Surds

Key Stage 4 Revision Programme

Pupil Name

Handbook Designed by Jacopo Scalise

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1 (Baseline assessment)			
2			
3			
4			
5			
6 (Final assessment)			
7 (Feedback)			

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Course Rationale

The aim of this course is to engage with lots of different problems involving **surds**, in order to reach a point in which you really master the topic. The skills needed to manipulate these special number are numerous and complex: you need to be able to expand brackets, understand deeply the meaning of roots and how they work, rationalise denominators and deal with roots emerging in geometric problems. In fact, the topic is famous for being largely misunderstood by students, who very frequently fall into surd traps in GCSE questions. Indeed, many students think of surds as an isolated topic, with its own specific questions – instead, surds are just a way to represent a number, a bit like fractions, and thus they might appear in a huge variety of questions, like this one:

The mean (average) of $\sqrt[3]{24}$ and $\sqrt[3]{192}$ is $\sqrt[3]{n}$. Find the value of n.

So, how are we going to train to be surd masters? Our main focus will be practice: indeed, research proves that the best way to permanently acquire a skill is to practice it in a lot of different contexts.
It is important to understand that we are not trying to solve every possible question that could come up in the GCSE papers: we will instead train to cope with unexpected questions and problems that might appear. We are going to work like orienteering experts: by training in a lot of different places, you will be able to find your way in completely new ones!

Subject Vocabulary

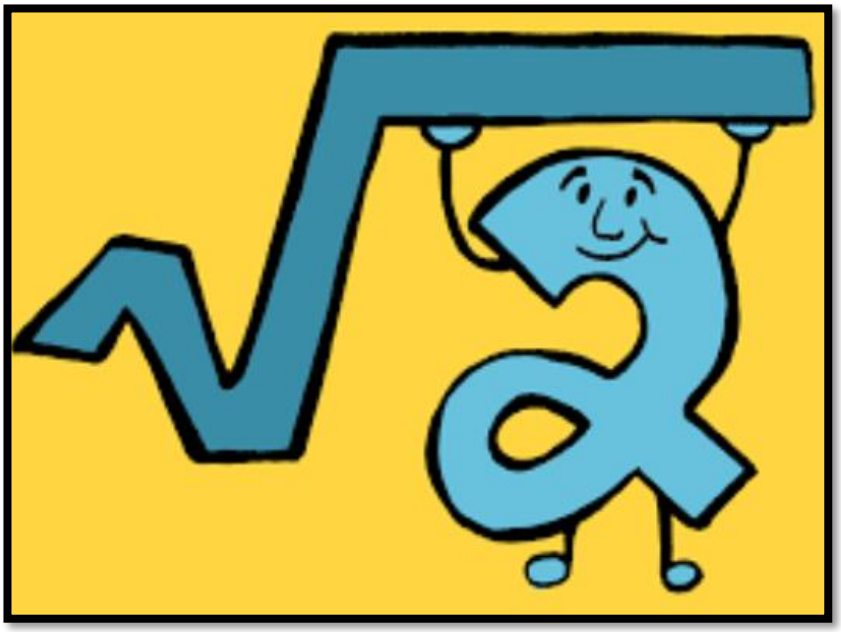
Word	Definition	In a sentence
Surd		
Rational/irrational		
Simplify		
Rationalise		
Fractional index		

Tutorial 1 - Baseline Test

What is the Purpose of Tutorial 1?

- Pre-SEF
- Baseline assessment

Tutorial 2 – Intro to surds



What is the Purpose of Tutorial 2?

- Objective A: to distinguish a surd and understand the basic surd notation.
- Objective B: to simplify a surd expression by adding like terms.
- Objective C: to divide and multiply two simple surds.

OA1: WHAT IS A SURD?
A surd is a number which features a square (or cube, fourth…) root. Examples are: $\sqrt{2}$, $\sqrt[3]{5}$, $1-\sqrt{71}$. If you plug these numbers in your calculator, you will get decimal values, like $\sqrt{2}=1.4142857$. It is important to understand that these decimals are not the **exact** values of the surds: their digits actually continue forever. The reason why we use surds is to represent such exact values.
We call surds the numbers which can be represented **only** using roots. For example, $\sqrt{49}-3$ is not really a surd because, as $\sqrt{49}=7$, $\sqrt{49}-3$ is actually just 4.
Surds are **irrational** numbers: this means they can never be written as integers or fractions (which are called **rational numbers**).

OA2: THE FUNDAMENTAL RULE OF SURDS
While working with surds, we have to remember what square roots actually mean: they are the inverse operations of powers. For example, $\sqrt{25}=5$ because $5^2=25$ and $\sqrt[4]{16}=2$ because $2^4=16$. If we square the square root of a number, we obtain the number we started from: $\sqrt{16}\times\sqrt{16}=4\times4=16$, and more generally:

$$\sqrt{a}\times\sqrt{a}=a.$$

During the course, we will mostly be dealing with square roots, but it is heathy to note that similar rules work for every kind of root. For example: $\sqrt[3]{5}\times\sqrt[3]{5}\times\sqrt[3]{5}=\sqrt[3]{5}^3=5$, because taking the cube root is the opposite of raising to the third power.

OA3: EXPRESSIONS OF THE FORM $a\sqrt{b}$

When we write such expressions as $3\sqrt{19}$ – a number followed by a root – what we actually mean is $3 \times \sqrt{19}$. You should be familiar with this from algebra: 3a actually means "3 times a". In general:

$$a\sqrt{b} = a \times \sqrt{b}$$

WORKED EXAMPLE A1

Only one of the following number is actually a surd: $\sqrt[3]{1}$, $\frac{2}{3}\sqrt{7}$, $\frac{4}{\sqrt{64}}$. Explain why.

The first number is actually 1, because $1^3 = 1$. The third one is not a surd either, because as $\sqrt{64} = 8$, the fraction simplifies to $\frac{4}{8} = \frac{1}{2}$. As the square root of 7 is not a whole number, the middle one is a proper surd.

WORKED EXAMPLE A2

Calculate the value of the following surd expression: $(3 \div \sqrt{3})^4$

Solution: first, we deal with the bracket. As we know $\sqrt{3} \times \sqrt{3} = 3$, we must have $3 \div \sqrt{3} = \sqrt{3}$. Thus:

$$(3 \div \sqrt{3})^4 = (\sqrt{3})^4.$$

Then:

$$(\sqrt{3})^4 = \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3 \times 3 = 9.$$

The final answer is 9.

WORKED EXAMPLE A3

Calculate the value of the following surd expression: $3\sqrt{5} \times \frac{2}{3}\sqrt{5}$.

$3\sqrt{5} \times \frac{2}{3}\sqrt{5} = 3 \times \sqrt{5} \times \frac{2}{3} \times \sqrt{5}$. This can be rearranged as:

$$3 \times \frac{2}{3} \times \sqrt{5} \times \sqrt{5} = 2 \times 5 = 10.$$

WORKED EXAMPLE A4

ABC is a right-angled triangle. AB=6cm and BC=7cm. Find the length of AC in surd form.

Using Pythagoras, we have $AC^2 = BC^2 - AB^2$. Thus,

$$AC^2 = 7^2 - 6^2 = 49 - 36 = 13.$$

This means $AC = \sqrt{13}$.

PRACTICE A1
Which ones of the following numbers are proper surds? Circle them.

(a) $\sqrt{25}$ (b) $\sqrt{7}$ (c) $2\sqrt{2}$ (d) $\sqrt[3]{64}$
(e) $1-\sqrt{2}$ (f) $2-\sqrt{1}$ (g) $\frac{9}{\sqrt[3]{27}}$ (h) $-\sqrt{121}+\sqrt{144}$

PRACTICE A2
Calculate the value of the following expressions

(a) $\sqrt{7}\times\sqrt{7}$ (b) $\sqrt[3]{11}\times\sqrt[3]{11}\times\sqrt[3]{11}$ (c) $\sqrt{17}^2$

(d) $3\sqrt{5}\times\sqrt{5}$ (e) $2\sqrt{3}\times 3\sqrt{3}$ (f) $0.1\sqrt{a}\times 100\sqrt{a}$

(g) $15\div\sqrt{15}$ (h) $(\sqrt{6})^5$ (i) $(3\sqrt{3})^3$

(j) $\frac{\sqrt{2}}{\sqrt{7}}\times\frac{\sqrt{2}}{2\sqrt{7}}$ (k) $\frac{a\sqrt{b}}{\sqrt{z}}\times\frac{\sqrt{b}}{z\sqrt{z}}$ (l) $\sqrt[3]{x^6}=x^?$

PRACTICE A3 (OUT OF THE BOX)

(a) Calculate the length of the diagonal of a square with side 8. Give your answer in surd form.
(b) Calculate the 5th term of a geometric progression with first term 2 and common ratio $\sqrt{10}$.
(c) The ratio 1:x:y is equivalent to $\sqrt{2}:2:2\sqrt{2}$. Find x and y.

ASSESSMENT A
Cross the correct answer for each of the following.

1) Which one of these is not a surd?
(a) $\sqrt{144}-\sqrt{4}$ (b) $\sqrt{121}\div\sqrt{49}$ (c) $1-\sqrt{36}$ (d) $\sqrt{77}-\sqrt{49}$

2) The value of $\frac{12}{\sqrt{12}}$ is
(a) 144 (b) 1 (c) $\sqrt{12}$ (d) $\frac{1}{\sqrt{12}}$

3) The value of $\sqrt{3}^5$ is
(a) $\sqrt{15}$ (b) $9\sqrt{3}$ (c) $3\sqrt{3}$ (d) 27

OB1: CAN WE ADD TWO DIFFERENT SURDS?
We already know that surds behave a bit like algebraic unknowns. In particular, we can sometimes simplify surd expressions adding like terms. For example, the expression $2\sqrt{5}+6\sqrt{5}$ can be simplified to $8\sqrt{5}$, exactly because $2x+6x=8x$. A more complicated example could be $2\sqrt{17}-5\sqrt{2}+4\sqrt{2}-11\sqrt{17}$. Considering like terms, this simplifies to $-9\sqrt{17}-\sqrt{2}$.

OB2: DO NOT ADD APPLES AND ORANGES
As a general rule, you can simplify such surd expressions this way only if you add together roots of the same number. For example, you **cannot** simplify $\sqrt{2}+\sqrt{14}$ to $\sqrt{16}$: a quick check on your calculator will show that the expressions d not have the same numerical value.

WORKED EXAMPLE B1
Simplify the expression $30\sqrt{30}-11\sqrt{3}-(5\sqrt{30}-6\sqrt{3})$
First, let's get rid of the bracket by changing the sign to both $5\sqrt{30}$ and -6. We end up with

$$30\sqrt{30}-11\sqrt{3}-5\sqrt{30}+6\sqrt{3}$$

Collecting like terms, we get

$$25\sqrt{30}-5\sqrt{3}$$

WORKED EXAMPLE B2
The perimeter of the following isosceles trapezium is $8\sqrt{17}+6\sqrt{7}$. The parallel sides are in the ratio 1:3 and the length of the equal sides is $3\sqrt{7}$. Calculate the length of the parallel sides.

We can start by finding the sum of the parallel sides: we then just need to take away from the perimeter the two equal sides.

$$p-2s=8\sqrt{17}+6\sqrt{7}-2\times 3\sqrt{7}=8\sqrt{17}.$$

In order to find the two parallel sides, we may now share their sum in the ratio 1:3. This gives $8\sqrt{17}\div 4=2\sqrt{17}$ and $(8\sqrt{17}\div 4)\times 3=6\sqrt{17}$.

PRACTICE B1

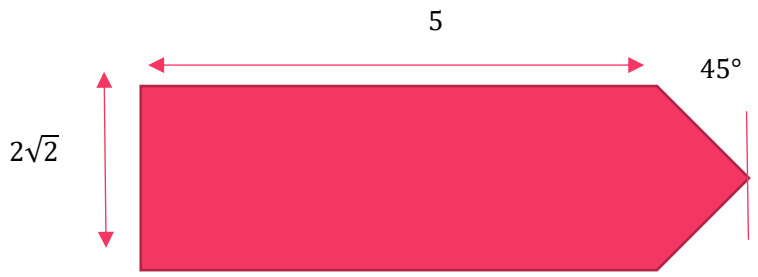
Simplify the following expressions.

(a) $\sqrt{3}-\sqrt{2}+24\sqrt{3}-4\sqrt{2}$ (b) $\sqrt{7}+\sqrt{49}-3\sqrt{7}+1$ (c) $3\sqrt[3]{2}+2\sqrt{3}-2\sqrt[3]{3}$

(d) $(\sqrt{7})^3-(\sqrt{7})^5$ (e) $\sqrt{2}-\sqrt{3}-\sqrt{5}$ (f) $\frac{11}{3}\sqrt{7}-\frac{4\sqrt{7}}{5}$ (g) $\frac{14}{\sqrt{6}}-\frac{8}{\sqrt{6}}$

PRACTICE B2 (OUT OF THE BOX)

(a) Calculate the perimeter of the following shape. Give your answer in simplified surd form.



(b) The first four terms of a quadratic sequence are

$$\sqrt{2} \quad 4\sqrt{2}-1 \quad 9\sqrt{2}-2 \quad 16\sqrt{2}-3.$$

Find an expression for the n-th term.

ASSESSMENT B

Cross the correct answer for each of the following.

1) Simplify $8\sqrt{3}-2\sqrt{6}+5\sqrt{3}-2\sqrt{36}+\sqrt{2}-\sqrt{6}$

(a) $\sqrt{39}-\sqrt{18}-12+\sqrt{2}$ (b) $13\sqrt{3}-3\sqrt{6}-12+\sqrt{2}$

(c) $3\sqrt{3}-\sqrt{6}-12+\sqrt{2}$ (d) $\sqrt{11}$

2) The value of $3\sqrt{12}-\frac{12}{\sqrt{12}}$ is

(b) $2\sqrt{12}$ (b) $\frac{2}{\sqrt{12}}$ (c) $\frac{3}{\sqrt{12}}$ (d) Can't be simplified

3) Find the odd one out

(a) $\sqrt{121}+\sqrt{4}=\sqrt{169}$ (b) $\sqrt{100}-\sqrt{64}=\sqrt{4}$

(c) $\sqrt{4}+\sqrt{4}=\sqrt{8}$ (d) $\sqrt{36}-\sqrt{25}=1$

OC1: MULTIPLYING AND DIVIDING ROOTS

Let us consider the expression $\sqrt{25} \times \sqrt{4}$: its value is $5 \times 2 = 10$. Note that we could have proceeded differently: $25 \times 4 = 100$, and then take the square root $\sqrt{100} = 10$. We have just discovered that $\sqrt{25} \times \sqrt{4} = \sqrt{25 \times 4}$. More generally, roots behave quite well when it comes to multiplications and divisions. We have indeed the general rules:

$$\sqrt{\boldsymbol{a}} \times \sqrt{\boldsymbol{b}} = \sqrt{\boldsymbol{a} \times \boldsymbol{b}} \text{ and } \frac{\sqrt{\boldsymbol{a}}}{\sqrt{\boldsymbol{b}}} = \sqrt{\frac{\boldsymbol{a}}{\boldsymbol{b}}}$$

The same formulas work for any kind of root (square, cube, etc).

OC2: MULTIPLYING SIMPLE SURDS

The above rules allow us to multiply very quickly simple surds. For example, if we want to multiply $3\sqrt{24}$ and $5\sqrt{2}$, we can just write:

$$3\sqrt{24} \times 5\sqrt{2} = 3 \times \sqrt{24} \times 5 \times \sqrt{2} = 3 \times 5 \times \sqrt{24} \times \sqrt{2} = 15 \times \sqrt{48} = 15\sqrt{48}$$

(of course you do not need to write down all the passages every time).

OC3: MULTIPLIYNG "HARDER" SURDS

We already know that surds can be more complicated than this, like for example $2 - 5\sqrt{17}$. If we want to multiply surds like those, we just need to proceed exactly like when expanding brackets. For example,

$$3\sqrt{6} \times \left(2 - 5\sqrt{17}\right) = 6\sqrt{6} - 15 \times \sqrt{17} \times \sqrt{6} = 6\sqrt{6} - 15\sqrt{102}.$$

WORKED EXAMPLE B1

Show that the expression

$$\frac{\sqrt{\boldsymbol{15}}^{\boldsymbol{3}}}{\sqrt{\boldsymbol{5}}^{\boldsymbol{5}}}$$

can be written in the form $\boldsymbol{a\sqrt{b}}$, where a is a fraction and b is an integer.

We need to work out the powers first, and then deal with the division (BIDMAS).

$$\sqrt{15}^3 = \sqrt{15} \times \sqrt{15} \times \sqrt{15} = 15\sqrt{15}$$

and, similarly, $\sqrt{5}^5 = 25\sqrt{5}$. Our fraction then becomes

$$\frac{15\sqrt{15}}{25\sqrt{5}} = \frac{15}{25} \times \frac{\sqrt{15}}{\sqrt{5}} = \frac{3}{5} \times \frac{\sqrt{15}}{\sqrt{5}}.$$

Now, using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, we see that $\frac{\sqrt{15}}{\sqrt{5}} = \sqrt{3}$, so the final answer is $\frac{3}{5}\sqrt{3}$.

WORKED EXAMPLE B2

Expand and simplify $(\sqrt{2} - 6\sqrt{6})(4\sqrt{3} - 3)$.

First, let's patiently write down the full expansion, paying attention to signs.

$$\sqrt{2} \times 4\sqrt{3} - \sqrt{2} \times 3 - 6\sqrt{6} \times 4\sqrt{3} + 6\sqrt{6} \times 3$$

Using $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$, the whole thing simplifies to

$$4\sqrt{6} - 3\sqrt{2} - 24\sqrt{18} + 18\sqrt{6}.$$

Since we are asked to **simplify,** we need to take a further step: two terms in the resulting expression may be added together ($4\sqrt{6}$ and $18\sqrt{6}$). The final answer is:

$$22\sqrt{6} - 3\sqrt{2} - 24\sqrt{18}.$$

WORKED EXAMPLE B3

The area of the following trapezium is $5\sqrt{56}$. The shortest parallel side is equal to the height, and measures $\sqrt{7}$. Find the length of the longest parallel side.

Since the area of a trapezium is $\frac{1}{2}(a + b)h$ where a and b are the parallel sides, we know that $5\sqrt{56} = \frac{1}{2}(a + \sqrt{7})\sqrt{7}$. Here, we have used the fact that height and shortest parallel sides are the same. Dividing both sides by $\sqrt{7}$ and multiplying by 2, we get

$$5\sqrt{56} \div \sqrt{7} = \frac{1}{2}(a + \sqrt{7})$$

$$5\sqrt{8} = \frac{1}{2}(a + \sqrt{7})$$

$$10\sqrt{8} = a + \sqrt{7}$$

$$a = 10\sqrt{8} - \sqrt{7}.$$

PRACTICE C1

Fill the following multiplication tables with the missing numbers.

x	$\sqrt{10}$	$\sqrt{6}$
$\sqrt{8}$	$\sqrt{}$	$2\sqrt{}$
$\sqrt{6}$	$2\sqrt{}$	

x	$\sqrt{9}$	$2\sqrt{3}$
$2\sqrt{3}$	___$\sqrt{}$	
$3\sqrt{3}$	___$\sqrt{}$	

x	$\sqrt{6}$	$\sqrt{3}$
$\sqrt{12}$	___$\sqrt{8}$	
$\sqrt{7}$	$\sqrt{3} \times \sqrt{}$	$\sqrt{}$

x	$3\sqrt{6}$	$2\sqrt{8}$
$\sqrt{6}$		___$\sqrt{48}$
$\sqrt{3}$	___$\sqrt{2}$	$4\sqrt{}$

PRACTICE C2

Expand and simplify the following brackets.

(a) $\sqrt{2}(3+\sqrt{5})$ (b) $\sqrt{2}(3\sqrt{2}+5)$ (c) $2\sqrt{5}(\sqrt{5}+3\sqrt{2})$ (d) $\sqrt[3]{2}(\sqrt[3]{4}-4\sqrt[3]{2})$
(e) $(3\sqrt{2}+1)(2-\sqrt{3})$ (f) $(5+\sqrt{3})(\sqrt{6}-2)$ (g) $(2\sqrt{2}-5)(1+4\sqrt{2})$
(h) $(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})$ (i) $(2\sqrt{2}-\sqrt{3})^2$

PRACTICE C3 (OUTSIDE THE BOX)

(a) Using the surds $\sqrt{2}$ $\sqrt{8}$ $\sqrt{10}$ $\sqrt{160}$ $\sqrt{320}$ and the operations $\div$ and $\times$, in how many ways can you make 4 ? You can use each surd more than once. Find at least 5 of them.
(b) Expand and simplify $(\sqrt{5}-\sqrt{3})^3-(2\sqrt{5}-3\sqrt{3})^2$
(c) For two integers e and f, we have that $(1+\sqrt{e})(3+\sqrt{e}) = f+4\sqrt{f-e+2}$. Find the values of e and f.

ASSESSMENT C

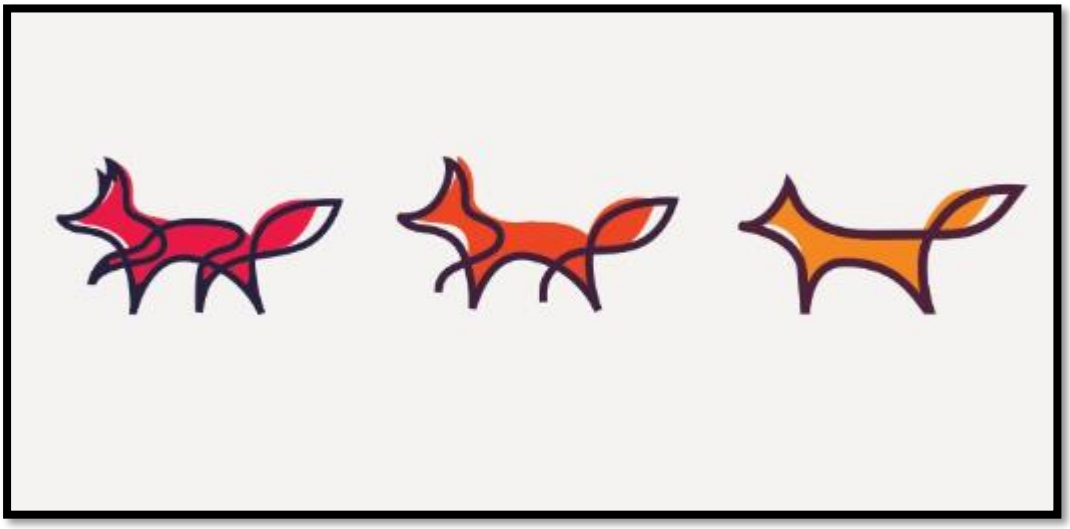
Cross the correct answer for each of the following.

1) In its simplest form, $2\sqrt{2}\times 3\sqrt{8} =$
(a) 96 (b) $\sqrt{96}$
(c) 24 (d) $6\sqrt{16}$

2) Expand $(1+\sqrt{2})(3-\sqrt{2})$
(c) 1 (b) $2\sqrt{2}$ (c) $2\sqrt{2}-1$ (d) $1+2\sqrt{2}$

3) If $a = 1+\sqrt{2}$ and $b = 1-\sqrt{2}$, which one of the following is a proper surd?
(a) $a+b$ (b) ab
(c) $(a+b)^2$ (d) $a-b$

Tutorial 3 – Simplifying a surd



What is the Purpose of Tutorial 3?

- Objective A: to simplify a pure surd by taking out square factors.
- Objective B: to learn the fractional index notation in relation to surds.

OA1: WHAT DOES IT MEAN TO SIMPLIFY?
We have previously seen how to simplify a surd by collecting like terms. This way surd expressions may become "smaller". This is not the only way we have to simplify a surd. For example, the expression $\sqrt{2} \times \sqrt{2} \times \sqrt{2}$ can be written in two ways: $\sqrt{8}$ (as $2 \times 2 \times 2 = 8$) or $2\sqrt{2}$ (as $\sqrt{2} \times \sqrt{2} = 2$). Which one of the two should be considered the "simplest"? As a general rule, we prefer to deal with small numbers rather than large ones (think of ratio simplification). Thus, **the smaller the number under the root, the better.** We choose $2\sqrt{2}$ to be our simplified surd.

OA2: SIMPLIFYING BY TAKING OUT SQUARE FACTORS
If we have a surd of the form $\sqrt{a}$, we can try to simplify it to a form $b\sqrt{c}$, where $c < a$. To do so, there is a standard procedure. Consider for example $\sqrt{50}$. The first thing to do is to look for factors of 50 which are also square numbers (4, 9, 16, 25…) We see that 25 suits us, as $25 \times 2 = 50$. We can then split $\sqrt{50} = \sqrt{25} \times \sqrt{2}$, as we know how multiplication of surds work. Now, since we were very clever, we have that $\sqrt{25} = 5$. This means we can write $\sqrt{50} = 5 \times \sqrt{2} = 5\sqrt{2}$, which is what we were looking for.

OA3: HOW DO I KNOW WHEN I AM DONE WITH SIMPLIFYING?
Simply, when you cannot find any more square factors in the number under the root. For example, $\sqrt{15}$ cannot be simplified further as it has no square factors. Vice versa, if we consider the simplification $\sqrt{200} = \sqrt{25} \times \sqrt{8} = 5\sqrt{8}$, this is not complete. Indeed, $\sqrt{8} = \sqrt{4} \times \sqrt{2}$, so $5\sqrt{8} = 5 \times 2\sqrt{2} = 10\sqrt{2}$.

WORKED EXAMPLE A1

Fully simplify $3\sqrt{32}$.

We look at factors of 32 which are also square numbers: we have 4 and 16. To speed up the process, we choose the largest. We have:

$$3\sqrt{32} = 3 \times \sqrt{16 \times 2} = 3 \times \sqrt{16} \times \sqrt{2} = 3 \times 4 \times \sqrt{2} = 12\sqrt{2}$$

WORKED EXAMPLE A2

Write $5\sqrt{200} - 4\sqrt{128} - 5\sqrt{8}$ in the form $a\sqrt{b}$, where b is the smallest possible integer.

We see that all the numbers under the roots can be written as twice a square number: $200 = 2 \times 100, 128 = 2 \times 64, 8 = 2 \times 4$. Thus, the expression simplifies to

$$5\sqrt{2} \times \sqrt{100} - 4\sqrt{2} \times \sqrt{64} - 5\sqrt{2} \times \sqrt{4} =$$
$$= 50\sqrt{2} - 32\sqrt{2} - 10\sqrt{2} = 8\sqrt{2}$$

WORKED EXAMPLE A3

Simplify fully $\sqrt{\frac{27}{125}}$. Write your answer in the form $a\sqrt{b}$ where both a and b are fractions.

We can rewrite the surd as $\frac{\sqrt{27}}{\sqrt{125}}$ and simplify numerator and denominator separately as follows: $\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$ and $\sqrt{125} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$. The surd then becomes

$$\frac{\sqrt{27}}{\sqrt{125}} = \frac{3\sqrt{3}}{5\sqrt{5}} = \frac{3}{5} \times \frac{\sqrt{3}}{\sqrt{5}} = \frac{3}{5}\sqrt{\frac{3}{5}}.$$

WORKED EXAMPLE A4

Simplify fully $\frac{\sqrt{700}}{14}$.

This is a deceptive one. As the 14 at the denominator is outside the square root, we cannot cancel out with the 700. Instead, we need to simplify the surd at the numerator: $\sqrt{700} = \sqrt{7 \times 100} = 10\sqrt{7}$. Now we can simplify the fraction:

$$\frac{\sqrt{700}}{14} = \frac{10\sqrt{7}}{14} = \frac{5\sqrt{7}}{7}$$

We cannot simplify further.

PRACTICE A1

(a) Write $\sqrt{98}$ in the form $k\sqrt{2}$, where k is an integer.
(b) Write $8\sqrt{8}$ in the form $k\sqrt{2}$, where k is an integer.
(c) Write $6\sqrt{3}$ as $\sqrt{b}$, where b is an integer.
(d) Write $4\sqrt{8}$ as $2\sqrt{a}$, where a is an integer.
(e) Fully simplify $\sqrt{12}+\sqrt{3}$.
(f) Fully simplify $2\sqrt{32}-3\sqrt{8}$.
(g) Fully simplify $2\sqrt{5}+3\sqrt{20}$.

PRACTICE A2

(a) Simplify $(7+2\sqrt{50})(5-2\sqrt{2})$, leaving your answer in the form $a+b\sqrt{2}$.
(b) Expand and simplify $(\sqrt{20}-2)(6+\sqrt{5})$,
(c) Find the mean of $\sqrt{80}$, $\sqrt{5}$ and $\sqrt{500}$, in its simplest form.
(d) In the right-angled triangle ABC, AC is the hypotenuse with length $8cm$, and AB=$6cm$. Work out the length of BC, giving your answer as a simplified surd.
(e) In a right angled triangle, the hypotenuse is $6cm$ long and the shortest side is $2\sqrt{3}cm$ long. Show that the area of the triangle can be written as $k\sqrt{2}cm^2$, where k is an integer to be found.
(f) $\sqrt{5}(\sqrt{8}+\sqrt{18})$ can be written as $a\sqrt{10}$ where a is an integer. Find the value of a.
(g) An integer a satisfies $\sqrt[3]{a}=5\sqrt[3]{2}$. Find a.
(h) An integer a satisfies $\sqrt[4]{a}=\sqrt{3}$. Find a.
(i) Simplify $\sqrt{20^2+15^2}$
(j) Simplify $\frac{\sqrt{12}}{\sqrt{48}}$.
(k) Prove that $\frac{\sqrt{192}}{2\sqrt{3}}$ simplifies to an integer to be found.

PRACTICE A3 (OUTSIDE THE BOX)

(a) Rectangle A has sides of length $3\sqrt{6}$m and $\sqrt{5}m$. Rectangle B has sides of length $\sqrt{10}m$ and $\sqrt{12}m$. Prove that rectangle A is $1\frac{1}{2}$ times bigger than rectangle B.
(b) Without a calculator, determine how many of the numbers

$$3\sqrt{11},\ 4\sqrt{7},\ 5\sqrt{5},\ 6\sqrt{3},\ 7\sqrt{2}$$

are greater than 10. Tip: use the reversed simplifying process.
(c) The positive integer k satisfies $\sqrt{k}=\sqrt{2}+\sqrt{18}+\sqrt{8}$. Find k.

ASSESSMENT A

Cross the correct answer for each of the following.

1) Simplify fully $\sqrt{12}+\sqrt{48}$
(a) $\sqrt{60}$ (b) $6\sqrt{6}$ (c) $6\sqrt{3}$ (d) Can't simplify

2) If $\sqrt{x}=6\sqrt{2}$ then x is
(d) 12 (b) 24 (c) 144 (d) 72

4) Simplify $(\sqrt{3}+\sqrt{27})^2$
(a) 48 (b) 30 (c) $\sqrt{30}$ (d) 738

OB1: THE INDEX $\frac{1}{2}$

We may write the square root of a number using indices in the following form:

$$\sqrt{a} = a^{\frac{1}{2}}$$

This has a very reasonable explanation: if we consider the basic identity $\sqrt{a} \times \sqrt{a} = a$ with this notation, we get

$$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1 = a$$

We can also notice

$$(a^{\frac{1}{2}})^2 = a^{\frac{1}{2}\times 2} = a$$

Following a similar reasoning, we can also write other roots this way:

$$\sqrt[a]{x} = x^{\frac{1}{a}}$$

For example, $\sqrt[4]{x} = x^{\frac{1}{4}}$.

OB2: INDICES OF THE FORM $\frac{b}{a}$

Fractional indices are used to express a root of a power. We have:

$$x^{\frac{b}{a}} = \sqrt[a]{x^b} = (\sqrt[a]{x})^b$$

Once again, we have a simple explanation for this. Using $\sqrt[a]{x} = x^{\frac{1}{a}}$, we have

$$\sqrt[a]{x^b} = (x^b)^{\frac{1}{a}} = x^{b\times\frac{1}{a}} = x^{\frac{b}{a}}$$

OB3: RELATIONS WITH SURDS

Sometimes, you might need to move from the index notation to the usual roots. A simple example would be to write $2^{\frac{3}{2}}$ in the fully simplified form $a\sqrt{b}$. We have

$$2^{\frac{2}{3}} = \sqrt{2^3} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}.$$

WORKED EXAMPLE B1

Work out $64^{\frac{2}{3}}$.

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$. It is important to note that we could have calculated $\sqrt[3]{64^2}$ and get the same result, but we deliberately chose not to do it, as it was not convenient in this situation.

WORKED EXAMPLE B2

Evaluate $81^{-0.5}$. Give your answer in the simplest form.

First, we need to remember that negative indices flip fractions. We get:

$$81^{-0.5} = \frac{1}{81^{0.5}}$$

Now, as $0.5 = \frac{1}{2}$ and $81^{\frac{1}{2}} = 9$, we get $81^{-0.5} = \frac{1}{9}$.

WORKED EXAMPLE A3

$9^x = 3\sqrt[3]{3}$. Find x.

We write the right hand side in fractional index form: $3 \times 3^{\frac{1}{3}} = 3^{1+\frac{1}{3}} = 3^{\frac{4}{3}}$. We also want to write the left hand side as a power of 3: $9^x = (3^2)^x = 3^{2x}$. Now, $2x = \frac{4}{3}$, so $x = \frac{2}{3}$.

PRACTICE B1
Write in simplified surd form:

(a) $2^{\frac{1}{2}}$ (b) $3^{\frac{5}{2}}$ (c) $9^{\frac{1}{4}}$ (d) $10^{\frac{3}{2}} \div 5^{\frac{1}{2}}$

PRACTICE B2
Work out:

$(2\tfrac{1}{4})^{-\frac{1}{2}} \times (3\tfrac{3}{8})^{\frac{1}{3}}$

$(1.25)^{-\frac{1}{3}} \times (10)^{\frac{1}{3}}$

$(1.5)^{-2} \times (16)^{-\frac{1}{2}} \times (81)^{\frac{3}{4}}$

$(48)^{\frac{1}{2}} \times (9)^{\frac{1}{6}} \times (3)^{-\frac{5}{6}}$

$(20)^{\frac{2}{3}} \div (100)^{\frac{1}{6}} \times (\frac{5}{2\sqrt{2}})^{\frac{2}{3}}$

$(12)^{\frac{1}{3}} \div (81)^{\frac{1}{6}} \times (162)^{\frac{1}{3}}$

PRACTICE B2 (OUTSIDE THE BOX)

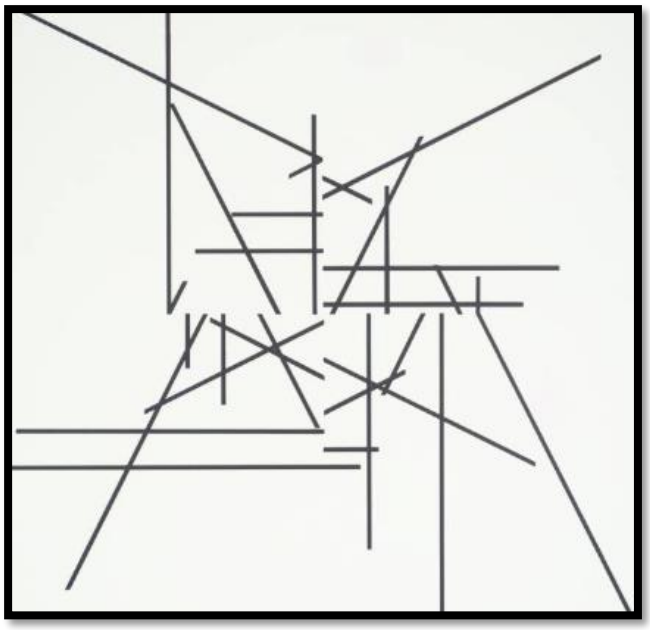
(a) For which value of k is $\sqrt{2016} + \sqrt{56}$ equal to 14^k?
(b) Evaluate $(64^{-\frac{1}{4}} - 3^{-\frac{1}{2}})(16^{-0.25} + (3\sqrt{3})^{-\frac{1}{3}})$

ASSESSMENT B
Cross the correct answer for each of the following.

1) $125^{-\frac{2}{3}}$ is equal to
(a) $\frac{1}{10}$ (b) $\frac{1}{25}$ (c) -25 (d) $-\frac{1}{25}$

2) $\sqrt{5} \times 5^{\frac{1}{2}}$ is equal to
(e) 5^2 (b) $\sqrt{\frac{5}{2}}$ (c) $\frac{5}{2}$ (d) 5

Tutorial 4 – Rationalising the denominator



What is the Purpose of Tutorial 4?

- Objective A: to be able to rationalise a surd with denominator $a\sqrt{b}$
- Objective B: to be able to rationalise a surd with more complex denominators.

OA1: WHAT DOES IT MEAN TO RATIONALISE THE DENOMINATOR?

An informal rule in maths says that it is preferable to work with hard numerators rather than hard denominators. For example, the equation $\frac{x}{5} - 1 = 2$ is more manageable than the (perfectly equivalent) equation $\frac{5}{x-5} = \frac{1}{2}$, because the second one requires extra steps to be solved. Surds are not an exception: if the denominator of a fraction has roots (like $\frac{2\sqrt{7}}{\sqrt{3}-1}$), we might want to transform it into a fraction which has roots only at the numerator. This particular simplification process is called **rationalisation of the denominator**, because we are making the denominator a rational number.

OA2: RATIONALISING SIMPLE SURD DENOMINATORS

The key thing we need to remember now is that if we multiply numerator and denominator of a fraction by the same number, we are going to get an equivalent fraction. We can use this to our advantage: for example, to rationalise the denominator of $\frac{1}{\sqrt{7}}$, we can multiply numerator and denominator by $\sqrt{7}$:

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$$

The resulting fraction has roots only in the numerator. We can use the same idea to rationalise more complicated denominators:

$$\frac{\sqrt{2}}{3\sqrt{5}} = \frac{\sqrt{2}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{2} \times \sqrt{5}}{3 \times 5} = \frac{\sqrt{10}}{15}.$$

However, this only works if the denominator is of the form $a\sqrt{b}$. We shall soon see that for fractions like $\frac{1}{\sqrt{2}-3}$ we will have to use something more sophisticated.

WORKED EXAMPLE A1

Write $\frac{10}{\sqrt{6}}$ as $a\sqrt{b}$ where a is rational.

The question is actually asking you to rationalise, as you need to move the root away from the denominator. We multiply by $\frac{\sqrt{6}}{\sqrt{6}}$ to get

$$\frac{10}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$$

To get the precise form required in the exercise, we need to write $\frac{5}{3}\sqrt{6}$.

WORKED EXAMPLE A2

Rationalise the denominator of

$$\frac{6 - 3\sqrt{2}}{\sqrt{3}}$$

Fully simplify your answer.

Once again, we want to make the denominator rational, so:

$$\frac{6 - 3\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3} - 3\sqrt{6}}{3}$$

To simplify the answer, we might note that we can divide both numerator and denominator by 3, and get $2\sqrt{3} - \sqrt{6}$.

WORKED EXAMPLE A3

Work out the mean of $\sqrt{75}, \sqrt{75}$ and $\frac{6}{\sqrt{3}}$. Fully simplify your answer.

First, it is a good idea to rationalise the denominator of the third number:

$$\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

The 3 surds cannot be nicely added together right now, so we should probably try to simplify $\sqrt{75}$, which is indeed $\sqrt{25 \times 3} = 5\sqrt{3}$. Now, we add together

$$5\sqrt{3} + 5\sqrt{3} + 2\sqrt{3} = 12\sqrt{3}$$

and, dividing by 3, we get $4\sqrt{3}$.

PRACTICE A1

Write the following in the form $a\sqrt{b}$, where a is rational.

(a) $\frac{5}{\sqrt{2}}$ (b) $\frac{20}{\sqrt{4}}$ (c) $\frac{9}{\sqrt{3}}$ (d) $\frac{4}{\sqrt{2}}$ (e) $\frac{14}{\sqrt{7}}$ (f) $\frac{21}{\sqrt{14}}$

PRACTICE A2

(a) Rationalise the denominator and fully simplify

$$\frac{8+2\sqrt{5}}{\sqrt{2}}$$

(b) Write in the form $a+b\sqrt{c}$

$$\frac{6+4\sqrt{2}}{\sqrt{2}}$$

(c) Write as a single fraction with rational denominator. Fully simplify your answer.

$$\frac{5}{3\sqrt{5}}-\frac{\sqrt{5}}{3}$$

(d) Write as a single fraction with rational denominator. Fully simplify your answer.

$$\left(\sqrt{3}+\frac{3}{\sqrt{3}}\right)^2$$

(e) Write as a single fraction with rational denominator. Fully simplify your answer.

$$\frac{3}{\sqrt{45}}+\frac{5}{\sqrt{125}}+\frac{12}{\sqrt{80}}$$

(f) Show that

$$\frac{\sqrt{20}+\sqrt{80}}{\sqrt{3}}$$

can be written as $\sqrt{a}$ where a is an integer.

PRACTICE A3 (OUTSIDE THE BOX)

(a) 3 surfaces of a cuboid have areas of $10cm^2$, $8cm^2$and $4cm^2$. It has a volume of $a\sqrt{b}cm^3$ in its simplest form. What are the values of a and b?

(b) Prove that the mean of the two numbers $(2+\sqrt{3})^2$ and $7-\frac{12}{\sqrt{3}}$ is an integer.

(c) Write in the form $a\sqrt{b}$ with a and b rational.

$$\frac{\sqrt{5}}{\sqrt{2}} \Big/ \frac{\sqrt{11}}{2}$$

ASSESSMENT A

Cross the correct answer for each of the following.

1) Which is a good first step to rationalise the denominator of $\frac{3\sqrt{2}}{\sqrt{5}}$?

(a) $\frac{3\sqrt{2}}{\sqrt{5}}\times\frac{5}{\sqrt{5}}$ (b) $\frac{3\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{2}}{\sqrt{5}}$ (c) $\frac{3\sqrt{2}}{\sqrt{5}}\times\sqrt{5}$ (d) $\frac{3\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$

2) Rationalise the denominator of $\frac{7}{\sqrt{6}}$

(a) $\frac{\sqrt{7}}{6}$ (b) $\sqrt{\frac{7}{6}}$ (c) $\frac{7\sqrt{6}}{\sqrt{6}}$ (d) $\frac{7\sqrt{6}}{6}$

OB1: RECALLING THE DIFFERENCE OF TWO SQUARES

In order to rationalise surd denominators which are not in the simple form $a\sqrt{b}$ (such as $2-\sqrt{3}$ or $4\sqrt{7}-\sqrt{2}$) we need to remind ourselves how the difference of two squares works. The general formula is:

$$\boldsymbol{a^2 - b^2 = (a-b)(a+b)}$$

In words, this means that multiplying the sum and difference of two numbers, we are going to get the difference of their squares. An example with surds, with $a=\sqrt{5}$ and $b=\sqrt{2}$:

$$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = \sqrt{5}^2 - \sqrt{2}^2 = 5-2=3$$

OB2: RATIONALISING MORE COMPLEX DENOMINATORS

To rationalise the denominator of

$$\frac{1}{\sqrt{3}-1}$$

we can still multiply numerator and denominator by the same number. However, $\sqrt{3}$ or even $\sqrt{3}-1$ will not solve the problem. Instead, we will multiply by $\boldsymbol{\sqrt{3}+1}$**:**

$$\frac{1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{\sqrt{3}^2-1^2} = \frac{\sqrt{3}+1}{2}$$

What happened here is that, using the difference of two squares formula, we were able to get rid of the square roots at the denominator. A more complex example:

$$\frac{2}{\sqrt{7}-2\sqrt{2}} = \frac{2}{\sqrt{7}-2\sqrt{2}}\times\frac{\sqrt{7}+2\sqrt{2}}{\sqrt{7}+2\sqrt{2}} =$$

$$= \frac{2\sqrt{7}+4\sqrt{2}}{\sqrt{7}^2-(2\sqrt{2})^2} = \frac{2\sqrt{7}+4\sqrt{2}}{7-8} = -(2\sqrt{7}+4\sqrt{2})$$

As a general rule, when the denominator is a sum (or a difference), multiply by the corresponding difference (or sum) to rationalise.

WORKED EXAMPLE B1

Rationalise the denominator and simplify

$$\boldsymbol{\frac{5\sqrt{5}-2}{2\sqrt{5}-3}}$$

We need to multiply numerator and denominator by $2\sqrt{5}+3$ to pull the trick of the difference of squares and get rid of the square roots. This is going to involve a double bracket to expand and simplify at the numerator, but we are already ridiculously good at that, so no problem.

$$\frac{5\sqrt{5}-2}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3} = \frac{(5\sqrt{5}-2)(2\sqrt{5}+3)}{(2\sqrt{5})^2-3^2} =$$

$$= \frac{50-4\sqrt{5}+15\sqrt{5}-6}{20-9} = \frac{44+11\sqrt{5}}{11} = 4+\sqrt{5}$$

WORKED EXAMPLE B2

Solve $y(\sqrt{3}-1)=8$. Give your answer in the form $a+\sqrt{b}$ where a and b are integers.

We must resist the temptation to expand the bracket, as what we really want is "y = a number". Thus, we divide by the entire bracket $(\sqrt{3}-1)$ to get:

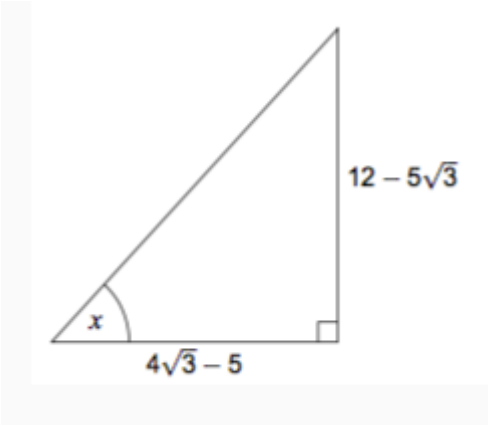
$$y=\frac{8}{\sqrt{3}-1}$$

The next step is to rationalise:

$$\frac{8}{\sqrt{3}-1}=\frac{8}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{8\sqrt{3}+8}{3-1}=\frac{8\sqrt{3}+8}{2}=4\sqrt{3}+4$$

WORKED EXAMPLE B3

Find the value of x.



This is a tough one. As we are given a right angled triangle and asked for an angle, we could use trigonometry – in particular, the TOA triangle, and rationalise to simplify. We have that

$$\frac{Opp}{Adj}=\frac{12-5\sqrt{3}}{4\sqrt{3}-5}=\frac{12-5\sqrt{3}}{4\sqrt{3}-5}\times\frac{4\sqrt{3}+5}{4\sqrt{3}+5}=$$
$$=\frac{48\sqrt{3}+60-60-25\sqrt{3}}{48-25}=\frac{23\sqrt{3}}{23}=\sqrt{3}$$

This tells us that $\tan(x)=\sqrt{3}$. This is the tangent of a fundamental angle: $x=60°$.

PRACTICE B1
Rationalise the denominator

(a) $\frac{1}{2-\sqrt{2}}$ (b) $\frac{2}{\sqrt{5}-2}$ (c) $\frac{9}{2\sqrt{3}+\sqrt{5}}$ (d) $\frac{\sqrt{7}}{\sqrt{7}+\sqrt{2}}$ (e) $\frac{11\sqrt{3}-2\sqrt{6}}{4\sqrt{6}-6\sqrt{3}}$

PRACTICE B2

(a) Write

$$\frac{5\sqrt{2}}{3\sqrt{6}-7}$$

In the form $\sqrt{k}+\sqrt{w}$ where k and w are integers.

(b) Simplify fully

$$\frac{24-\sqrt{300}}{4\sqrt{3}-5}$$

Write your answer in the form $a\sqrt{b}$, with a and b are integers.

(c) The fraction

$$\frac{1+\sqrt{2}}{(3-\sqrt{2})^2}$$

can be written in the form $a+b\sqrt{2}$. Find a and b.

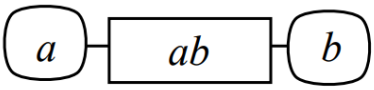
(d) Express each one of the following in the form $a\sqrt{x}$ with a an integer.

$$\frac{6}{\frac{1}{\sqrt{5}}+\sqrt{5}} \qquad \frac{8}{\sqrt{3}-\frac{1}{\sqrt{3}}} \qquad \frac{\frac{4}{\sqrt{3}}+2\sqrt{3}}{\sqrt{3}+\frac{2}{\sqrt{3}}}$$

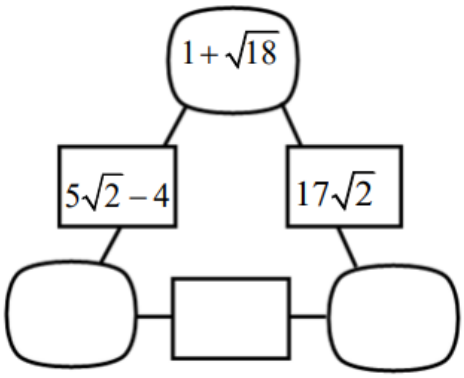
(e) Expand and simplify

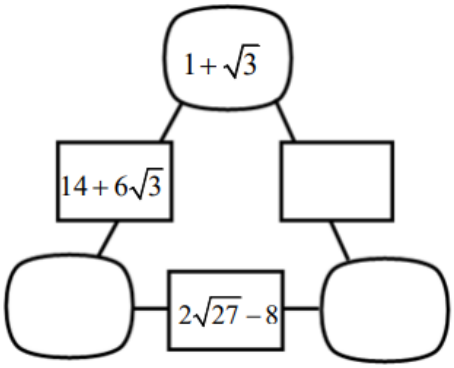
$$\left(\frac{2}{\sqrt{2}}+\frac{3}{\sqrt{3}}\right)\times\frac{1}{\sqrt{3}+\sqrt{2}}$$

PRACTICE B3 (OUTSIDE THE BOX)
Surd mystery – the key is:



Fill the empty boxes in simplified surd form.





ASSESSMENT B

Cross the correct answer for each of the following.

1) Which is a good first step to rationalise the denominator of $\frac{5}{1+\sqrt{2}}$?

(a) $\frac{5}{1+\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ (b) $\frac{5}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$ (c) $\frac{5}{1+\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ (d) $\frac{5}{1+\sqrt{2}} \times \frac{5}{1+\sqrt{2}}$

2) Which is a good first step to rationalise the denominator of $\frac{\sqrt{2}}{\sqrt{11}-7}$?

(a) $\frac{\sqrt{2}}{\sqrt{11}-7} \times \frac{7-\sqrt{11}}{7-\sqrt{11}}$ (b) $\frac{\sqrt{2}}{\sqrt{11}-7} \times \frac{\sqrt{11}+7}{\sqrt{11}+7}$ (c) $\frac{\sqrt{2}}{\sqrt{11}-7} \times \frac{\sqrt{11}-7}{\sqrt{11}-7}$ (d) $\frac{\sqrt{2}}{\sqrt{11}-7} \times \frac{\sqrt{11}}{\sqrt{11}}$

3) Rationalise the denominator of $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

(a) $\frac{5+2\sqrt{6}}{5}$ (b) $\frac{1}{5}$ ` (c) $5+2\sqrt{6}$ (d) $5-2\sqrt{6}$

Tutorial 5 – Surds in algebra and geometry problems

What is the Purpose of Tutorial 5?

- Objective: to systematically practise surd manipulation in apparently unrelated contexts

SURDS IN ALGEBRA
We always need to remember that **surds are just numbers**, and thus they can appear as coefficient in any algebra problem: in an expansion of a set of brackets, as terms in a geometric progression…
You might also have to work with irrational coordinates of points in the plane: manipulating these numbers might be a step to solve problems with graphs (like quadratic graphs).

A TOUGH EXAMPLE
Find the equation of the line perpendicular to the line $y = \sqrt{2}x$ passing through the point A=($3\sqrt{2} - 1, 4\sqrt{2}$). Give gradient and y-intercept in simplified surd form.
First of all, the gradient of the perpendicular line is found by considering the opposite of the reciprocal of the gradient $\sqrt{2}$, which is $-\frac{1}{\sqrt{2}}$. We can rationalise the denominator:

$$-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}.$$

Now, our line will be of the form $y = -\frac{\sqrt{2}}{2}x + c$, so we need to find c. To do this, we substitute the coordinates of the point A and get $3\sqrt{2} - 1 = -\frac{\sqrt{2}}{2} \times 4\sqrt{2} + c$. After multiplyin and rearranging for c, we get $c = 3\sqrt{2} + 3$.

PRACTICE SURDS IN ALGEBRA

(a) Here is a sequence

$2 \quad 2\sqrt{7} \quad 14 \quad 14\sqrt{7}$

(i) Find the next term

(ii) Write an expression for the nth term

(iii) Find the mean of the first 6 terms

(b) Solve $(3-\sqrt{2})x-\sqrt{2}=0$.

(c) Expand $(1-\sqrt{3})(\sqrt{3}+4)(2-\sqrt{3})$.

(d) Expand and simplify $(\sqrt{2}+\sqrt{x})(2\sqrt{2x}+1)$

(e) Find the number a that satisfies $9^{2a}=\sqrt{9}$

(f) Solve the equation $2x^2-64=0$, giving your answer in simplified surd form.

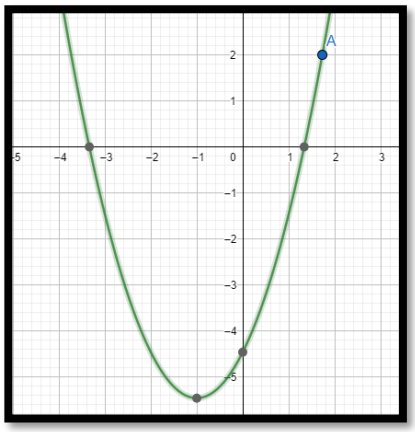
(g) Find the gradient of the straight line through the points $(7,3\sqrt{2})$ and $(-7,4\sqrt{2}-3)$. Give your answer in simplified surd form.

(h) Solve $5x+3=\sqrt{5}x+\sqrt{3}$.

(i) Factorise $x^2+2\sqrt{3}x+3$.

(j) For what value of x is the equation $2x^3=108$ satisfied? Give your answer in simplified surd form.

(k) The quadratic graph $y=x^2+2x+\sqrt{3}k$ passes through the point $A=(\sqrt{3},2)$. Find the value of k.



(l) Given that p is a prime number, rationalise the denominator of

$$\frac{7\sqrt{p}-p^2}{\sqrt{p^3}}$$

SURDS IN GEOMETRY

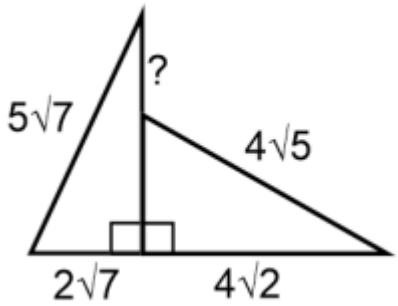
Surds may appear frequently as lengths, areas and volumes in problems with shapes – if you are sked to find the area of a shape whose sides have irrational length, you will need to be able to multiply and simplify surds.

We have already seen that questions about right angled triangles might involve surds, due to Pythagoras. There is also another related connection between right angled triangles and surds, which is the fact that several of the exact trig values you have to memorise for the GCSE are indeed surds:

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined

A TOUGH EXAMPLE

Find the missing length in the diagram. Write your answer in simplified surd form.



The missing length is obtained by subtracting the two missing sides of the right angled triangles, so the first step would be to find those. We can use Pythagoras. For the longest one:

$$\sqrt{(5\sqrt{7})^2 - (2\sqrt{7})^2} = \sqrt{25 \times 7 - 4 \times 7} = \sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}.$$

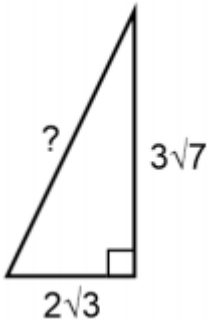
For the shortest one:

$$\sqrt{(4\sqrt{5})^2 - (4\sqrt{2})^2} = \sqrt{16 \times 5 - 16 \times 2} = \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}.$$

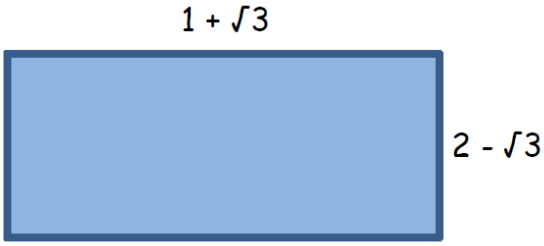
The missing length is: $7\sqrt{3} - 4\sqrt{3} = 3\sqrt{3}$.

PRACTICE SURDS IN GEOMETRY – ALL ANSWERS NEED TO BE GIVEN IN SIMPLIFIED SURD FORM

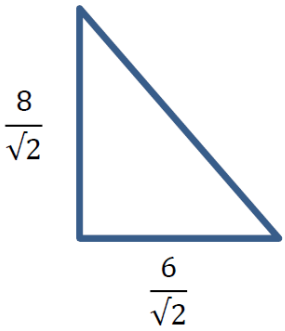
(a) Calculate the missing length.



(b) Calculate the area.

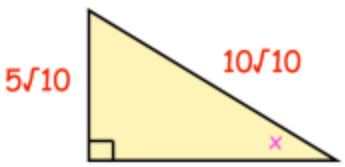


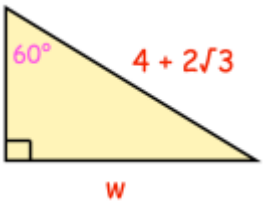
(c) Calculate perimeter and area.



(d) Find the perimeter of a square with area $162cm^2$.

(e) Find the angle x and the length of w without using a calculator.

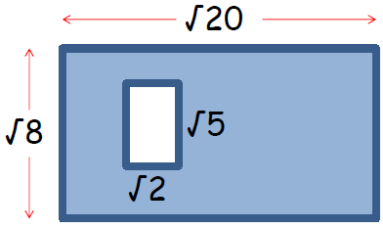




(f) The sides of a rectangle are in the ratio $2:\sqrt{5}$ and its area is $(1+\sqrt{10})cm$. Calculate its perimeter.

(g) Calculate the area of an equilateral triangle with perimeter $6\sqrt{15}$.

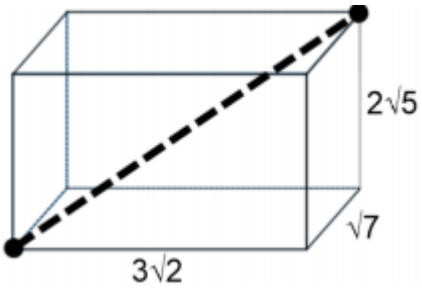
(h) Express the area of the blue shape as a percentage of the area of the whole large rectangle.



(i) The area of a shape is $10\sqrt{5}$. It is enlarged by a scale factor $3\sqrt{2}$. What is the area of the enlarged shape?

(j) Calculate the radius of the cross section of a cylinder with volume $24\sqrt{2}\pi\ cm^3$ and height $\sqrt{288}cm$.

(k) Calculate the length of the dotted line.



(l) Prove that

$$\frac{\sin(30) \times \sin(45) \times \sin(60) - 1}{1 - \cos(30) \times \cos(45) \times \cos(60)}$$

Is equal to -1.

Tutorial 6 – Final Assessment

Tutorial 7 - Feedback

What is the Purpose of Tutorial 7?

- To receive feedback on final assessments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assessment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
• • •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Notes

Notes

Notes

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