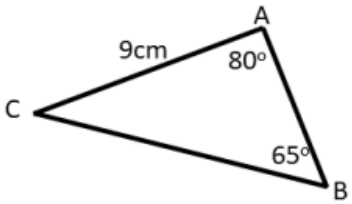
Find the length of AB to 2dp.



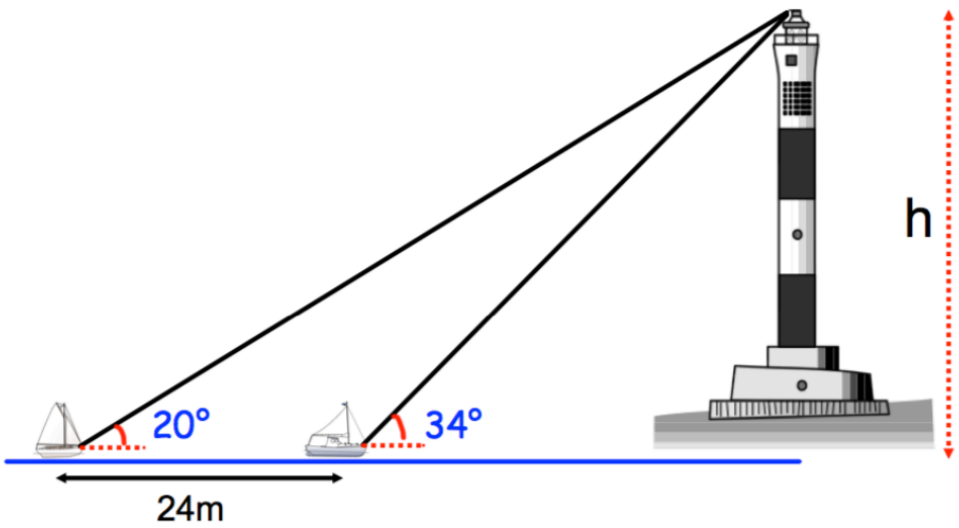
The area of the triangle is 9.23cm^2 .
Find the size of angle ACB to 3sf.



Find the size of angle BAC to 1dp.



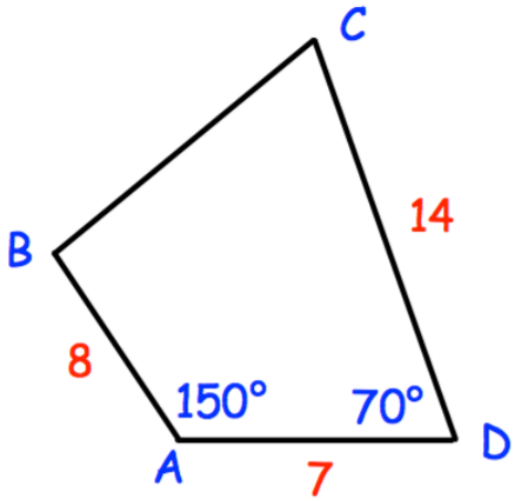
Find the perimeter of the triangle to 2dp.



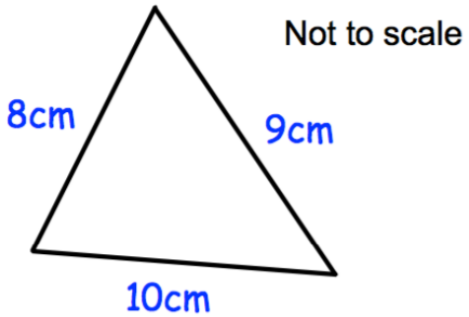
Two small boats are 24m apart.
The angle of elevation of the boats to the top of a lighthouse are 20° and 34° respectively.

Calculate the height of the lighthouse.

In a quadrilateral ABCD, AD = 7cm, AB = 8cm and CD = 14cm.
Angle BAD = 150° and Angle ADC = 70°



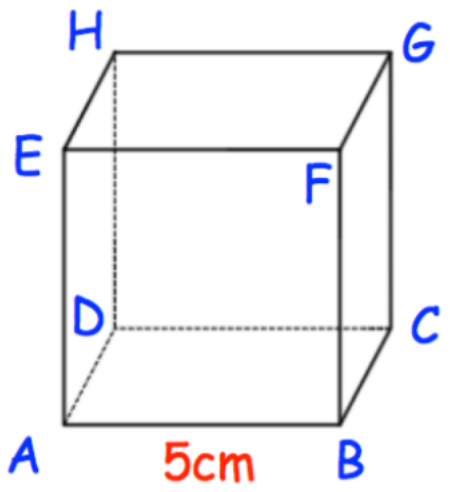
Calculate the length BC.



Find the area of the triangle.

3D Trig problems

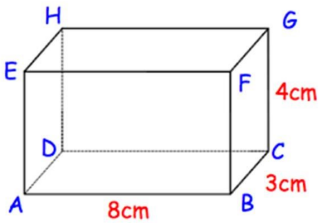
Calculate the value of the angle CAG



I= Eedi

A question asks you to calculate the angle between the line FD and the plane ABCD.

Which angle is this?



A FAD

B FDA

C FDC

D FDB

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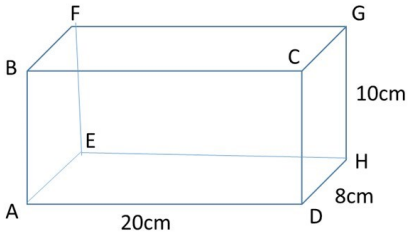

Which of the following is not a right-angled triangle?

A FCD

B GCA

C HAF

D CHE

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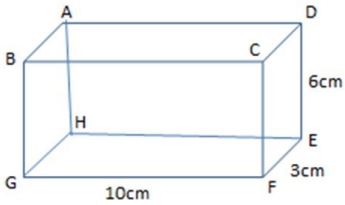
Calculate the size of the angle between the base EFGH and line DG.



!! NOT TO SCALE !!

A 16.7°

B 29.9°

C 3.2°

D 35.1°

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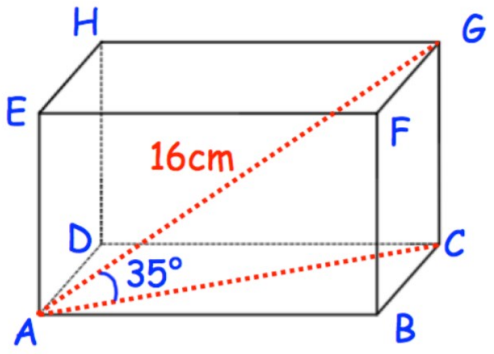
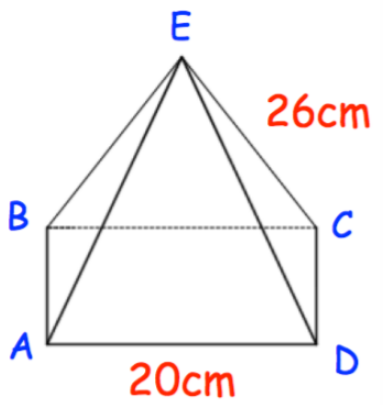
Shown below is a cuboid.



Length AG = 16cm
Angle CAG is 35°

Work out the length of EG.

Shown below is a square based pyramid.
The apex E is directly over the centre of the base.



AD = 20cm
CE = 26cm

(a) Work out the length of AC

.....cm
(2)

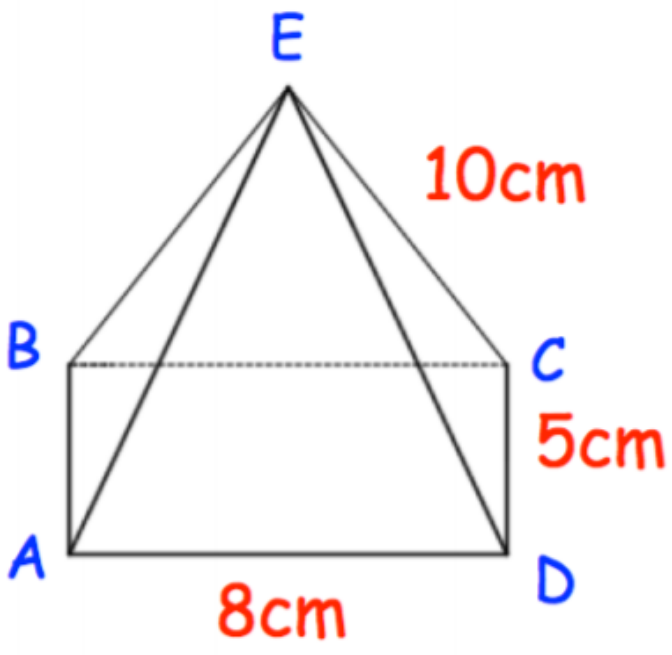
(b) Calculate angle CAE

..... °
(2)

(c) Work out the height of the pyramid

.....cm
(2)

Shown below is a rectangular based pyramid.
The apex E is directly over the centre of the base.



AD = 8cm
CD - 5cm
CE = 10cm

(a) Calculate the height of the pyramid

.....cm
(4)

(b) Calculate angle between the face ABE and the base ABCD

..... °
(3)

Notes

Notes

Tutorial 6 – Revision Techniques/Final Assignment

Notes

Notes

Tutorial 7 – Feedback tutorial

What is the Purpose of Tutorial 7?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.

To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well...	What I could have improved on...

My target for future work is...

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?

What did you find challenging about the programme?	How did you overcome these challenges?

Notes

Notes

Notes

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