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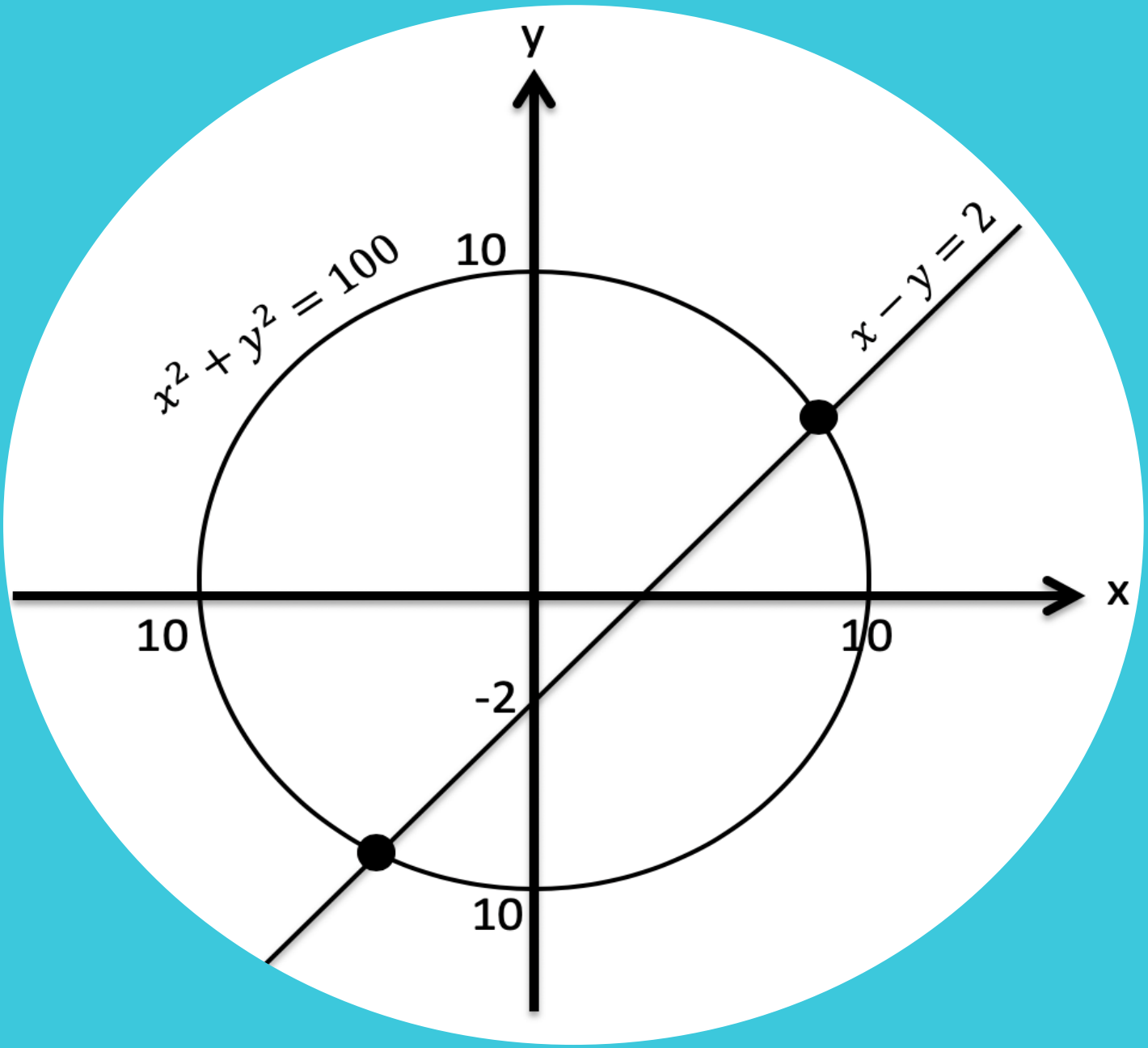


Simultaneous Equations

Key Stage 4 Programme

Pupil Name	
Handbook Designed by	Dr Martin

Contents

Course Rationale **P3**

Tutorial 1 **P4**

Linear simultaneous equations by elimination

Tutorial 2 **P13**

Solving simultaneous equations by substitution

Tutorial 3 **P23**

Solving simultaneous equations graphically

Tutorial 4 **P38**

Solving simultaneous inequalities

Feedback **P51**

Course Rationale

This course is designed to help you master simultaneous equations from the mathematics GCSE curriculum. We will work our way through the topic step by step, practising each element along the way. Mathematics requires constant practise in order to gain a good understanding and develop problem-solving skills. Good quality practice involves doing the same thing, but in a variety of contexts that range in difficulty. This Uni Pathways course is developed around the importance of practice and revision.

The tutorials start by solving linear simultaneous equations, moving on to solving quadratic simultaneous equations and ending with solving simultaneous inequalities. At the end of this Uni Pathways you should feel comfortable and confident whenever you come across a question involving simultaneous equation.

Tutorial 1 – Solving simultaneous equations by elimination

Objectives for this tutorial

A. Solve linear simultaneous equations via elimination.
B. Manipulate variables in order to solve simultaneous equations via elimination.
C. Identify and set up simultaneous equations in order to solve a problem.

Objective A: Solving linear simultaneous equations via elimination

Worked example 1: Solve the following to find x and y.

$3x + y = 17,\ \ 3x + 4y = 23$

$3x + y = 17$ (a)
$3x + 4y = 23$ (b)

Label the equations.

$3x + y = 17$ (a)
$3x + 4y = 23$ (b)

Check for the same coefficients for x or y.

In this case the x coefficient is 3 in both equations.

$3x + 4y = 23$ (b)
$3x + y = 17$ (a)

Once identified, eliminate the common coefficient, $3x - 3x = 0$.

In this case I have done (b) – (a) since the y coefficient in (b) is greater than the y coefficient in (a).

$3y = 6$

Now solve for y

$(\div 3)$ $(\div 3)$

$y = 2$

Divide by 3 on both sides

If you did (a)-(b) you will still get the same answer at the end!

$3x + y = 17$ (a)

To find x you substitute the value of y into one of the original equations.

$3x + 2 = 17$

(-2) (-2)

Now solve for x

In this case I substituted y=2 into (a).

$3x = 15$

Subtract 2 from both sides.

$(\div 3)$ $(\div 3)$

$x = 5$

Divide by 3 on both sides.

$3x + y = 17$ (a)
$3x + 4y = 23$ (b)

Now check you answer by substituting you values of x and y into the equations.

$3(5) + 2 = 17$
$3(5) + 4(2) = 23$

If both sides of the equation are not equal there is a mistake somewhere!

Answer: $x = 5, y = 2$

Worked example 2: Solve the following to find a and b.

$2a + b = 10, \ 4a - b = 8$

$2a + b = 10$ (e1) Label the equations.
$4a - b = 8$ (e2)

Don't confuse the variables with the labelling. Use different letters to a and b! In this case I have used e1 and e2.

$2a + b = 10$ (e1) Check for the same coefficients for a or b.
$4a - b = 8$ (e2)

In this case the b coefficient is 1 in both equations.

$2a + b = 10$ (e1) Once identified, eliminate the common coefficient, $b + -b = 0$.
$4a - b = 8$ (e2) $+$
$-$

Since addition is commutative, (e1) + (e2) = (e2) + (e1), it doesn't matter which was you add the equations, you will get the same answer.

$6a = 18$ Now solve for a

$(\div 6)$ $(\div 6)$

$a = 3$ Divide by 6 on both sides

$2a + b = 10$ (e1) To find b you substitute the value of a into one of the original equations.

$2(3) + b = 10$

In this case I substituted a=3 into(e1).

$6 + b = 10$ Now solve for b

(-6) (-6) Subtract 6 from both sides.

$b = 4$

$2a + b = 10$ (e1) Now check you answer by substituting you values of x and y into the equations.
$4a - b = 8$ (e2)

$2(3) + 4 = 10$
$4(3) - 4 = 8$

If both sides of the equation are not equal there is a mistake somewhere!

Answer: $a = 3, \ b = 4$

Worked example 3: Solve the following to find x and y.

$3x = 10 + y,\ \ 4x - y = 16$

Are these equations in the form ax+by=c?

$3x - y = 10$
$4x - y = 16$

You may have to rearrange the equations.

$3x - y = 10$ (a) Label the equations.
$4x - y = 16$ (b)

$3x - y = 10$ (a) Check for the same coefficients for x or y.
$4x - y = 16$ (b)

In this case the y coefficient is -1 in both

$4x - y = 16$ (b) Once identified, eliminate the common coefficient, $-y - -y = 0$.
$3x - y = 10$ (a) $-$

In this case I have done (b) – (a) since the x coefficient in (b) is greater than the x coefficient in (a).

If you did (a)-(b) you will still get the same answer at the end!

$x = 6$ Now solve for x

$3x - y = 10$ (a)

To find y you substitute the value of x into one of the original equations.

$3(6) - y = 10$

In this case I have substituted x=6 into(a).

$18 - y = 10$

Now solve for y

(-18) (-18)

Subtract 18 from both sides.

$-y = -8$

$(\div -1)$ $(\div -1)$

Divide by -1 on both sides.

$y = 8$

$3x - y = 10$ (a)
$4x - y = 16$ (b)

Now check you answer by substituting you values of x and y into the equations.

$3(6) - 8 = 10$
$4(6) - 8 = 16$

If both sides of the equation are not equal there is a mistake somewhere!

Answer: $x = 6, y = 8$

Practice A: Solving linear simultaneous equations via elimination

Solve the following simultaneous equations:

1) $5a + 2b = 27,\ \ 5a + 4b = 49$

2) $4a + 3b = 44,\ \ 3a + 3b = 39$

3) $2x + y = 36,\ \ x - y = 9$

4) $9x = 19 + 4y,\ \ 4x + 4y = 20$

5) $6x - 3y = 12,\ \ 4x = 2 + 3y$

6) $a - 5b = 65,\ \ 2a - 5b = 85$

Assessment A: Solving linear simultaneous equations via elimination

Solve the following simultaneous equations for find the value of a and b.

$$10a - 10b + 40 = 0,\ \ 10a + 4b = 16$$

A	B	C	D
$a = \frac{-17}{3}, b = \frac{-296}{3}$	$a + b = 4$	The sum of a and b is 2.5	$b - a = -4$

Objective B: Manipulate variables in order to solve simultaneous equations via elimination.

Worked example 1: Solve the following to find x and y.

$7x + 3y = 23, \ 4x + 2y = 14$

$7x + 3y = 23$ (a)
$4x + 2y = 14$ (b)

Label the equations.

In this case none of the coefficients are the same.

$7x + 3y = 23$ (a) $(\times 2)$
$4x + 2y = 14$ (b) $(\times 3)$

We need to manipulate the equations so that the coefficients of x or y and the same in both equations.

In this case I will make the y coefficients the same. To do this I multiply equation (a) by the y coefficient in equation (b) and multiply equation(b) by the y coefficient in equation (a).

$14x + 6y = 46$ (c)
$12x + 6y = 42$ (d)

Label the equations.

Now the y coefficient is 6 in both equations.

$14x + 6y = 46$ (c)
$12x + 6y = 42$ (d)
$-$

Now eliminate the common coefficient, $6y - 6y = 0$.

In this case I have done (c) – (d) since the x coefficient in (c) is greater than the x coefficient in (d).

$2x = 4$ Now solve for x

$(\div 2)$ $(\div 2)$

$x = 2$ Divide by 2 on both sides

If you did (d)-(c) you will still get the same answer at the end!

$4x + 2y = 14$ (b)

To find y you substitute the value of x into one of the original equations.

In this case I substituted x=2 into (b).

$4(2) + 2y = 14$

$8 + 2y = 14$

Now solve for y.

(-8) (-8)

$2y = 6$

Subtract 8 from both sides.

$(\div 2)$ $(\div 2)$

$y = 3$

Divide by 2 on both sides.

$7x + 3y = 23$ (a)
$4x + 2y = 14$ (b)

Now check you answer by substituting you values of x and y into the equations.

$7(2) + 3(9) = 23$
$4(2) + 2(3) = 14$

If both sides of the equation are not equal there is a mistake somewhere!

Answer: $x = 2, y = 3$

Practice B: Manipulate variables in order to solve simultaneous equations via elimination.

1) $4a + 2b = 34,\ \ 3a + b = 21$

2) $9a - 4b = 59,\ \ 2a - b = 12$

3) $8a + 4b = -28,\ \ 3a - 12b = 30$

4) $12x + 3y = 9,\ \ 2x + 11y = -9$

5) $15s - 4t = 82,\ \ 5s - 9t = 12$

6) $8x = 4 + y,\ \ 3x + 8y = -166$

Assessment B: Manipulate variables in order to solve simultaneous equations via elimination.

Solve the following simultaneous equations for find the value of a and b.

$$5a + 2b = 38,\ \ 2a = 19 + 3y$$

A	B	C	D
$a + b = \frac{19}{11}$	$a = 1\,, b = -8$	$a - b = 9$	The product of a and b is -8

Objective C: Identify and set up simultaneous equations in order to solve a problem.

Worked example 1:

The cost of buying a coffee and a tea in a café is £4. The cost of buying a coffee and three teas in a café is £7. Work out the cost of buying a coffee and the cost of buying a tea.

Highlight the information required to set up simultaneous equations.

The cost of buying a coffee and a tea in a café is £4. The cost of buying a coffee and three teas in a café is £7. Work out the cost of buying a coffee and the cost of buying a tea.

Extract this information and using suitable variables write two equations.

1 coffee and 1 tea = £4 C + T = £4
1 coffee and 3 teas = £7 C + 3T = £7

In this case I have used C for coffee and T for tea.

$C + T = 4$ (a) Label the equations.
$C + 3T = 7$ (b)

$C + T = 4$ (a) Check for the same coefficients for x or y.
$C + 3T = 7$ (b)

Remember that if they do not have a coefficient the same you will have to manipulate the equations.

$C + 3T = 7$ (b)
$C + T = 4$ (a) −

Now eliminate the common coefficient, $C - C = 0$.

In this case I have done (b) – (a) since the T coefficient in (b) is greater than the T coefficient in (a).

$2T = 3$

Now solve for T.

$(\div 2)$ $(\div 2)$

Divide by 2 on both sides.

If you did (a)-(b) you will still get the same answer!

$T = 1.5$

$C + T = 4$ (a)

To find the value of C substitute the value of T into one of the equations.

In this case I have chosen (a).

$C + 1.5 = 4$

Now solve for C.
Subtract 1.5 from both sides.

(-1.5) (-1.5)

$C = 2.5$

Don't forget to check your answer by substituting the values of C and T into both equations!

$C + T = 4$ (a)
$C + 3T = 7$ (b)

If both sides of the equation are not equal there is a mistake somewhere!

$2.5 + 1.5 = 4$
$2.5 + 3(1.5) = 7$

Refer back to the question to see what it is asking for.

The cost of buying a coffee and a tea in a café is £4. The cost of buying a coffee and three teas in a café is £7. Work out the cost of buying a coffee and the cost of buying a tea.

Answer: Coffee costs £2.50, Tea costs £1.50.

Practice C: Identify and set up simultaneous equations in order to solve a problem.

1) The sum of Rosemary's age and Hannah's age is 102 years. The difference between Rosemary's age and Hannah's age is 52 years. Rosemary is older than Hannah. Find the age of each woman.

2) Five adult tickets and three child tickets for a movie cost £58. Two adult tickets and eight child tickets for a movie cost £47. Find the cost of each type of ticket.

3) Four chairs and two tables cost £218. Six chairs and seven tables cost £587. Find the total cost of buying twenty chairs and five tables.

4) A plumber charges a price for each hour, £h, and a fixed charge, £c. A 5 hour job costs £155 in total. An 8 hour job costs £230 in total. How much would a job that lasts 2 hours cost?

5) Barry buys 200 pieces of stationary for £76. Of the 200 pieces of stationary, x of them are rulers that cost 50p each and y of them are pens that cost 20p each. Find how many rulers Barry buys and how many pens he buys.

6) In a greengrocers, 4kg of bananas and 3kg of apples costs £7.50. In the same greengrocers, 3kg of bananas and 5kg of apples costs £8.10. How much would 2kg of bananas and 2kg of apples cost?

Assessment C: Identify and set up simultaneous equations in order to solve a problem.

When you book Bingham Hall for a conference, you pay a fixed booking fee plus a charge for each delegate at the conference. The total charge for a conference with 65 delegates was £192.50. The total change for a conference with 40 delegates was £180. What will be the charge for a conference with 70 delegates?

A	B	C	D
£160	£195	£197.50	£230

Amul and Kim have £10.70 between them. Amul has £3.70 more than Kim. How much does each person have?

A	B	C	D
Amul has £7.50 Kim has £3.70	Amul has £6.70 Kim has £5	Amul has £7.20 Kim has £3.50	Amul has £3.50 Kim has £7.20

Tutorial 2 – Solving simultaneous equations by substitution

Objectives for this tutorial

A. Solve equations using substitution.
B. Solve linear simultaneous equations via substitution.
C. Solve quadratic simultaneous equations via substitution.

Objective A: Solve equations using substitution

Worked example 1: Solve the following to find y when x=4.

$5x - 12 = y$ First, substitute $x = 4$ into the equation.

$5(4) - 12 = y$ Now evaluate.

Remember: 5(4) = 5 × 4 = 20.

$20 - 12 = y$

$y = 8$

Worked example 2: Solve the following to find y when x=4.

$4x - 20 = 3 + y$ First, substitute $x = 4$ into the equation.

$4(4) - 20 = 3 + y$ Now evaluate.

Remember: 4(4) = 4 × 4 = 16.

$16 - 20 = 3 + y$

$-4 = 3 + y$ Solve for y. To isolate y we must subtract 3 from both sides.

$-4 - 3 = 3 + y - 3$

$-7 = y$

$-4 - 3 = -7$

Worked example 3: Solve the following to find y when x=-4.

$3x - 2 = 7 + y$ First, substitute $x = -4$ into the equation.

$3(-4) - 2 = 7 + y$ Now evaluate.

Remember: 3(-4) = 3 × -4 = -12.

$-12 - 2 = 7 + y$

$-14 = 7 + y$ Solve for y. To isolate y we must subtract 7 from both sides.

$-14 - 7 = 7 + y - 7$

$-21 = y$

$-4 - 3 = -7$

Worked example 4: Solve the following to find y when x=-4.

$3 - 2x = 1 - y$ First, substitute $x = -4$ into the equation.

$3 - 2(-4) = 1 - y$ Now evaluate.

Remember: 2(-4) = 2 × -4 = -8

$3 - -8 = 1 - y$

Remember: 3 - - 8 = 3 + 8 = 11

$11 = 1 - y$ Solve for y. To isolate y we must first subtract 1 from both sides.

$11 - 1 = 1 - y - 1$

$10 = -y$ Now multiply by -1 on both sides.

Remember: -y × - 1 = y

$10 \times -1 = -y \times -1$

$-10 = y$

Practice A: Solve equations using substitution

Solve the following using substitution.

1) $9x - 10 = 26 + y$ when $x = 2$

2) $2 - 4x = y$ when $x = -2$

3) $12 - x = y - 7$ when $x = 20$

4) $-2x + y = 26$ when $x = -3$

5) $2(x - y) = -10$ when $x = 1$

6) $\frac{x}{5} - 2(y + 3) = -4$ when $x = -0.5$

Assessment A: Solve equations using substitution

Solve the following equation find the value of b when a = $-\frac{1}{4}$.

$$2(b - a) = a^2$$

A	B	C	D
$\frac{9}{16}$	$\frac{9}{32}$	$\frac{7}{16}$	$\frac{-7}{16}$

Objective B: Solve linear simultaneous equations via substitution.

Worked example 1: Solve the following to find x and y.

$3x + y = 17,\ \ 3x + 4y = 23$

$3x + y = 17$ (a) Label the equations.
$3x + 4y = 23$ (b)

Rearrange one of the equations to be of the form $y =$ or $x =$

$3x + y - 3x = 17 - 3x$ (a)

In this case rearrange (a) to be of the form y = 17-3x by subtracting 3x from each side.

$y = 17 - 3x$ (a)
$3x + 4y = 23$ (b)

You would get the same answer if you rearranged (a) to be x= (17-y)/3

Once rearranged, substitute (a) into (b).

$3x + 4(17 - 3x) = 23$ (b)

Substitute 17-3x wherever there is a y!

$3x + 4(17 - 3x) = 23$ (b) Evaluate.

Multiply out the bracket.

$3x + 68 - 12x = 23$ (b)

Now solve for x

$68 - 9x = 23$

Collect like terms.

3x – 12x = -9x

$68 - 9x - 68 = 23 - 68$

Subtract 68 from both sides.

$-9x = -45$

Divide by -9 on both sides.

$-9x \div -9 = -45 \div -9$

$-9 \div -9 = 1$

$x = 5$

Substitute the value of x into (a) to find the value of y.

$y = 17 - 3x$ (a)

$y = 17 - 3(5)$ (a)

Remember $3(5) = 3 \times 5$

$y = 17 - 15$

$y = 2$

$y = 17 - 3x$ (a)
$3x + 4y = 23$ (b)

Now check you answer by substituting you values of x and y into the equations.

$2 = 17 - 3(5)$
$3(5) + 4(2) = 23$

If both sides of the equation are not equal there is a mistake somewhere!

Answer: $x = 5, y = 2$

Worked example 2: Solve the following to find x and y.

$7x + 3y = 23, \ 4x + 2y = 14$

$7x + 3y = 23$ (a)
$4x + 2y = 14$ (b)

Label the equations.

Rearrange one of the equations to be of the form $y =$ or $x =$

$4x + 2y = 14$ (b)

In this case rearrange (b) to isolate y.

You would get the same answer if you rearranged (b) to be x= (14-2y)/4

$4x + 2y - 4x = 14 - 4x$

Subtract $4x$ from both sides of the equation.

$2y = 14 - 4x$

Divide both side by 2.

$2y \div 2 = (14 - 4x) \div 2$

Remember, the entire RHS is divided by 2.

Use brackets if it helps!

$7x + 3y = 23$ (a)
$y = 7 - 2x$ (b)

Once rearranged, substitute (b) into (a).

$7x + 3y = 23$ (a)

Substitute 7 – 2x wherever there is a y!

$7x + 3(7 - 2x) = 23$

Evaluate.

Multiply out the bracket!

$7x + 21 - 6x = 23$

Now solve for x

$x + 21 = 23$

Collect like terms.

7x – 6x = x

$x + 21 - 21 = 23 - 21$

Subtract 21 from both sides.

$x = 2$

Substitute the value of x into (b) to find the value of y.

$y = 7 - 2x$ (b)

$y = 7 - 2(2)$ (b)

$y = 7 - 4$

$y = 3$

$7x + 3y = 23$ (a)
$y = 7 - 2x$ (b)

Now check you answer by substituting you values of x and y into the equations.

$7(2) + 3(3) = 23$ (a)
$3 = 7 - 2(2)$ (b)

If both sides of the equation are not equal there is a mistake somewhere!

Answer: $x = 2, y = 3$

Practice B: Solve linear simultaneous equations via substitution.

Solve the following using substitution.

1) $x - 4y = -10, \quad y = 2x - 1$

2) $2a + b = -6, \ 2a - b = -14$

3) $5x - 3y = 10, \ x - y = 4$

4) $4x + 5y = -5, \ x = 10 + y$

5) $63(x - y) + 9 = 0, \ y - 3x = 1$

6) $2a - b = -11, \ y - 5 = x$

Assessment B: Solve linear simultaneous equations via substitution.

Solve the following simultaneous equation to find the value of x and y.

$$2(x - y - 1) = 0, \quad -(x - y) = 1$$

A	B	C	D
The product of x and y is 2	$x - y = 1$	The product of x and y is -2	$x - y = -3$

Objective C: Solve quadratic simultaneous equations via substitution.

Worked example 1: Solve the following to find x and

$y - x = 3, \;\; y - 5x + 2 = x^2$

$y - x = 3$ (a) Label the equations.
$y - 5x + 2 = x^2$ (b)

Rearrange the linear equation such that it is of the form x= or y=

$y - x + x = 3 + x$ (a) Add x from both sides of the equation.

$y = 3 + x$ (a)

In this case rearrange (a) to isolate y.

Once rearranged, substitute (a) into (b).

It is usually easier to substitute the linear equation into the nonlinear equation.

$y = 3 + x$ (a)
$y - 5x + 2 = x^2$ (b)

Substitute 3 + x wherever there is a y!

$(3 + x) - 5x + 2 = x^2$

$3 + x - 5x + 2 = x^2$

$-4x + 5 = x^2$ Collect like terms.

$0 = x^2 - 4x - 5$ Rearrange the quadratic equations such that it is equal to 0.

Try factorising first!

$0 = (x - 1)(x + 5)$

Solve the quadratic equation to find two values for x.

$x = 1, \qquad x = -5$ Substitute each value of x into (a) to find the corresponding value of y.

You should have two values for x unless you have a repeated root!

Substituting $x = 1$

$y = 3 + x$ (a)

$y = 3 + (1)$

$y = 4$

Substituting $x = -5$

$y = 3 + x$ (a)

$y = 3 + (-5)$

$y = -2$

Don't forget to check your answers by substituting both sets of values for x and y into the equations.

Answer: $x = 1$ and $y = 4$

Answer: $x = -5$ and $y = -2$

Worked example 2: Solve the following to find x and y.

$-y + 5x = -2, \;\; y - 3 = 9x^2 + 11x$

$-y + 5x = -2$ (a)
$y - 3 = 9x^2 + 11x$ (b)

Label the equations.

Rearrange the linear equation such that it is of the form x= or y=

$5x + 2 = y$ (a)
$y - 3 = 9x^2 + 11x$ (b)

In this case rearrange (a) to isolate y.

$y - 3 = 9x^2 + 11x$ (b)

Once rearranged, substitute (a) into (b).

It is usually easier to substitute the linear equation into the nonlinear equation.

$(5x + 2) - 3 = 9x^2 + 11x$

$5x + 2 - 3 = 9x^2 + 11x$

Substitute 5x + 2 wherever there is a y!

$5x - 1 = 9x^2 + 11x$

Collect like terms.

$0 = 9x^2 + 6x + 1$

Rearrange the quadratic equations such that it is equal to 0.

Try factorising first!

$0 = (3x + 1)(3x + 1)$

Solve the quadratic equation to find two values for x.

$x = -\frac{1}{3}$

There is a repeated root! Only one value for x.

$y - 3 = 9x^2 + 11x$ (b)

Substituting $x = -\frac{1}{3}$ into b to find the value for y.

$y - 3 = 9\left(-\frac{1}{3}\right)^2 + 11\left(-\frac{1}{3}\right)$

$y = 9\left(-\frac{1}{3}\right)^2 + 11\left(-\frac{1}{3}\right) + 3$

$y = 1 - \frac{11}{3} + 3$

$y = \frac{1}{3}$

Don't forget to check your answers by substituting both sets of values for x and y into the equations.

Answer: $x = -\frac{1}{3}$ and $y = \frac{1}{3}$

Worked example 3: Solve the following to find x and y.

$y + 2x = 7,\ \ 8 = x^2 - y^2$

$y + 2x = 7$ (a)
$8 = x^2 - y^2$ (b)

Label the equations.

Rearrange the linear equation such that it is of the form x= or y=

In this case rearrange (a) to isolate y.

$y = 7 - 2x$ (a)
$8 = x^2 + y^2$ (b)

Once rearranged, substitute (a) into (b).

Substitute 7-2x wherever there is a y!

In this case you need to square (7-2x).

$8 = x^2 - (7 - 2x)^2$

$8 = x^2 - (49 + 4x^2 - 28x)$

$8 = x^2 - 49 - 4x^2 + 28x$

$0 = -3x^2 - 57 + 28x$

Collect like terms.

Rearrange the quadratic equations such that it is equal to 0.

$0 = 3x^2 - 28x + 57$

Solve the quadratic equation to find two values for x.

In this case you need to use the quadratic formula! a= 3, b= -28 and c=57.

Be careful when substituting a,b and c into the formula!

You should have two values when using the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(3)(57)}}{2(3)}$

$x = \frac{28 \pm \sqrt{784 - 684}}{6}$

$x = \frac{28 \pm \sqrt{100}}{6}$

$x = \frac{28 \pm 10}{6}$

$x = 3,\ \ x = \frac{19}{3}$

Substituting $x = 3$

$y + 2x = 7$ (a)

$y + 2(3) = 7$

$y = 1$

Substituting $x = \frac{19}{3}$

$y + 2x = 7$ (a)

$y + 2\left(\frac{19}{3}\right) = 7$

$y = \frac{-17}{3}$

Don't forget to check your answers by substituting both sets of values for x and y into the equations.

Answer: $x = 1$ and $y = 4$ Answer: $x = -5$ and $y = -2$

Practice C: Solve quadratic simultaneous equations via substitution.

Solve the following using substitution.

1) $y + 1 = 2x, \quad y = x^2 - 2x + 2$

2) $b = -a^2 + 5a + 2, \quad b = 3a^2 - a - 2$

3) $x + y = 7, \quad xy = 10$

4) $y = x^2 + x - 2, \quad 2x - y + 4 = 0$

5) $x - 4y + 1 = 0, \; x^2 - 4xy + y^2 = 13$

6) $b - a = -2, \quad 2a^2 - ab = 11$

Assessment C: Solve quadratic simultaneous equations via substitution.

Solve the following simultaneous equation to find the value of x and y.

$$2(2x + y + 0.5) = 0, \qquad y + 7 = x^2 + x$$

A

$x = -4.46 \;\&\; y = 8.42$
$x = 1.46 \;\&\; y = -3.42$

B

$x = 4.46 \;\&\; y = -8.42$
$x = 1.46 \;\&\; y = -3.42$

C

$x = 4.46 \;\&\; y = -8.42$
$x = -1.46 \;\&\; y = 3.42$

Tutorial 3 – Solving equations graphically

Objectives for this tutorial

A. Plot linear and quadratic graphs accurately.
B. Solve linear simultaneous equations graphically.
C. Solve linear and quadratic simultaneous equations graphically.

Objective A: Plot linear and quadratic graphs accurately.

Worked example 1: Plot the graph $2x - y = 4$

x	-1	0	1	2	3
y					

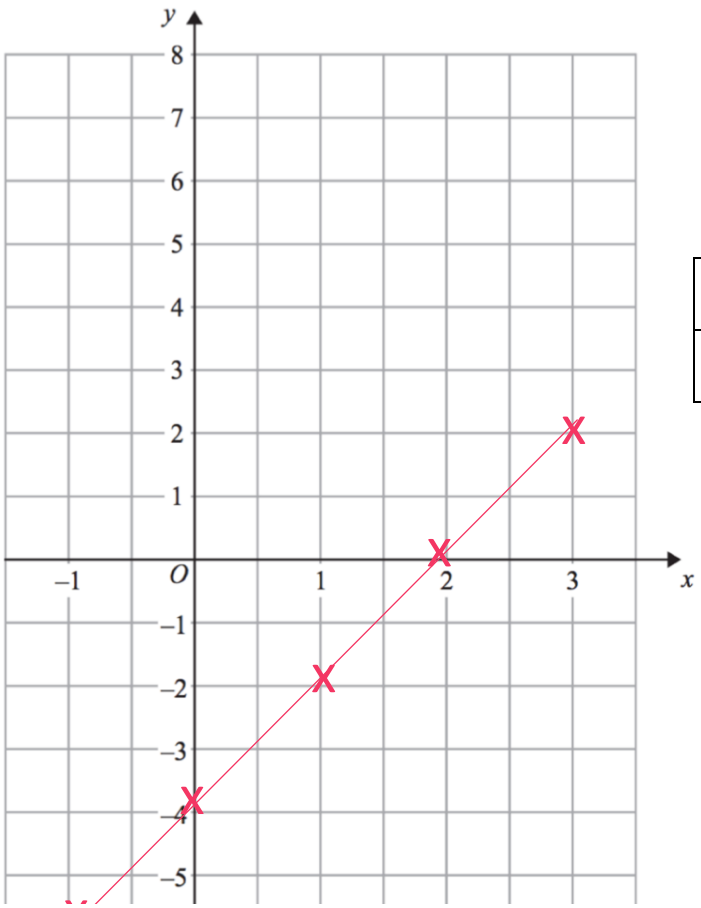
Make a table of values. Check the equation is of the form y=…

Rearrange the equation such that it is of the form y = 2x-4

$2x - y = 4$

Substitute the x values into the equation to find the corresponding y values.

x	-1	0	1	2	3
y	$2(-1) - 4 = -6$	$2(0) - 4 = -4$	$2(1) - 4 = -2$	$2(2) - 4 = 0$	$2(3) - 4 = 2$



Ensure that the axis is suitable for the table of values.

Plot the coordinates from the table of values.

x	-1	0	1	2	3
y	-6	-4	-2	0	2

Remember the x coordinate is first:
(x, y)
$(-1, -6), (0, -4), (1, -2), (2,0), (3,2)$

When you have plotted the coordinates connect them with a suitable line.

This is a linear graph so the line should be straight.

Worked example 2: Plot the graph $2x^2 - x - 3 = y$

Use a ruler when drawing linear graphs!

x	-1	0	1	2	3
y					

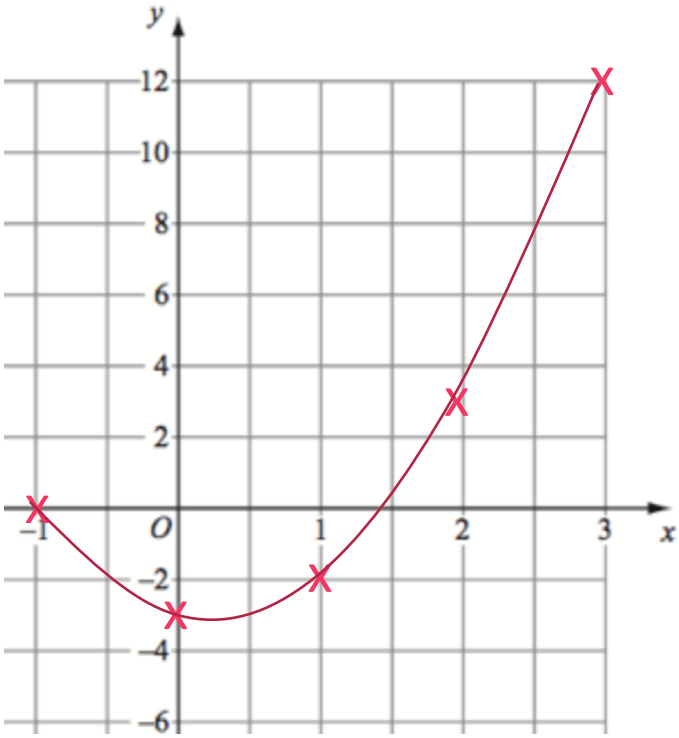
Make a table of values. Check the equation is of the form y=…

No rearrangement required.

$2x^2 - x - 3 = y$

Substitute the x values into the equation to find the corresponding y values.

x	-1	0	1	2	3
y	$2(-1)^2 - (-1) - 3 = 0$	$2(0)^2 - (0) - 3 = -3$	$2(1)^2 - (1) - 3 = -2$	$2(2)^2 - (2) - 3 = 3$	$2(3)^2 - (3) - 3 = 12$



Ensure that the axis is suitable for the table of values.

Plot the coordinates from the table of values.

x	-1	0	1	2	3
y	0	-3	-2	3	12

Remember the x coordinate is first:
(x, y)
$(-1,0), (0,-3), (1,-2), (2,3), (3,12)$

When you have plotted the coordinates connect them with a suitable line.

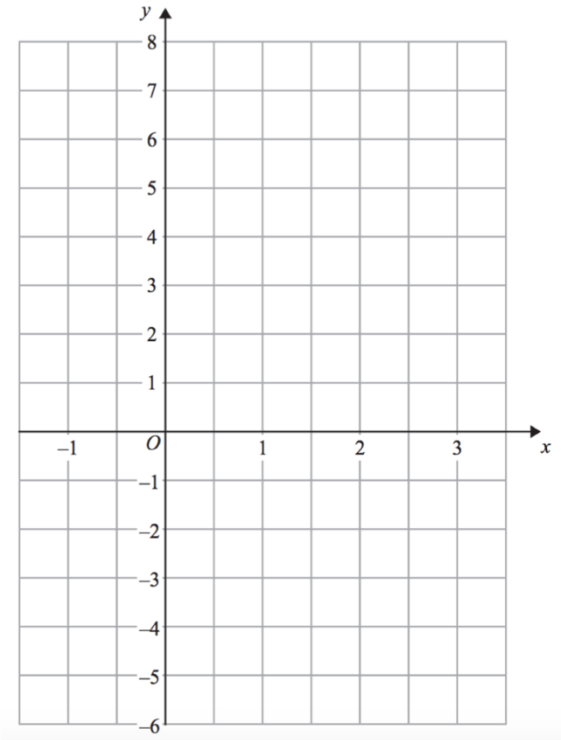
This is a quadratic graph so the line should be curved.

Make sure the line is smooth!

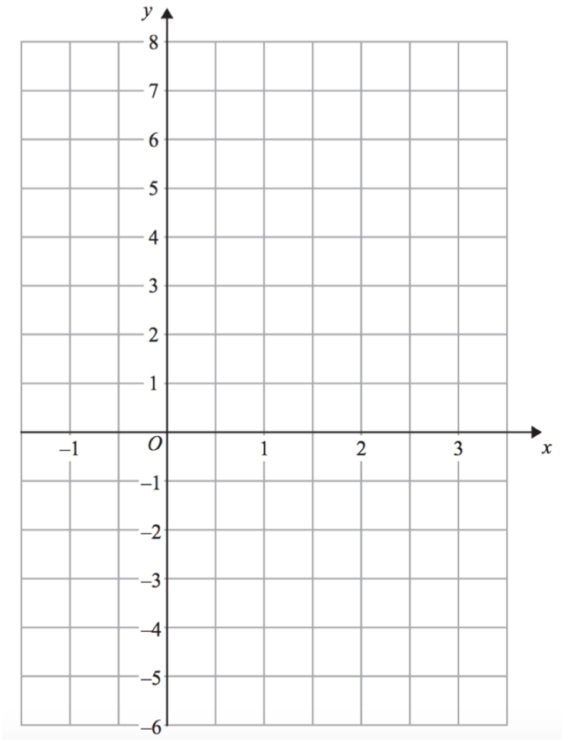
Practice A: Plot linear and quadratic graphs accurately.

Plot the following graphs.

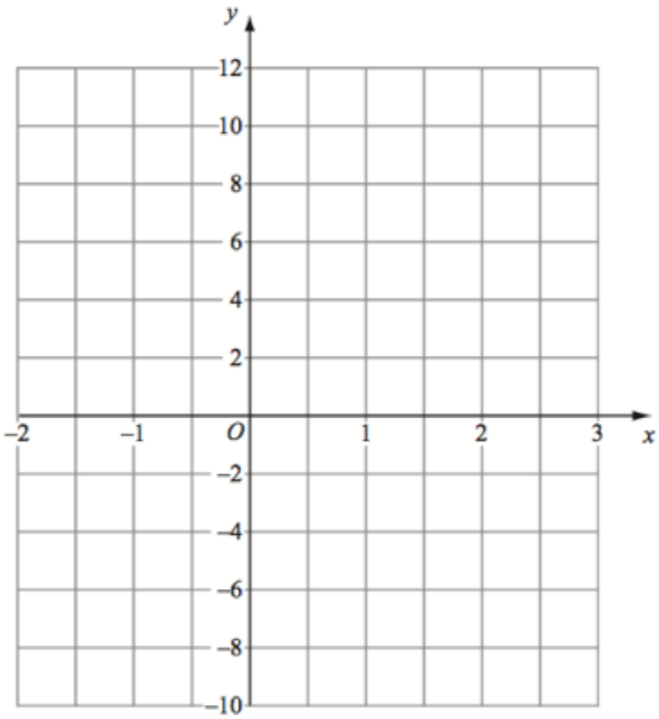
1) $x + y = 3$



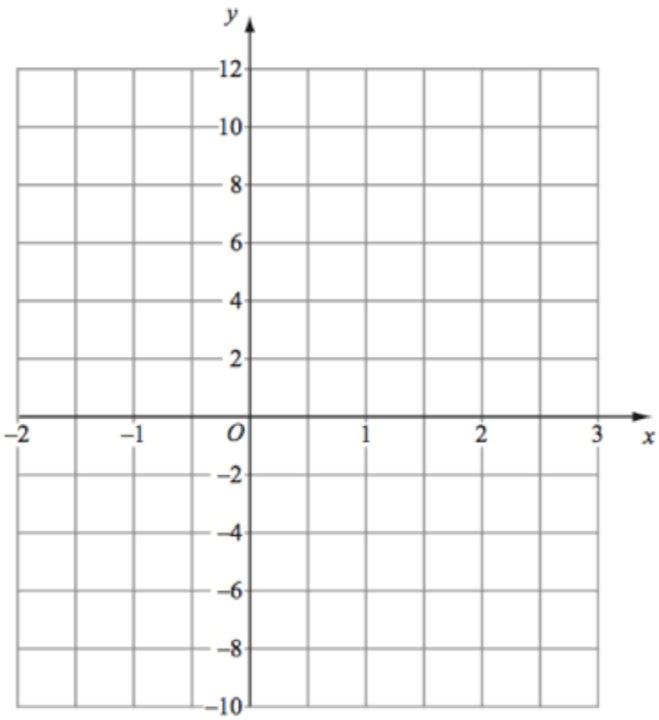
2) $x + 2y = -2$



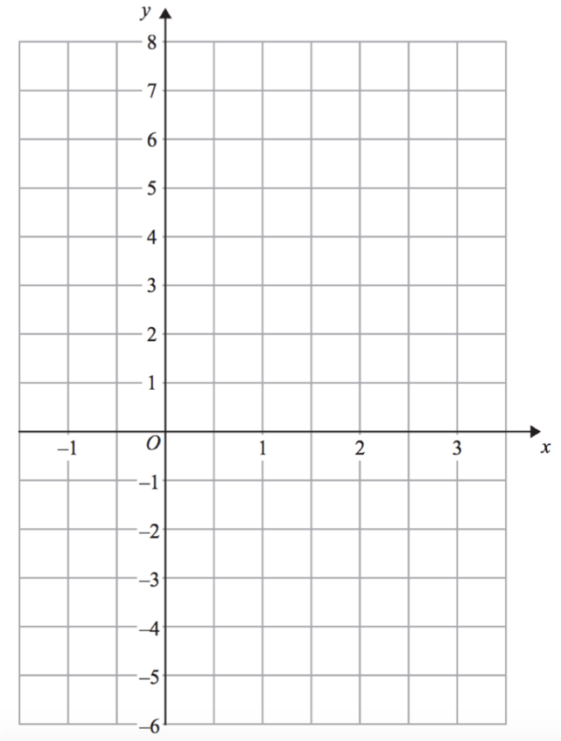
3) $x^2 - 3x + 2 = y$



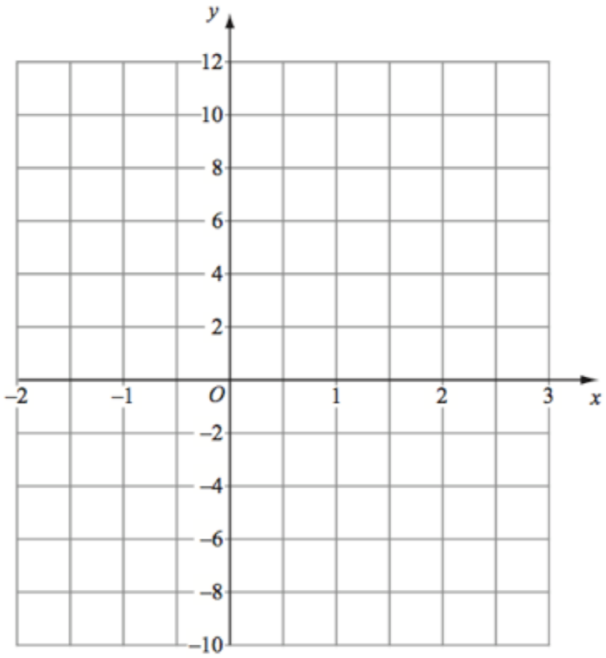
4) $x^2 + 3x - 6 = y$



5) $2x + y = 4$



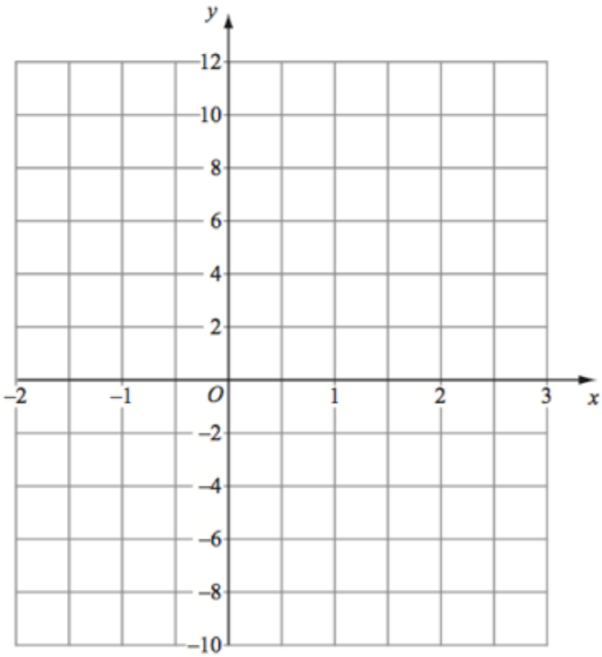
6) $x^2 - 2x = y$



Assessment A: Plot linear and quadratic graphs accurately.

Where does the graph $y = 8 - x^2$, cross the line $y = 4$?

x	-2	-1	0	1	2	3
y						



A

$x = -4$ and $x = 4$

B

$x = -2$ and $x = 2$

C

$x = 12$ and $x = 4$

D

$x = 4$

Objective B: Solve linear simultaneous equations graphically

Worked example 1: By drawing the graphs of $y = 3x + 1$ *and* $y + x = 7$, solve the simultaneous equations:
$y = 3x + 1, y + x = 7$

Make a table of values for each of the equations.

Remember to rearrange the equation if required.

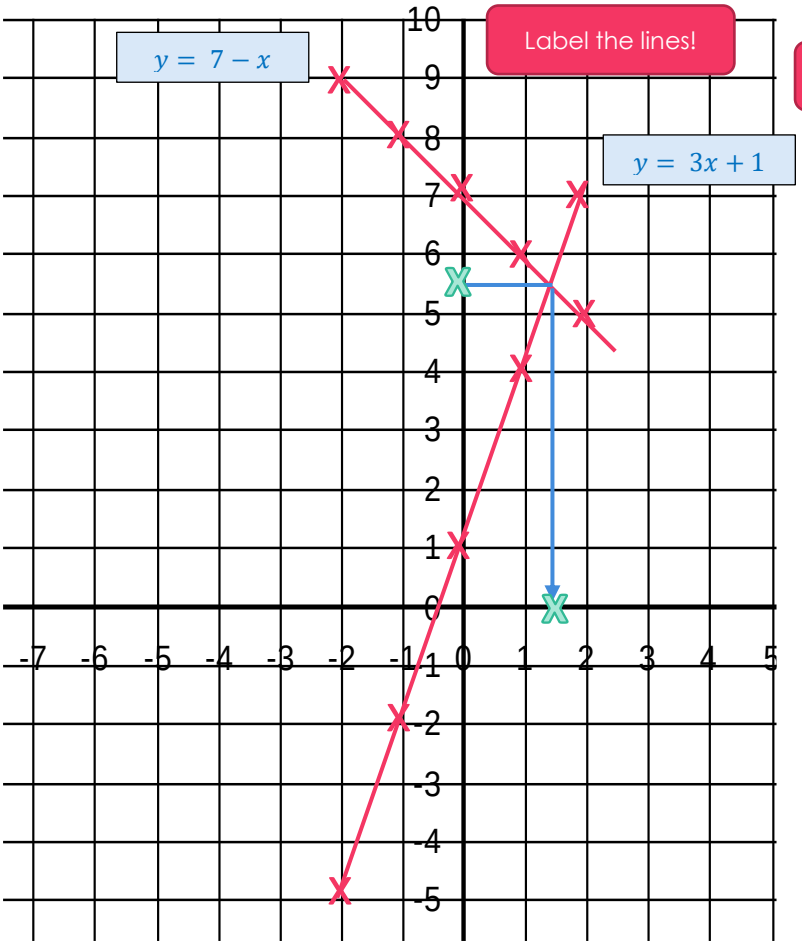
$y = 3x + 1$

x	-2	-1	0	1	2
y	3(-2)+1 = -5	3(-1)+1 = -2	3(0)+1 = 1	3(1)+1 = 4	3(2)+1 = 7

$y = 7 - x$

Substitute the x values into the equation to find the corresponding y values.

x	-2	-1	0	1	2
y	7-(-2) = 9	7-(-1) = 8	7-(0) = 7	7-(1) = 6	7-(2) = 5



Label the lines!

Ensure that the axis is suitable for the table of values.

Plot the coordinates from the table of values.

When you have plotted the coordinates connect them with a suitable line.

These are linear equations so make sure the lines are straight!

Once drawn the lines will intersect.

If the lines don't cross double check your coordinates!

Where the lines intersect draw a straight line horizontally to the y axis and vertically to the x axis.

Where the lines cross the axes is your approximate solution. X

Make sure you don't draw the lines too thick; you need to be as accurate as possible.

Answer: $(1.4, 5.5)$ $x = 1.4,\ y = 5.5$

Worked example 2: By drawing the graphs of $y = 3x + 5$ and $x - 2y + 6 = 0$, solve the simultaneous equations:
$y = 3x + 5, x - 2y + 6 = 0.$

Make a table of values for each of the equations.

Remember to rearrange the equation if required.

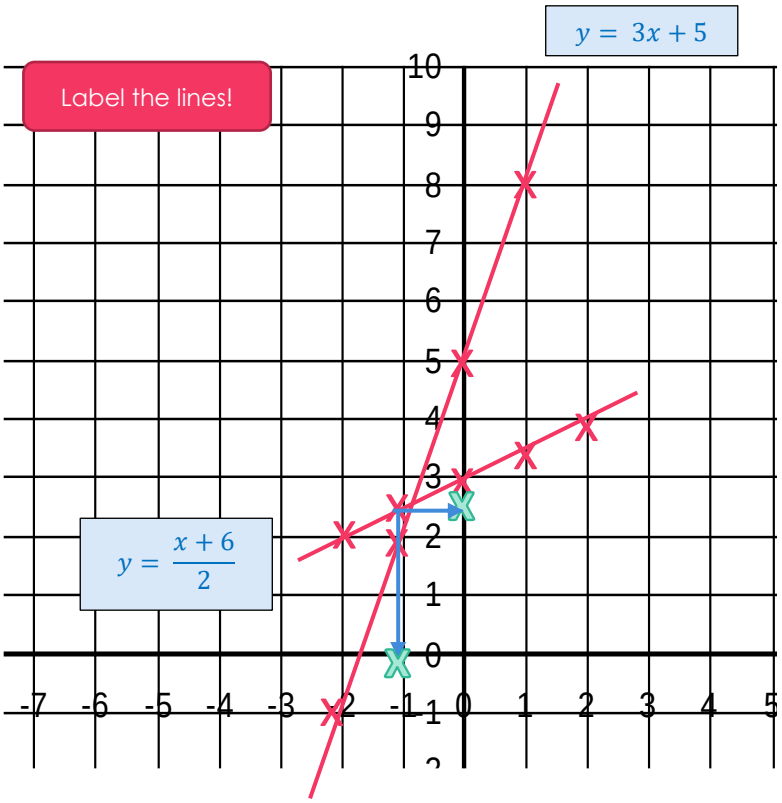
$y = 3x + 5$

x	-2	-1	0	1	2
y	3(-2)+5 = -1	3(-1)+5 = 2	3(0)+5 = 5	3(1)+5 = 8	3(2)+5 = 11

$y = \frac{x+6}{2}$

Substitute the x values into the equation to find the corresponding y values.

x	-2	-1	0	1	2
y	$\frac{-2+6}{2} = 2$	$\frac{-1+6}{2} = 2.5$	$\frac{0+6}{2} = 3$	$\frac{1+6}{2} = 3.5$	$\frac{2+6}{2} = 4$



Ensure that the axis is suitable for the table of values.

Plot the coordinates from the table of values.

When you have plotted the coordinates connect them with a suitable line.

These are linear equations so make sure the lines are straight!

Once drawn the lines will intersect.

If the lines don't cross double check your coordinates!

Where the lines intersect draw a straight line horizontally to the y axis and vertically to the x axis.

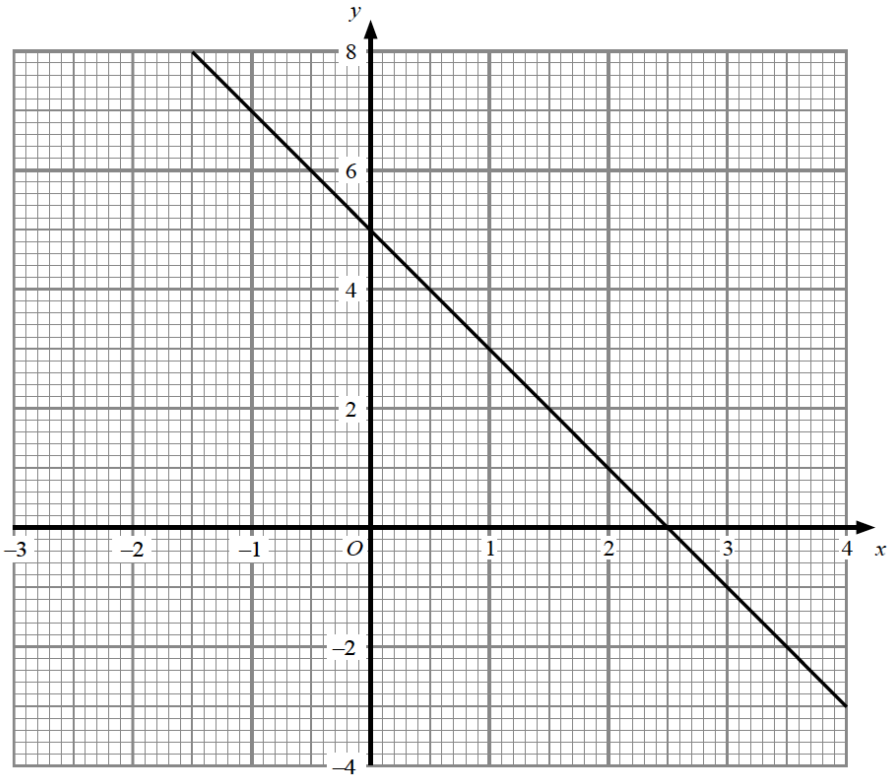
Where the lines cross the axes is your approximate solution.

Make sure you don't draw the lines too thick; you need to be as accurate as possible.

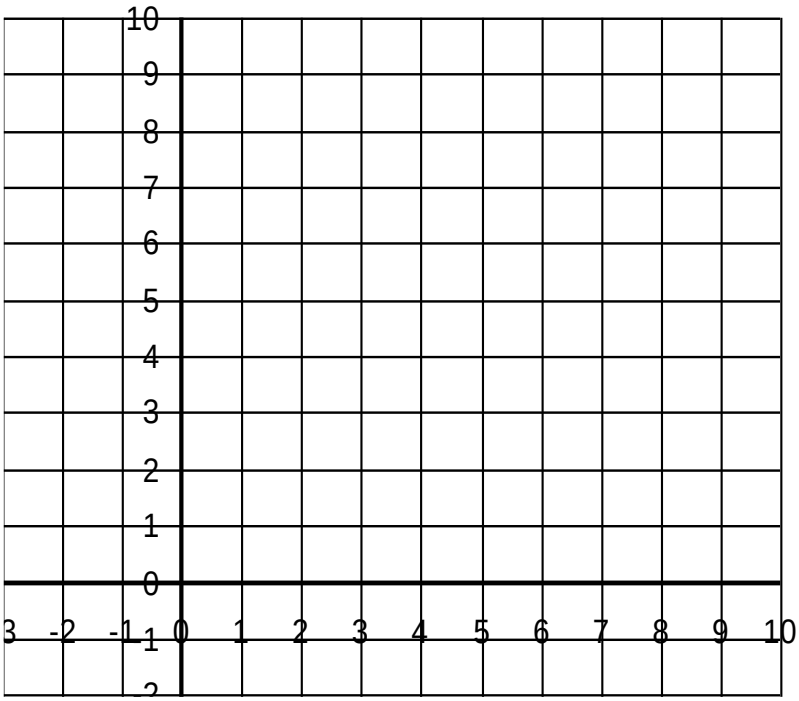
Answer: $(-1.1, 2.5)$ $x = -1.1, y = 2.5$

Practice B: Solve linear simultaneous equations graphically

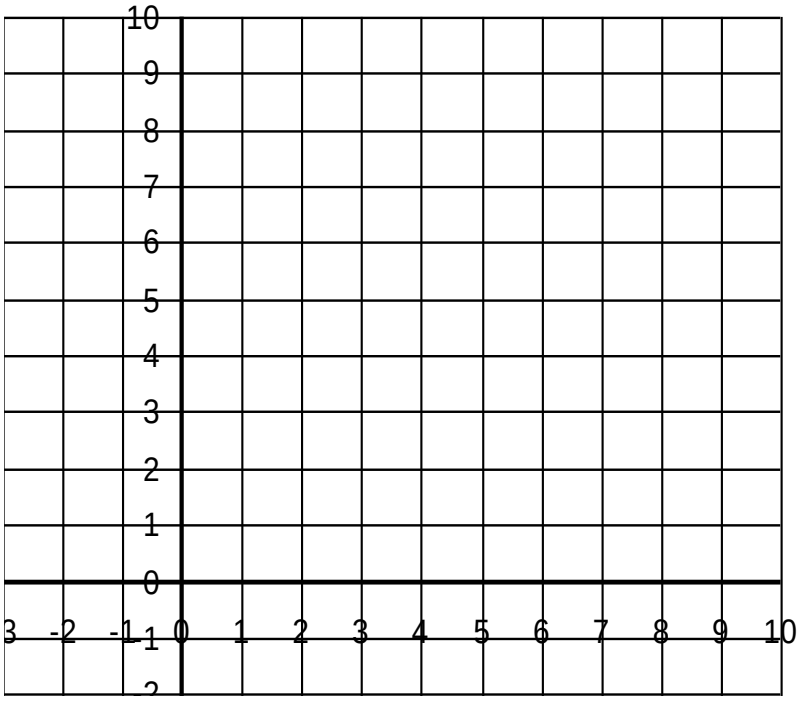
1) The graph $y + 2x = 5$ is plotted below. Draw the graph $y = 2x - 1$ to solve the simultaneous equations:
$y = 5 - 2x, y = 2x - 1$



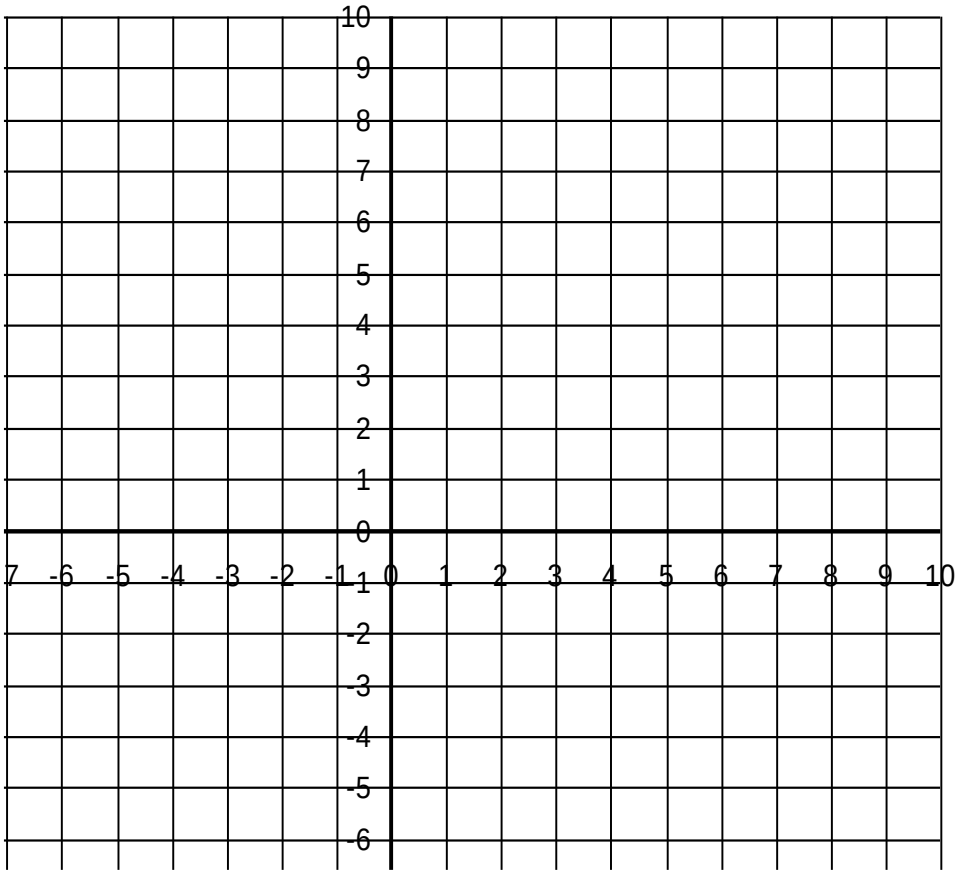
2) Use the graph to solve the simultaneous equations:
$3y + 2x = 12,\ y = x - 1$



3) By drawing a graph to solve the simultaneous equations:
$y = \frac{1}{2}x + 2,\ 2y + 3x = 12$

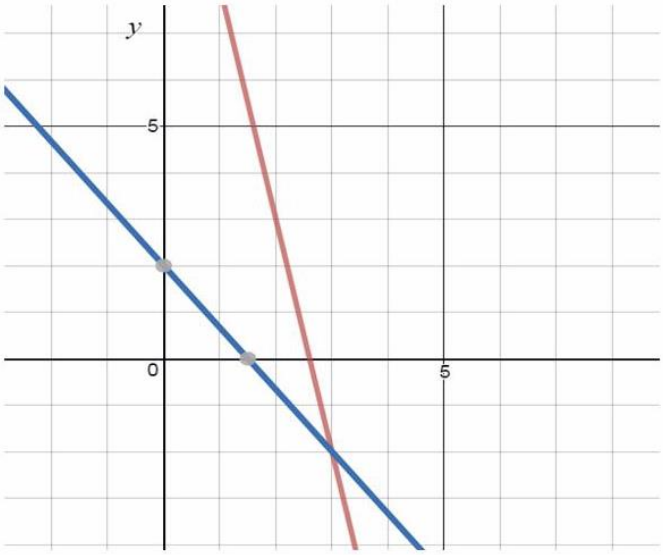


4) By drawing a graph to solve the simultaneous equations:
$y = 2x + 3,\ y + 3x = 8$



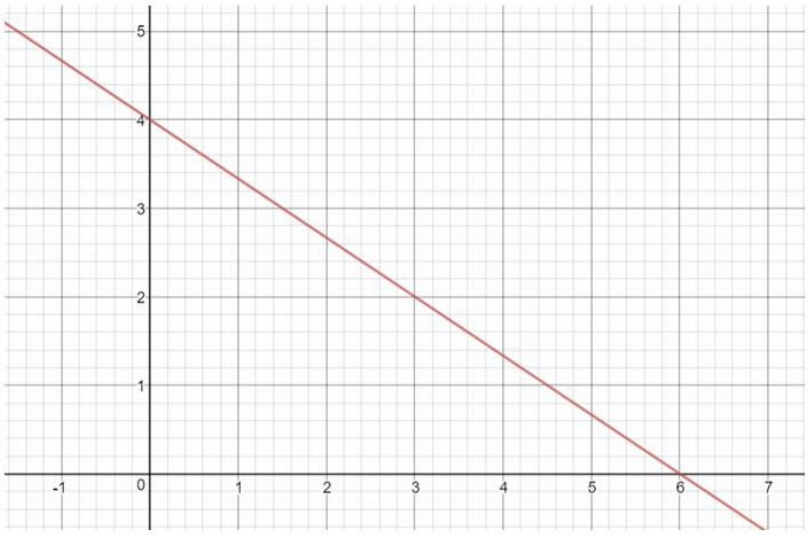
Assessment B: Solve linear simultaneous equations graphically

On the axes, the graphs of $y + 5x = 13$ and $4x + 3y = 6$ have been drawn.
Use the graph to solve the simultaneous equations: $y + 5x = 13,\ 4x + 3y = 6$.



A	B	C	D
$(2, 2.5)$	$(2-, 3)$	$(3-, 2)$	$(2.5, 2)$

This is the graph of $y = 4 - \frac{2}{3}x$. Use the graph to solve $2 = 4 - \frac{2}{3}x$.



A	B	C	D
$x = 2\frac{2}{3}$	$x = 3$	$x = 4$	$x = 6$

Objective C: Solve linear and quadratic simultaneous equations graphically

Worked example 1: Using graphical methods, find an approximate solution to $-4 = x^2 + 2x - 5$.

First, treat each side of the equation as two separate equations: $y = -4$ & $y = x^2 + 2x - 5$.

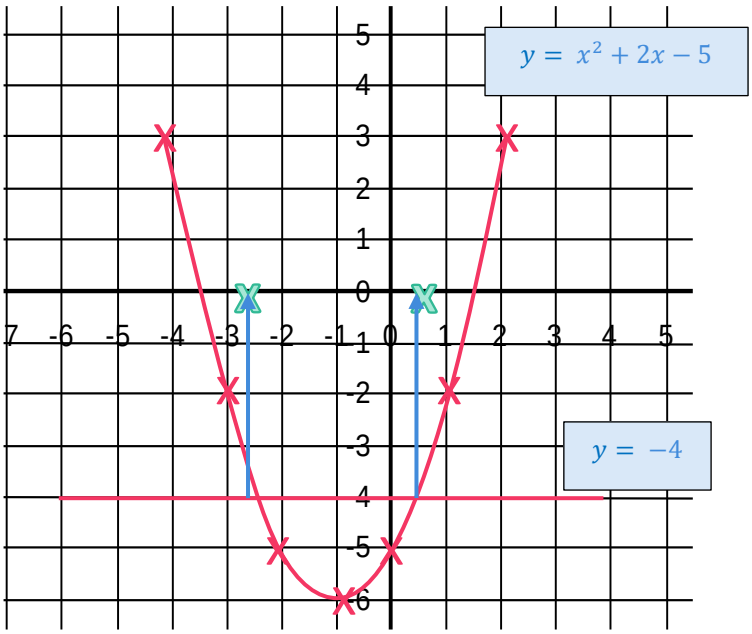
Make a table of values for each of the equations.

$y = x^2 + 2x - 5$

x	-4	-3	-2	-1
y	$(-4)^2 + 2(-4) - 5 = 3$	$(-3)^2 + 2(-3) - 5 = -2$	$(-2)^2 + 2(-2) - 5 = -5$	$(-1)^2 + 2(-1) - 5 = -6$
x	0	1	2	3
y	$(0)^2 + 2(0) - 5 = -5$	$(1)^2 + 2(1) - 5 = -2$	$(2)^2 + 2(2) - 5 = 3$	$(3)^2 + 2(3) - 5 = 10$

You don't need a table of values for $y = -4$

Ensure that the axis is suitable for the table of values.



Plot the coordinates from the table of values.

When you have plotted the coordinates connect them with a suitable line.

Once drawn the lines will intersect.

If the lines don't cross double check your coordinates!

Where the lines intersect draw a straight line vertically to the x axis.

The lines intersect at two places!

Where the lines cross the axes is your approximate solution. X

Make sure you don't draw the lines too thick; you need to be as accurate as

Answer: $(0.5, -4)$ $x = 0.5, y = -4$
$(-2.5, -4)$ $x = -2.5, y = -4$

Worked example 2: Using graphical methods, find an approximate solution to $x - 1 = x^2 + 2x - 5$.

First, treat each side of the equation as two separate equations: $y = x - 1$ & $y = x^2 + 2x - 5$.

Make a table of values for each of the equations.

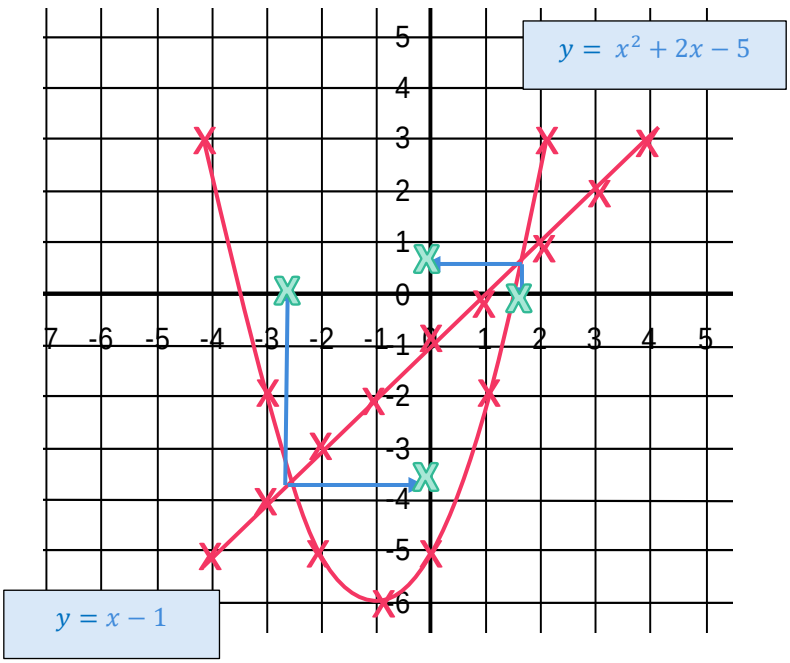
$y = x^2 + 2x - 5$

x	-4	-3	-2	-1
y	$(-4)^2 + 2(-4) - 5 = 3$	$(-3)^2 + 2(-3) - 5 = -2$	$(-2)^2 + 2(-2) - 5 = -5$	$(-1)^2 + 2(-1) - 5 = -6$
x	0	1	2	3
y	$(0)^2 + 2(0) - 5 = -5$	$(1)^2 + 2(1) - 5 = -2$	$(2)^2 + 2(2) - 5 = 3$	$(3)^2 + 2(3) - 5 = 10$

$y = x - 1$

x	-4	-3	-2	-1	0	1	2	3	4
y	$-4 - 1 = -5$	$-3 - 1 = -4$	$-2 - 1 = -3$	$-1 - 1 = -2$	$0 - 1 = -1$	$1 - 1 = 0$	$2 - 1 = 1$	$3 - 1 = 2$	$4 - 1 = 3$

Ensure that the axis is suitable for the table of values.



Plot the coordinates from the table of values.

When you have plotted the coordinates connect them with a suitable line.

Once drawn the lines will intersect.

If the lines don't cross double check your coordinates!

Where the lines intersect draw a straight line horizontally to the y axis and vertically to the x axis.

The lines intersect at two places!

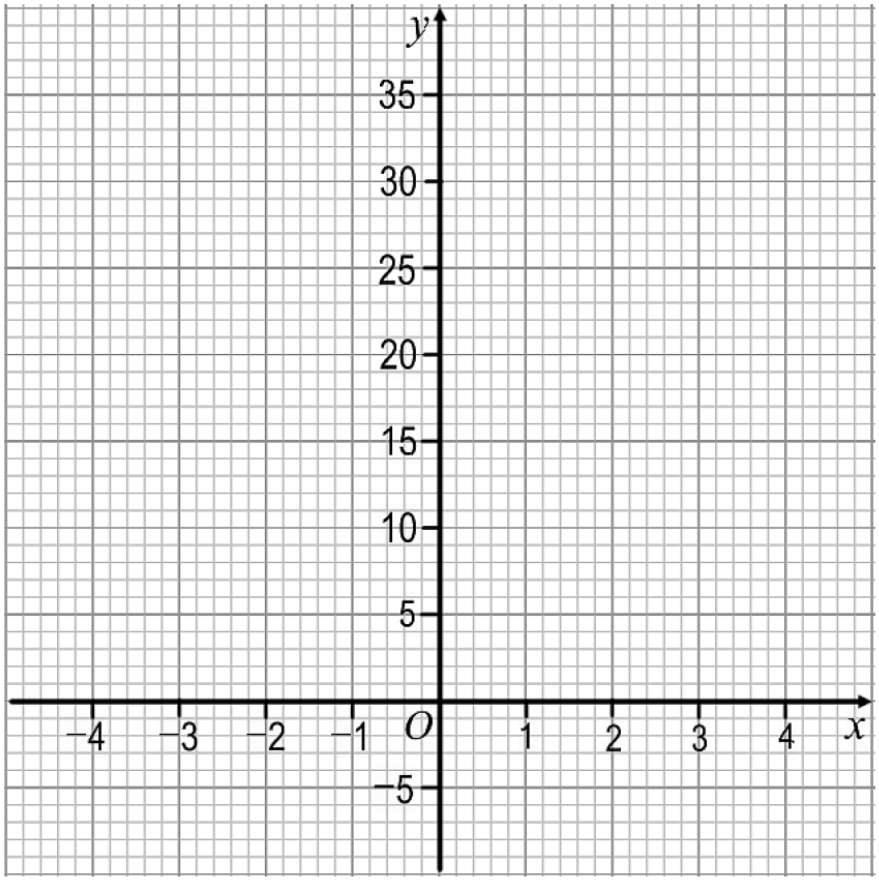
Where the lines cross the axes is your approximate solution. X

Make sure you don't draw the lines too thick; you need to be as accurate as possible.

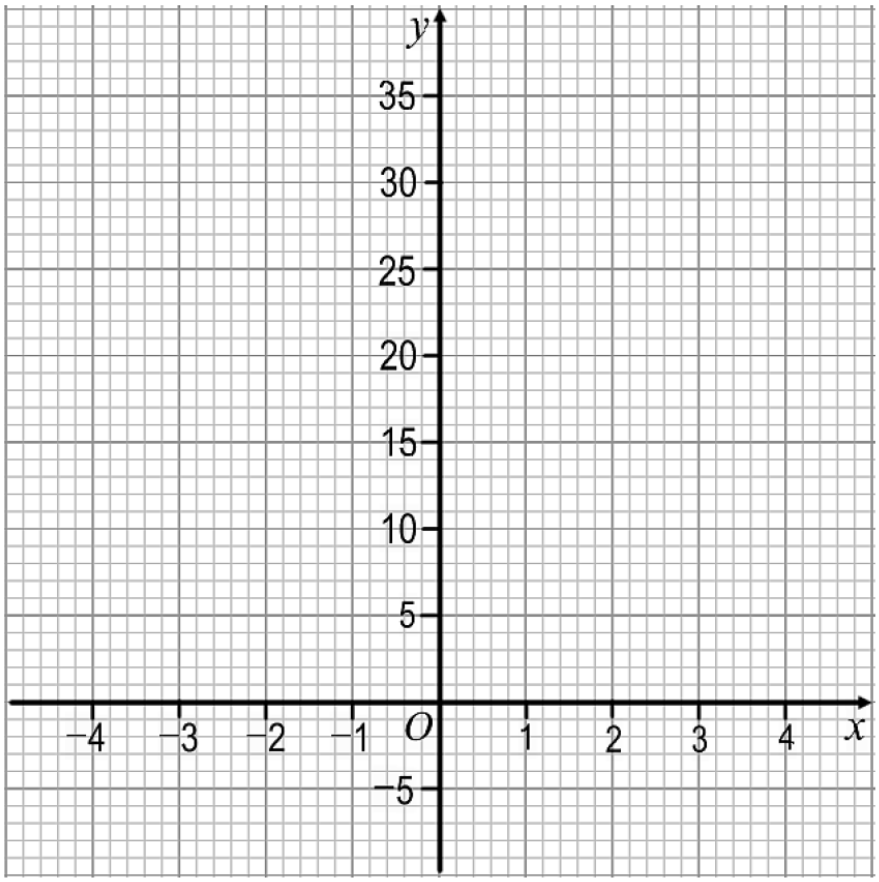
Answer: $(-2.7, -3.7)$ $x = -2.7, y = -3.7$
$(1.7, 0.7)$ $x = 1.7, y = 0.7$

Practice C: Solve linear and quadratic simultaneous equations graphically.

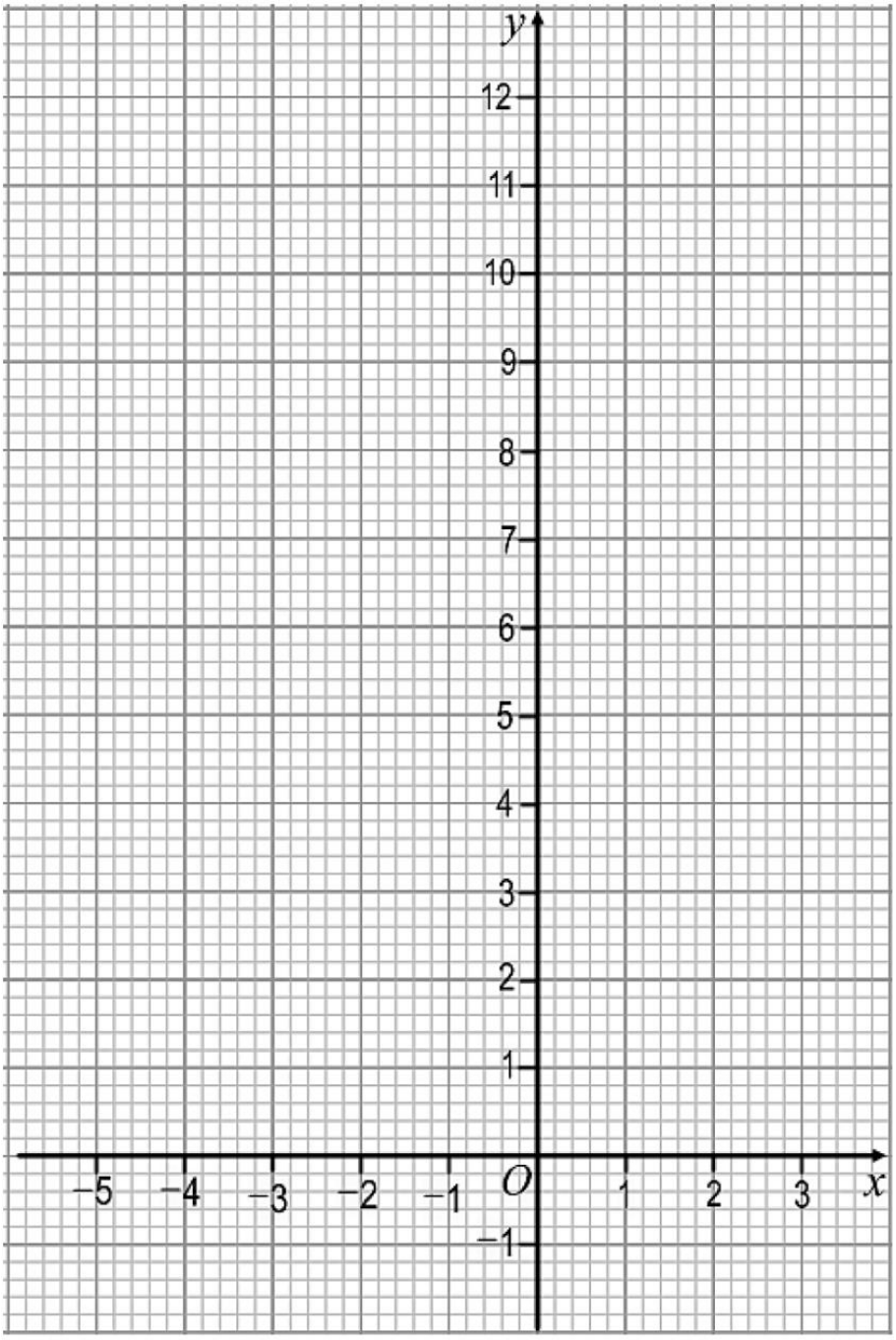
1) By drawing a graph to solve $x^2 + 4x + 1 = x - 1$



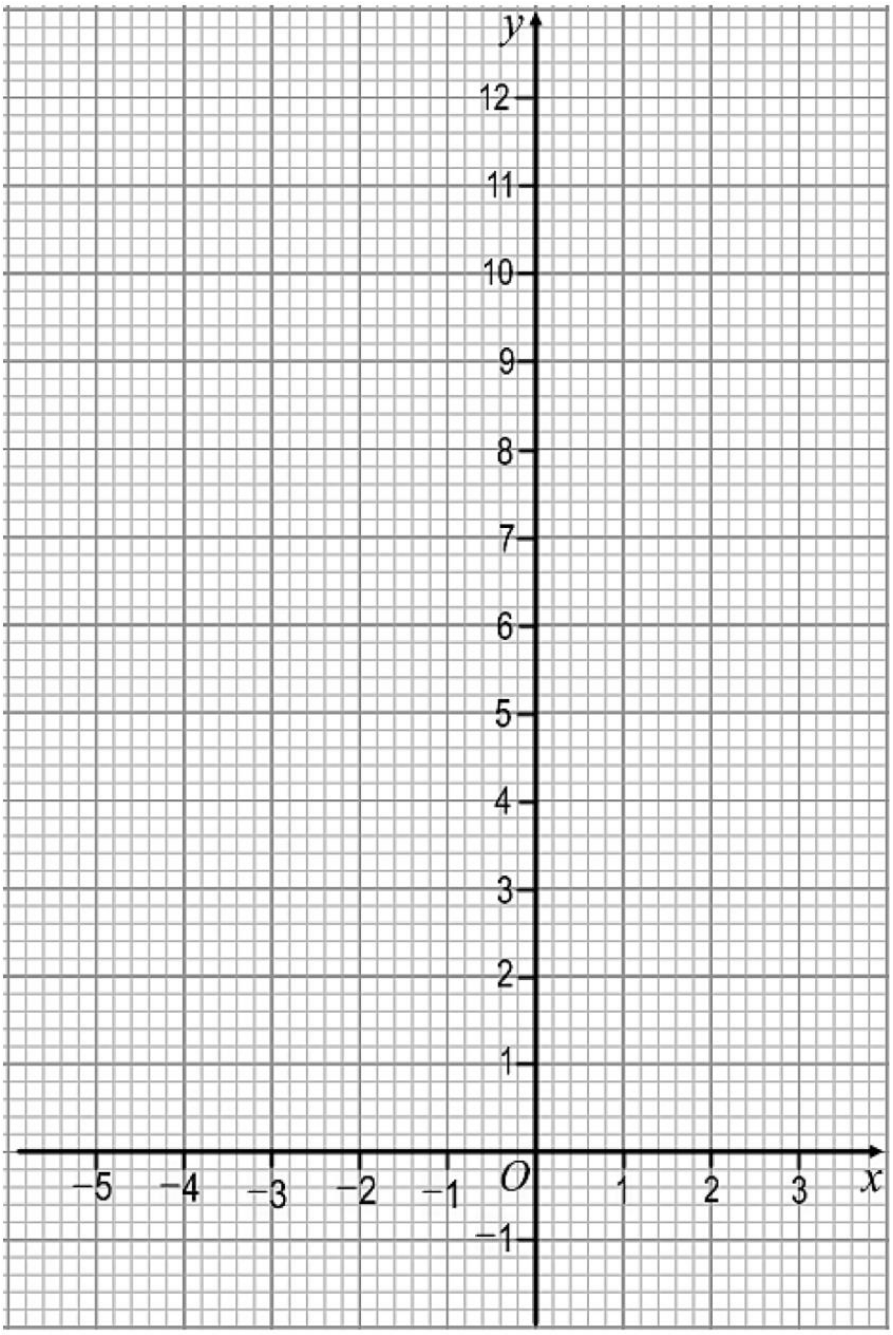
2) By drawing a graph to solve the simultaneous equations: $y = x^2 - 2x + 1,\ y + x = 3$



3) By drawing a graph to solve $x^2 + 3x + 1 = 2x + 5$.

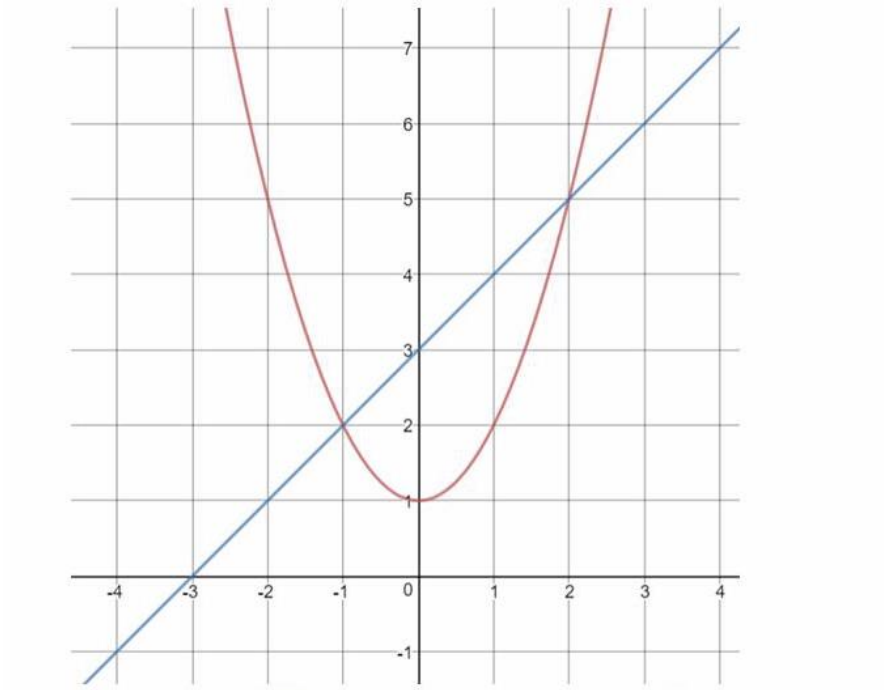


4) By drawing a graph to solve the simultaneous equations:
$4y = 6x^2 - 2,\ y = 2x + 1$



Assessment C: Solve linear and quadratic simultaneous equations graphically.

On the axes, the graphs of $y - 3 = x$ and $x^2 + 1 = y$ have been drawn.
Using the diagram, what is the solution to $x + 3 = x^2 + 1$?



A

$x = 5$

B

$x = 5$ & $x = 2$

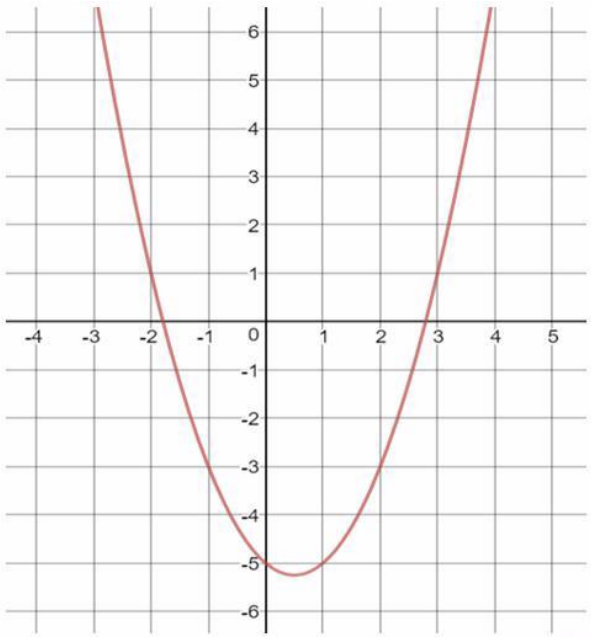
C

$x = 2$

D

$x = -1$ & $x = 2$

This is the graph of $y = f(x)$. What is the solution to $1 = f(x)$.



A

$x = 0$ & $x = -5$

B

$x = 0.5$

C

$x = -2$ & $x = 3$

D

$x = -3$

Tutorial 4 – Solving simultaneous inequalities graphically

Objectives for this tutorial

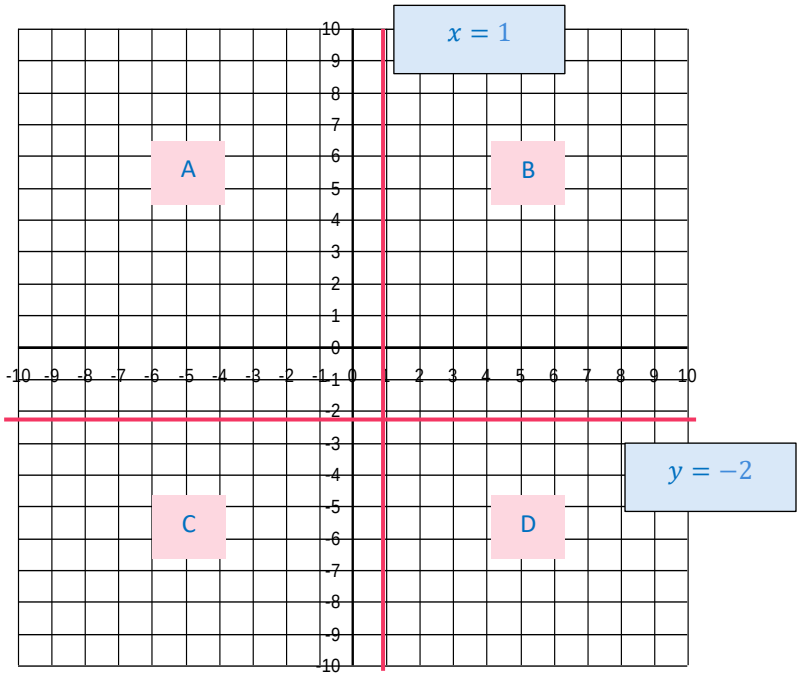
A. Identify regions on a graph which satisfy an inequality.
B. Solve linear simultaneous inequalities graphically.
C. Solve linear and quadratic simultaneous inequalities graphically.

Objective A: Identify regions on a graph which satisfy an inequality

Worked example 1: Shade the region which satisfies $y > -2,\ x < 1$.

First, draw the lines:
$y = -2$ and $x = 1$.

The lines should intersect.



Now study each region to see if it satisfies the inequalities.

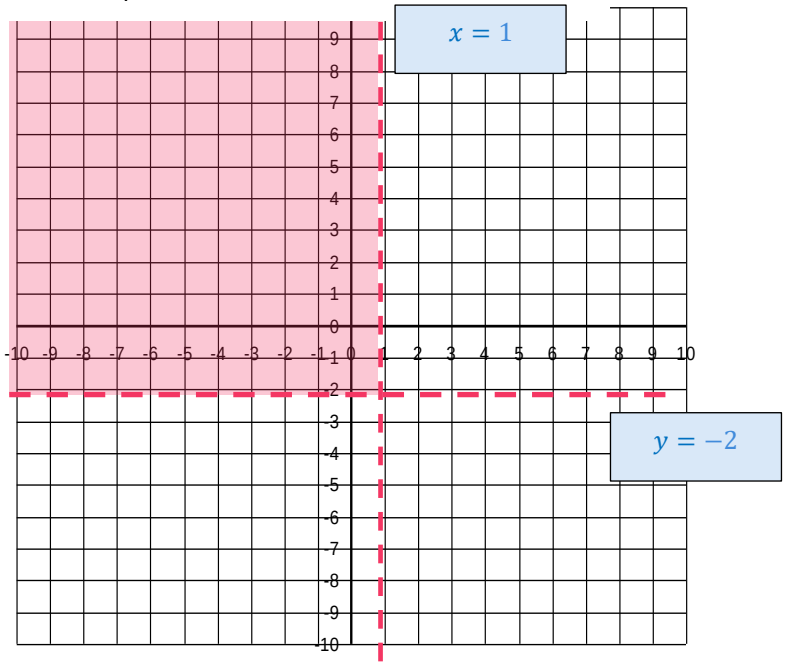
Both inequalities must be satisfied!

$$y > -2, \qquad x < 1$$

Test coordinates from each region.
Region A – test $(0,0)$. $0 > -2$ and $0 < 1$. The inequalities are both true.
Region B – test $(2,1)$. $1 > -2$ and $2 < 1$. The inequalities aren't both true.
Region C – test $(-2,-3)$. $-3 > -2$ and $-2 < 1$. The inequalities aren't both true.
Region D – test $(2,-3)$. $-3 > -2$ and $2 < 1$. The inequalities aren't true.

You can test any coordinates within each region you will come to the same conclusion.

Region A should be shaded!



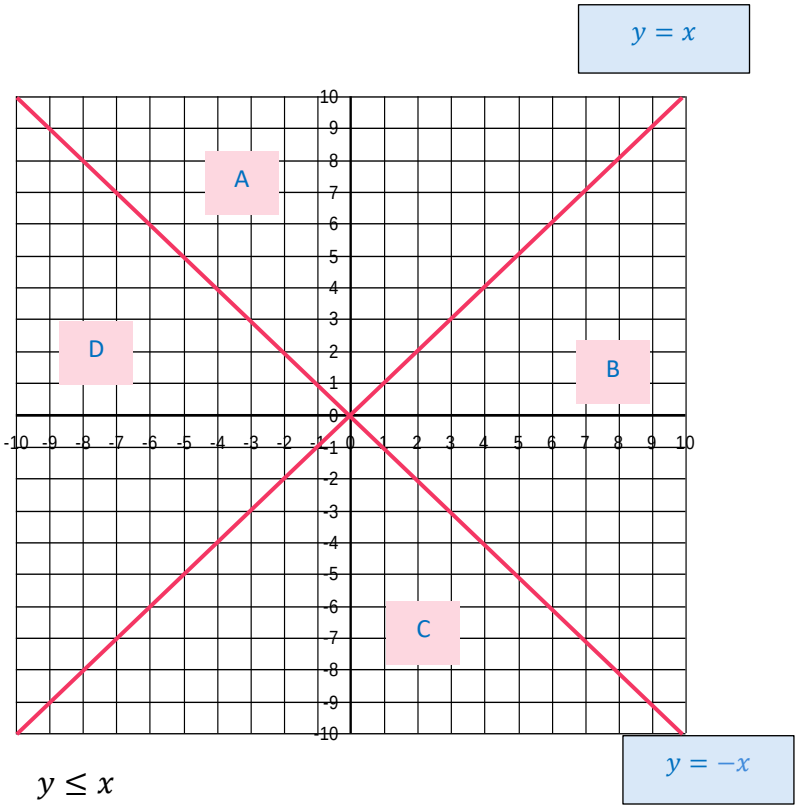
The lines must be dashed since the inequalities are strictly less than and greater than.

The inequalities do not include -2 and 1.

Worked example 2: Shade the region which satisfies $y > -x,\ y \leq x$.

First, draw the lines:
$y = -x$ and $y = x$.

The lines should intersect.



Now study each region to see if it satisfies the inequalities.

Both inequalities must be satisfied!

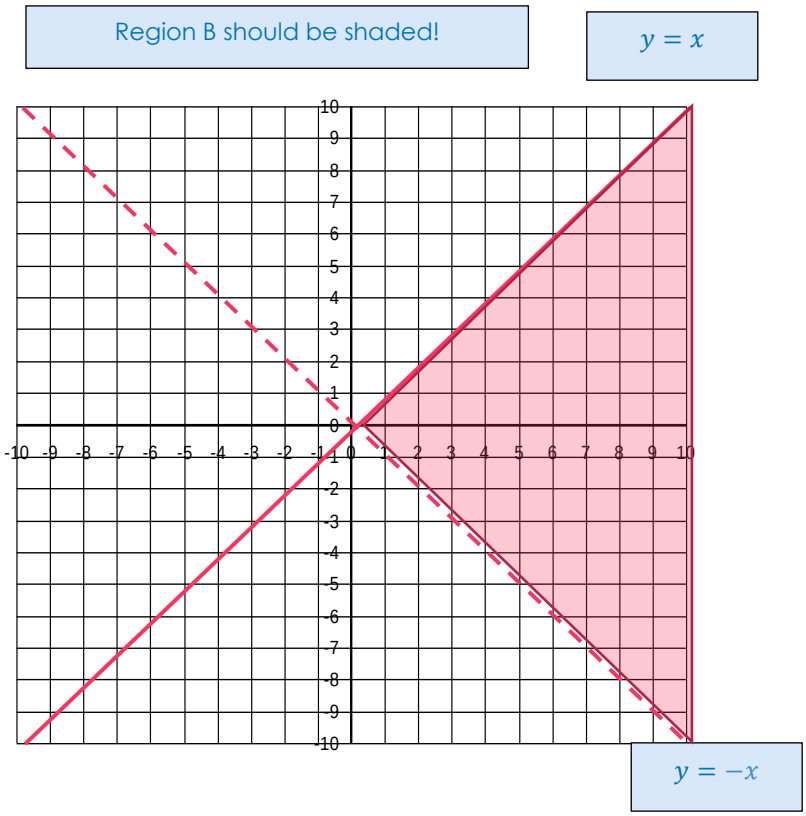
$$y > -x, \qquad y \leq x$$

Test coordinates from each region.
Region A – test $(1,2)$. $1 > 2$ and $2 \leq 1$. The inequalities aren't true.
Region B – test $(2,1)$. $2 > 1$ and $1 \leq 2$. The inequalities are both true.
Region C – test $(-1,-3)$. $-3 > 1$ and $2 - 3 \leq -1$. The inequalities aren't both true.
Region D – test $(-2,1)$. $1 > 2$ and $1 \leq -2$. The inequalities aren't true.

Don't forget –(-1) = 1

You can test any coordinates within each region you will come to the same conclusion.

Region B should be shaded!



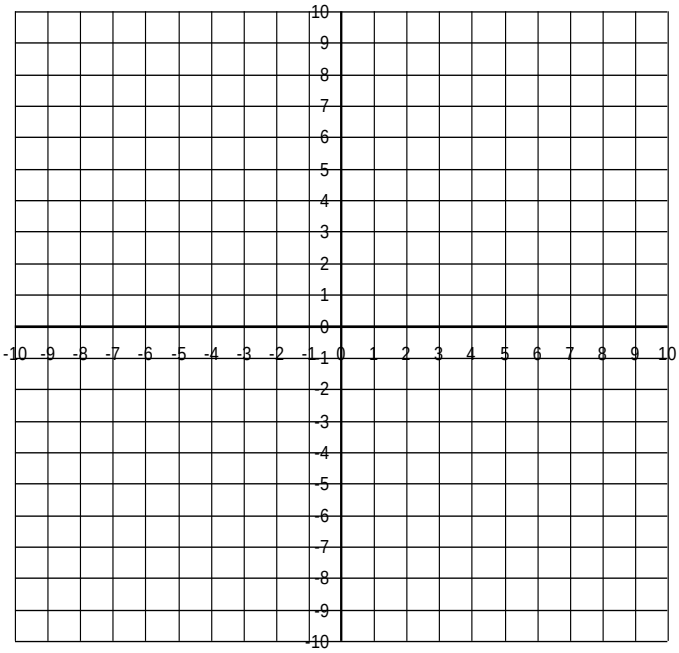
The line $y = -x$ must be dashed since the inequalities are strictly less than. The line $y = x$ must be solid since the inequality is greater than or equal to.

The inequality $y \leq x$ includes the values on the line $y = x$. The inequality $y > -x$ does not include the values on the line $y = -x$.

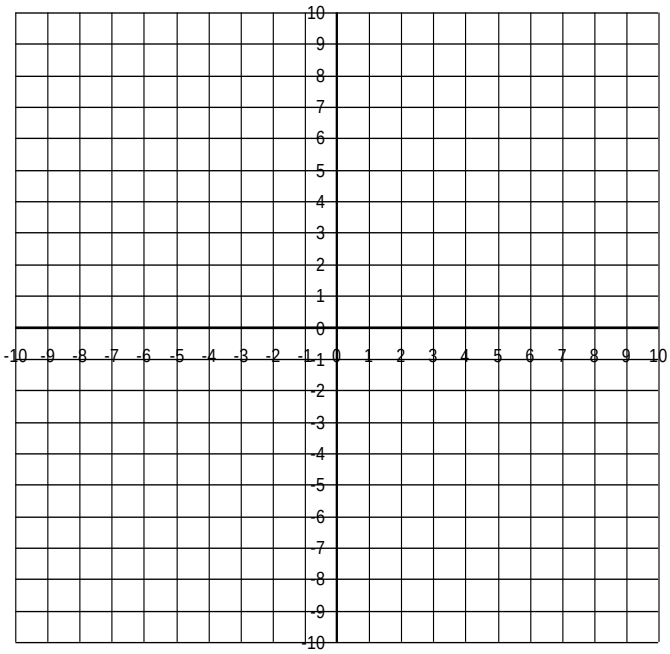
Practice A: Identify regions on a graph which satisfy an inequality

Shade the regions that satisfy the inequality/inequalities.

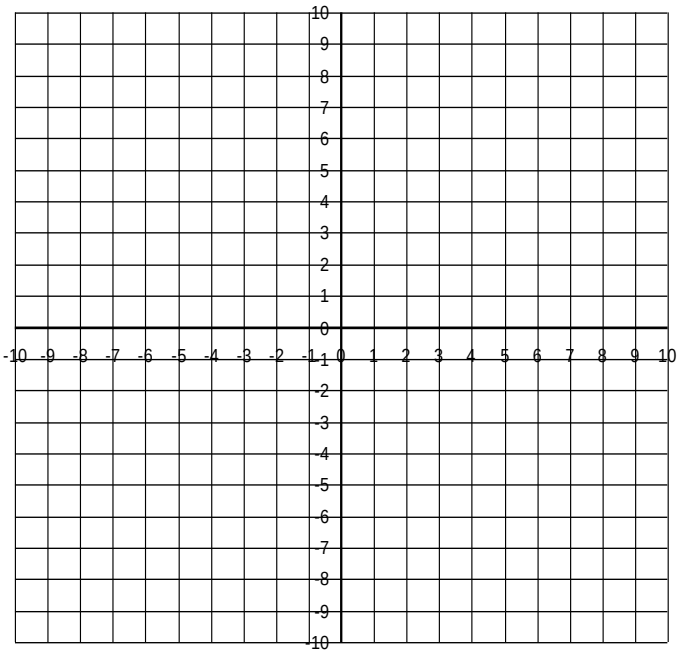
1) $y > 4$



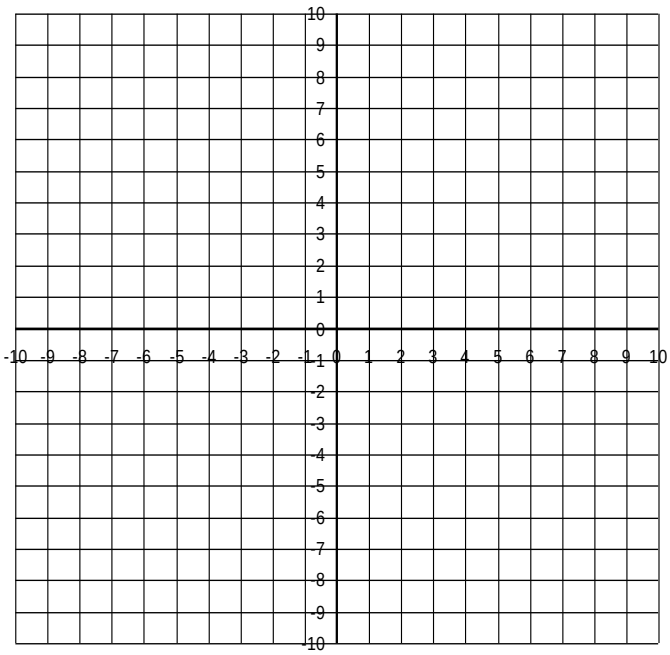
2) $x < -1$



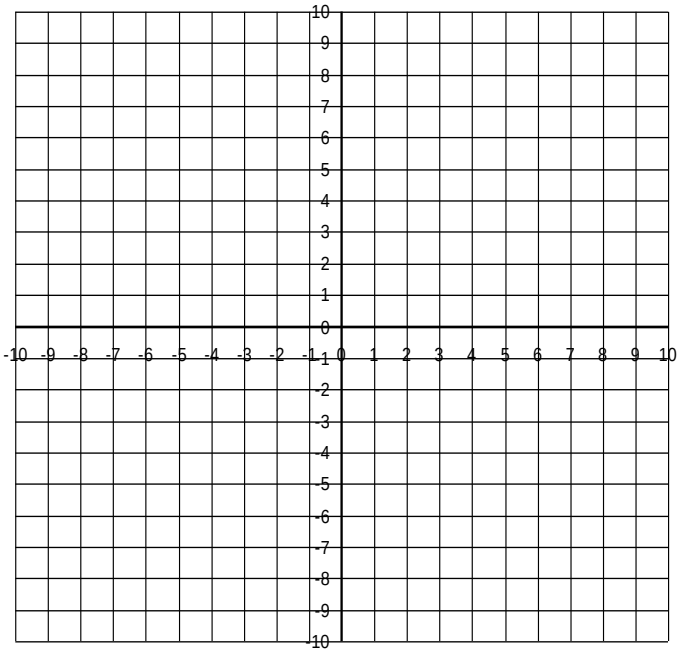
3) $y > 2,\ x < 3$



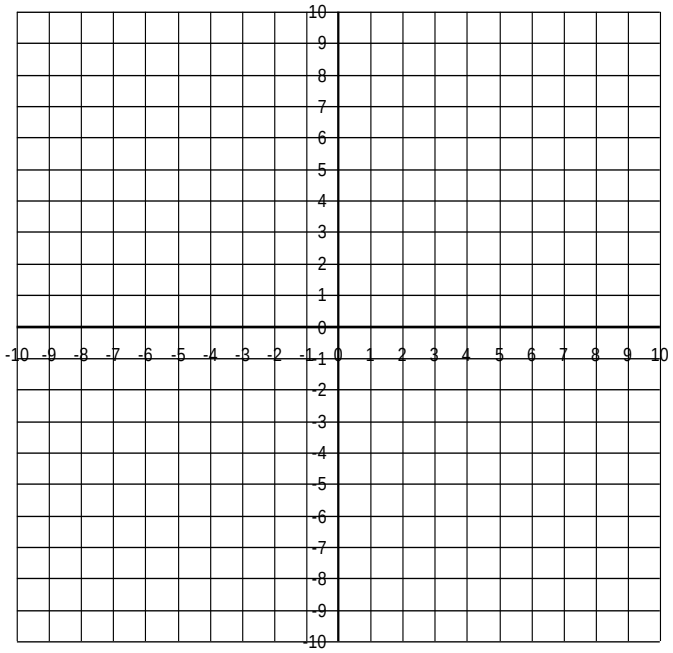
4) $y > x,\ x \geq 5$



5) $y > 2,\ x < 6,\ y < x$

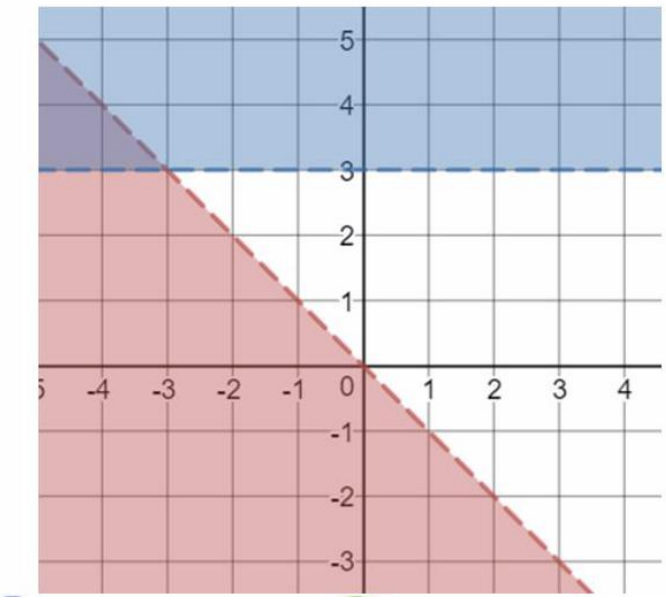


6) $x + 1 > 4,\ x > y$



Assessment A: Identify regions on a graph which satisfy an inequality

The unshaded region is given by:



A	B	C	D
$y > -x$ $y < 3$	$y > -x$ $y > 3$	$y < -x$ $y > 3$	$y > -x$ $y > 3$

Objective B: Solve linear simultaneous inequalities graphically

Worked example 1: Shade the region which satisfies $y \geq x - 4$, $y + 2x < 4$.

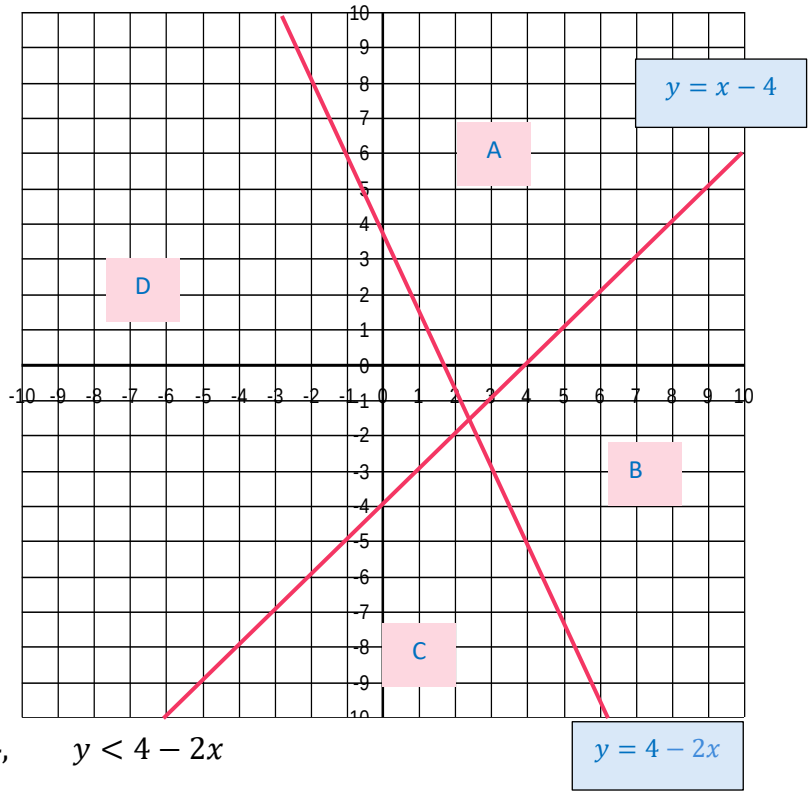
First rearrange the inequalities if required.
$y \geq x - 4$ and $y < 4 - 2x$.

It is easier if they are rearranged.

Draw the lines:
$y = x - 4$ and $y = 4 - 2x$.

Now study each region to see if it satisfies the inequalities.

Both inequalities must be satisfied!



$y \geq x - 4, \qquad y < 4 - 2x$

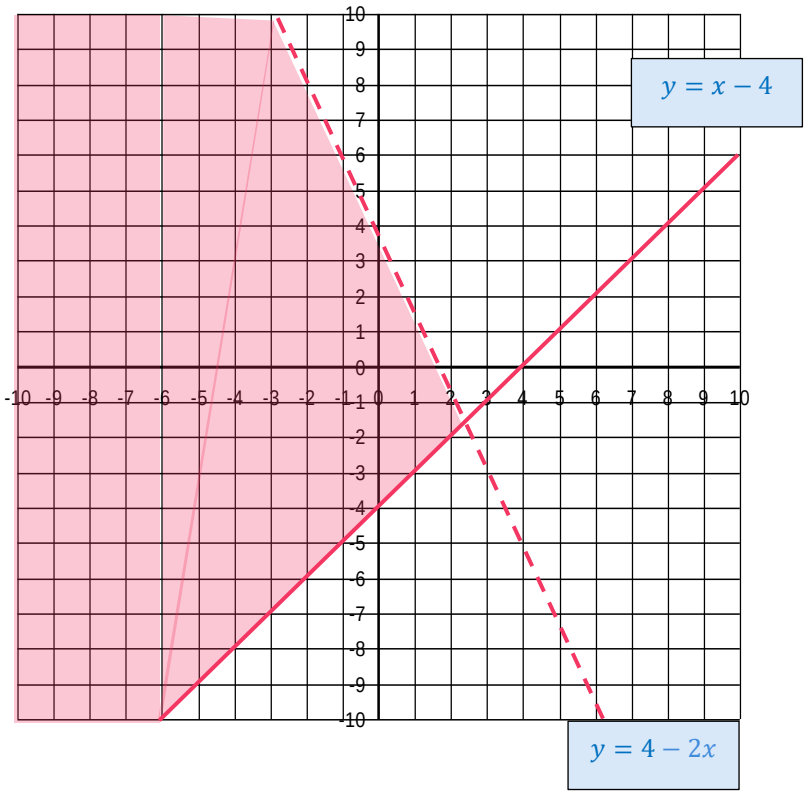
Test coordinates from each region.
Region A – test $(2, 3)$. $3 \geq 2 - 4$ and $3 < 4 - 2(2)$. The inequalities aren't both true.
Region B – test $(5,0)$. $0 \geq 5 - 4$ and $0 < 4 - 2(5)$. The inequalities aren't both true.
Region C – test $(1, -4)$. $-4 \geq 1 - 4$ and $-4 < 4 - 2(1)$. The inequalities aren't both true.
Region D – test $(0,0)$. $0 \geq 0 - 4$ and $0\ 4 - 2(0)$. The inequalities are true.

You can test any coordinates within each region you will come to the same conclusion.

Region D should be shaded!

The line $y = 4 - 2x$ must be dashed since the inequalities are strictly greater than. The line $y = x - 4$ must be solid since the inequality is less than or equal to.

The inequality $y \geq x - 4$ includes the values on the line $y = x - 4$. The inequality $y < 4 - 2x$ does not include the values on the line $y = 4 - 2x$.



Worked example 2: Shade the region which satisfies $y > 3x$, $y + x \leq 2$.

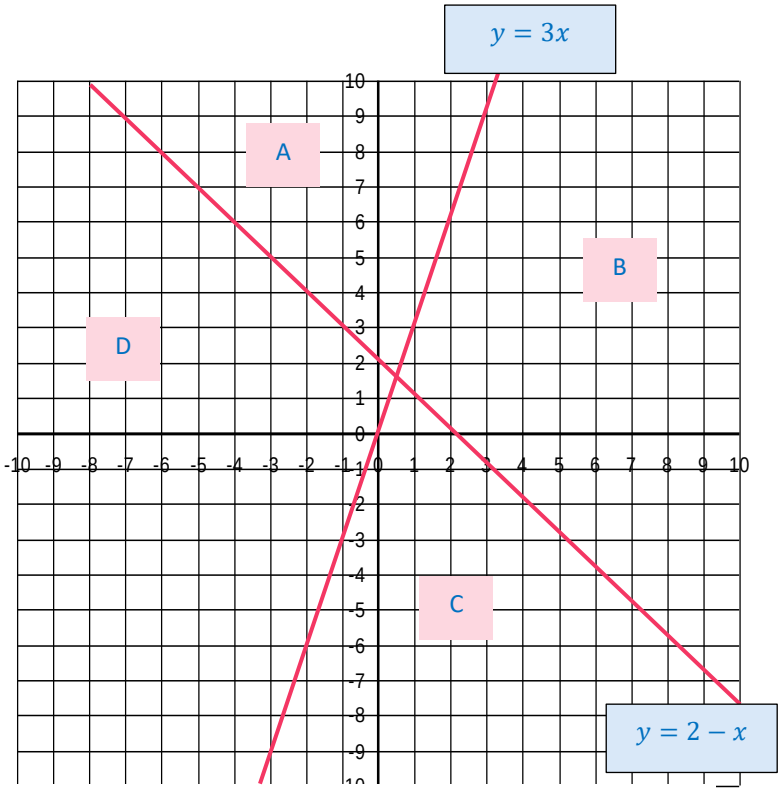
First rearrange the inequalities if required.
$y > 3x$ and $y \leq 2 - x$.

It is easier if they are rearranged.

Draw the lines:
$y = 3x$ and $y = 2 - x$.

Now study each region to see if it satisfies the inequalities.

Both inequalities must be satisfied!



$y > 3x, \quad y \leq 2 - x$

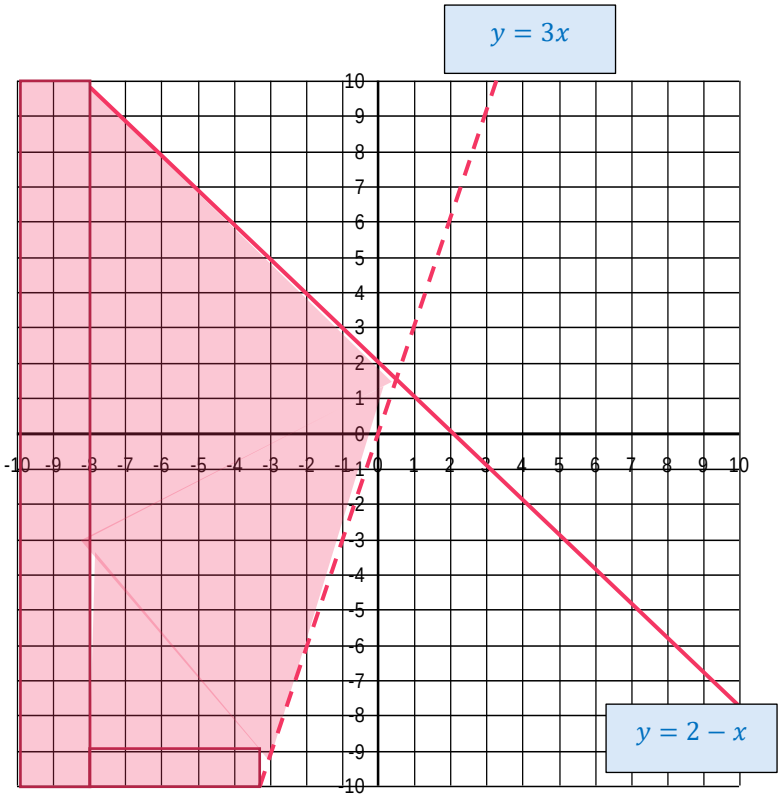
Test coordinates from each region.
Region A – test $(1, 5)$. $5 > 3(1)$ and $5 \leq 2 - 2(1)$. The inequalities aren't both true.
Region B – test $(1, 2)$. $2 > 3(1)$ and $2 \leq 2 - 2(1)$. The inequalities aren't true.
Region C – test $(1, 0)$. $0 > 3(1)$ and $0 \leq 2 - 2(1)$. The inequalities aren't true.
Region D – test $(-1, 1)$. $1 > 3(-1)$ and $1 \leq 2 - 2(-1)$. The inequalities are true.

You can test any coordinates within each region you will come to the same conclusion.

Region D should be shaded!

The line $y = 3x$ must be dashed since the inequalities are strictly less than. The line $y = 2 - 4$ must be solid since the inequality is greater than or equal to.

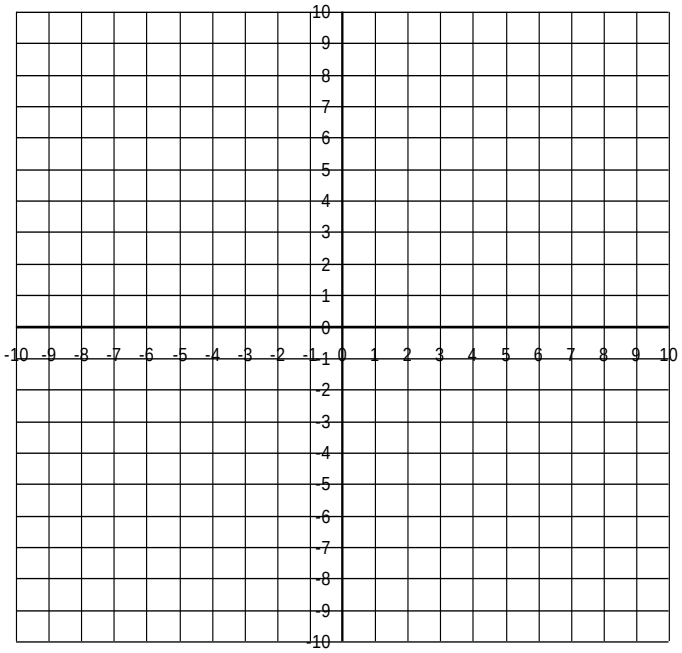
The inequality $y \geq 2 - x$ includes the values on the line $y = 2 - x$. The inequality $y > 3x$ does not include the values on the line $y = 3x$.



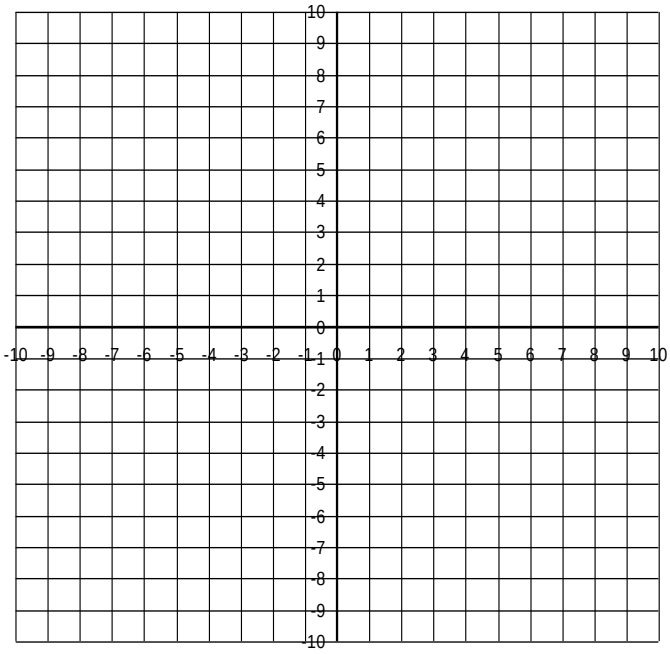
Practice B: Solve linear simultaneous inequalities graphically

Shade the regions that satisfy the following simultaneous inequalities.

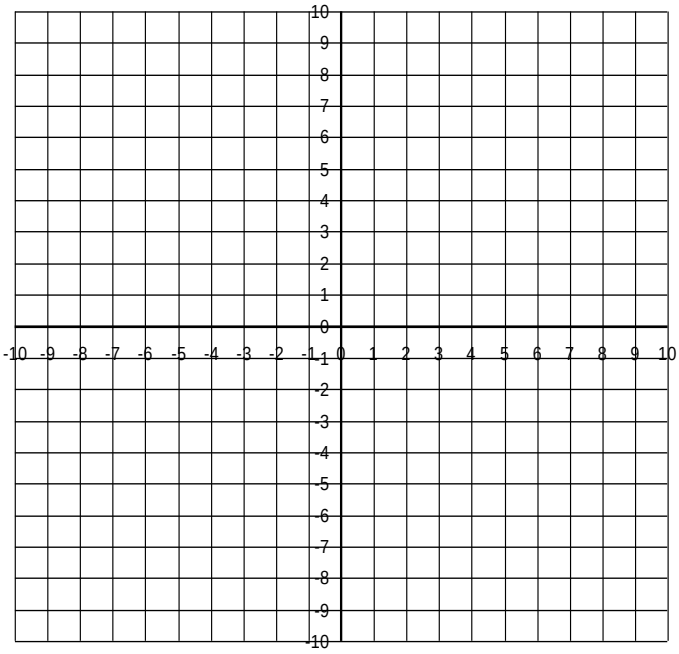
1) $y \geq x - 2,\ y < x + 1$



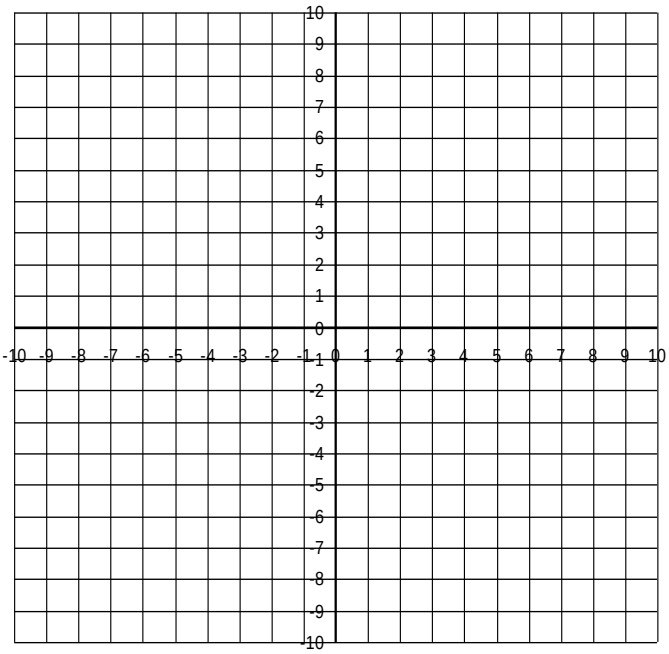
2) $y > 2x,\ y > \frac{x}{2}$



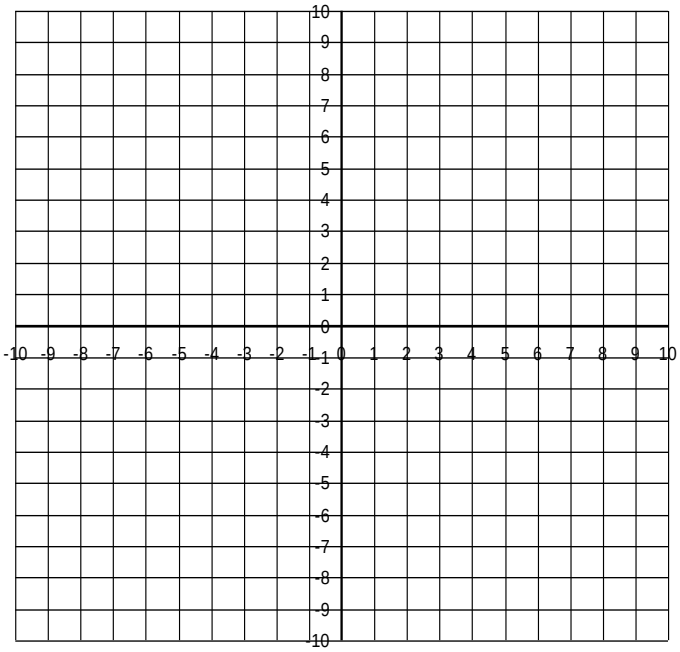
3) $y \geq 2x + 1,\ y < x + 1$



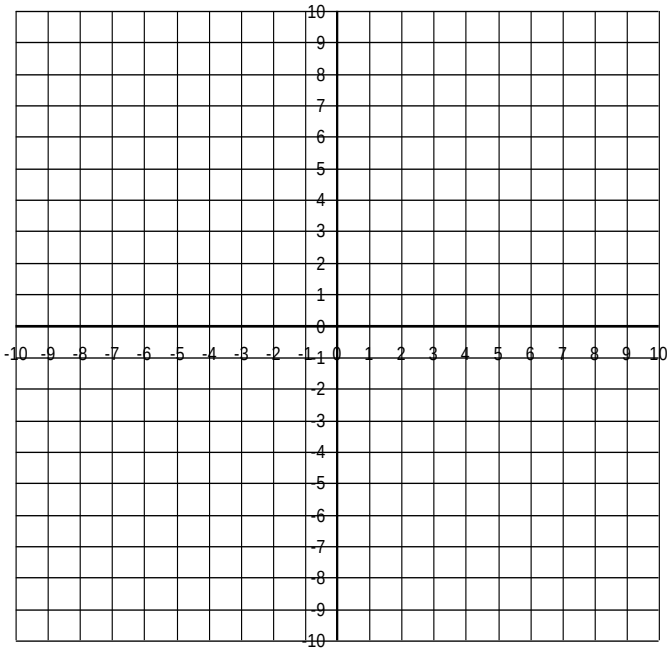
4) $y > 4x + 1,\ y < 6 - x - x$



5) $y + 4 > 3x, \;\; y \leq \frac{x}{2} + 2$

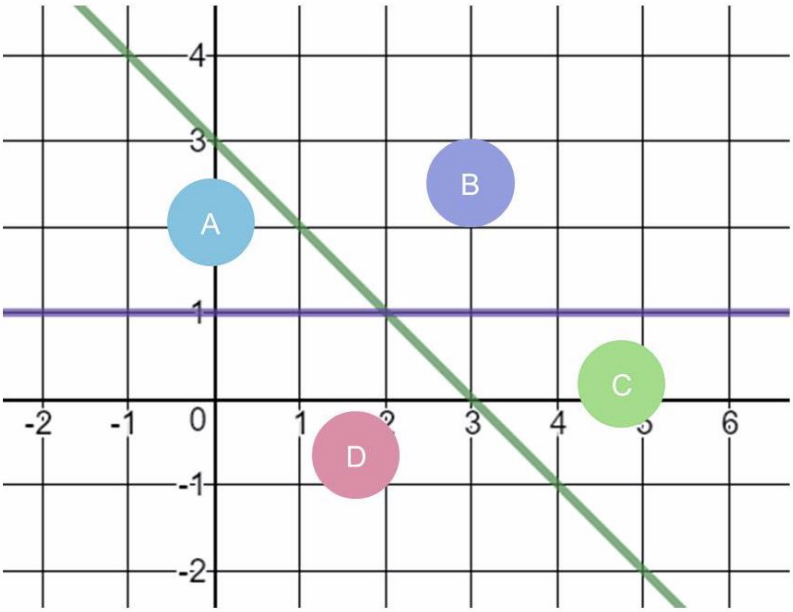


6) $y + 4 > \frac{x}{2}, \;\; y \leq 5 - 3x$

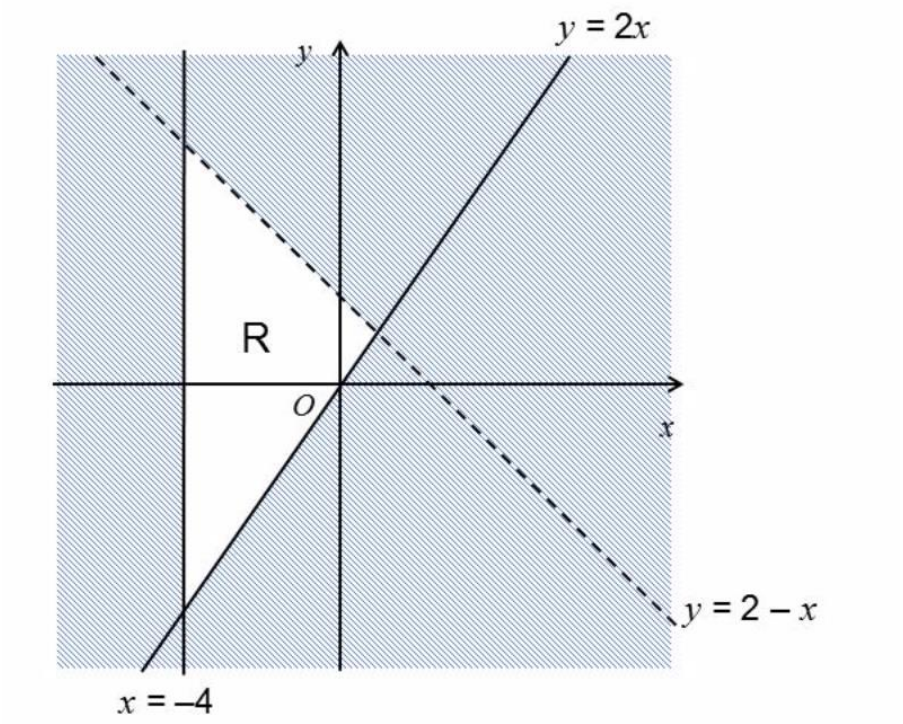


Assessment B: Solve linear simultaneous inequalities graphically

Which letter describes the region where $x + y \leq 3$ and $y \geq 1$.



The region R is satisfied by which of the following inequalities:



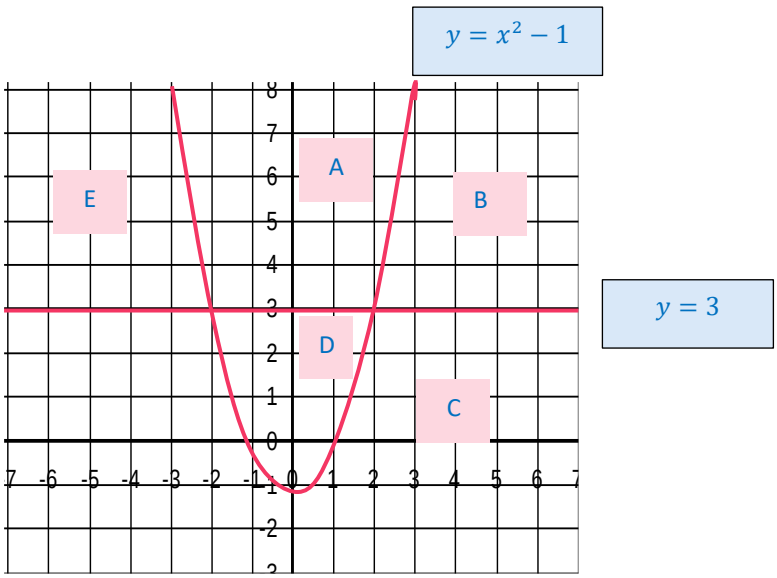
A	B	C	D
$y \geq 2x$ $y \leq 2 - x$ $x \geq -4$	$y \leq 2x$ $y < 2 - x$ $x < -4$	$y \geq 2x$ $y < 2 - x$ $x < -4$	$y \geq 2x$ $y < 2 - x$ $x \geq -4$

Objective C: Solve linear and quadratic simultaneous inequalities graphically

Worked example 1: Shade the region which satisfies $3 \leq x^2 - 1$.

Draw the lines:
$y = 3$ and $y = x^2 - 1$.

Now study each region to see if it satisfies the inequality.



$$3 \leq x^2 - 1$$

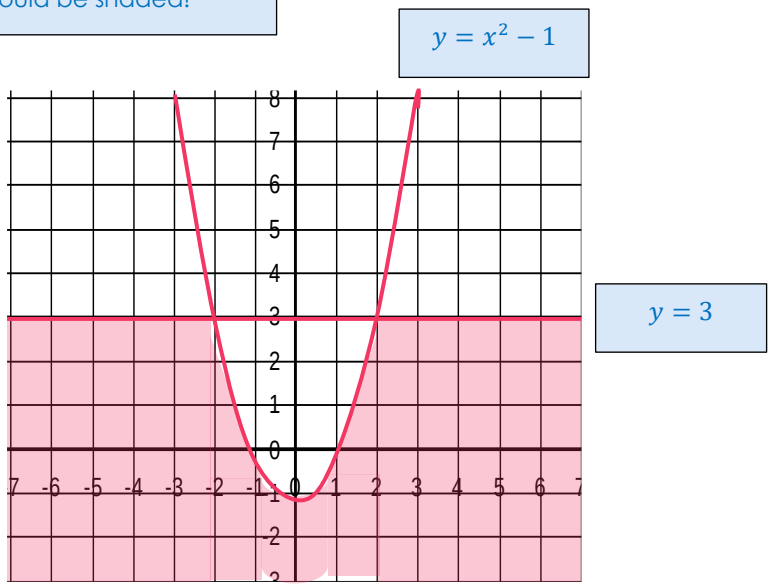
Test coordinates from each region.
Region A – test $(1, 4)$. $3 \leq (1)^2 - 1$. The inequality isn't true.
Region B – test $(5,4)$. $3 \leq (5)^2 - 1$. The inequality isn't true.
Region C – test $(2,0)$. $3 \leq (2)^2 - 1$. The inequality is true.
Region D – test $(0,0)$. $3 \leq (0)^2 - 1$. The inequality isn't true.
Region E – test $(-4,5)$. $3 \leq (-4) - 1$. The inequality isn't true.

Region C should be shaded!

You can test any coordinates within each region you will come to the same conclusion.

The lines must be solid since the inequality is greater than or equal to.

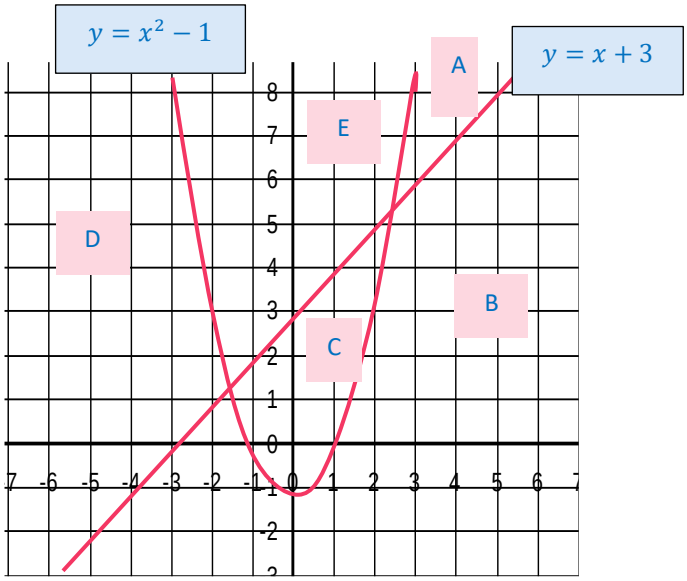
The inequality $3 \geq x^2 - 1$ includes the values on the lines $y = 3$ & $y = x^2 - 1$.



Worked example 2: Shade the region which satisfies $x + 3 \leq x^2 - 1$.

Draw the lines:
$y = x + 3$ and $y = x^2 - 1$.

Now study each region to see if it satisfies the inequality.



$$x + 3 \leq x^2 - 1$$

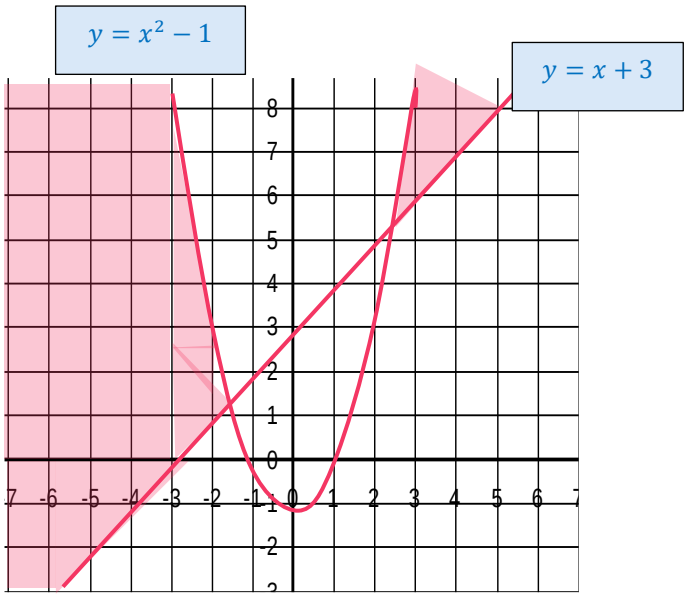
Test coordinates from each region.
Region A – test $(4, 8)$. $4 + 3 \leq 4^2 - 1$. The inequality is true.
Region B – test $(2,0)$. $2 + 3 \leq 2^2 - 1$. The inequality isn't true.
Region C – test $(0,0)$. $0 + 3 \leq 0^2 - 1$. The inequality is true.
Region D – test $(-4,0)$. $-4 + 3 \leq (-4)^2 - 1$. The inequality is true.
Region E – test $(0,5)$. $0 + 3 \leq 0^2 - 1$. The inequality isn't true.

Region A and D should be shaded!

You can test any coordinates within each region you will come to the same conclusion.

The lines must be solid since the inequality is greater than or equal to.

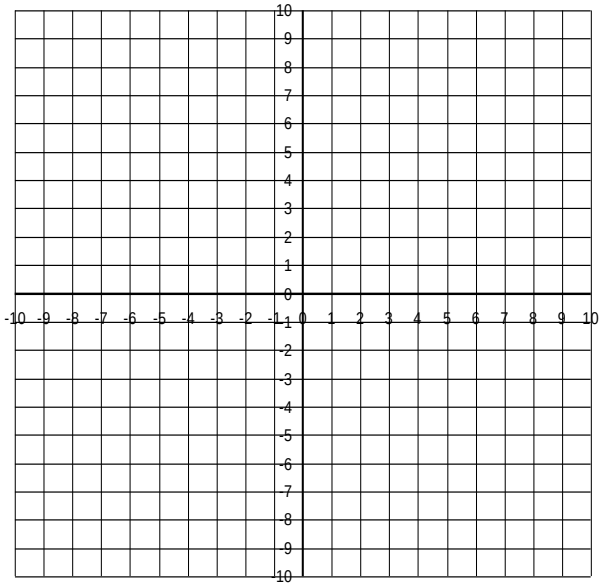
The inequality $x + 3 \geq x^2 - 1$ includes the values on the lines $y = x + 3$ & $y = x^2 - 1$.



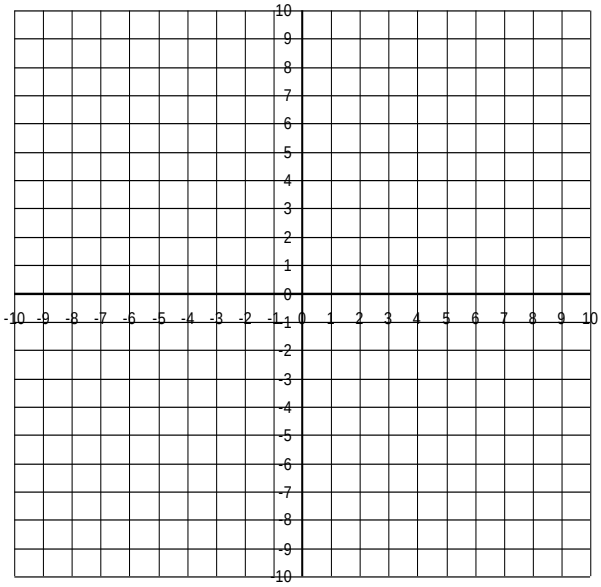
Practice C: Solve linear and quadratic simultaneous inequalities graphically

By drawing both a quadratic and linear graph, shade the regions that satisfy the following inequalities.

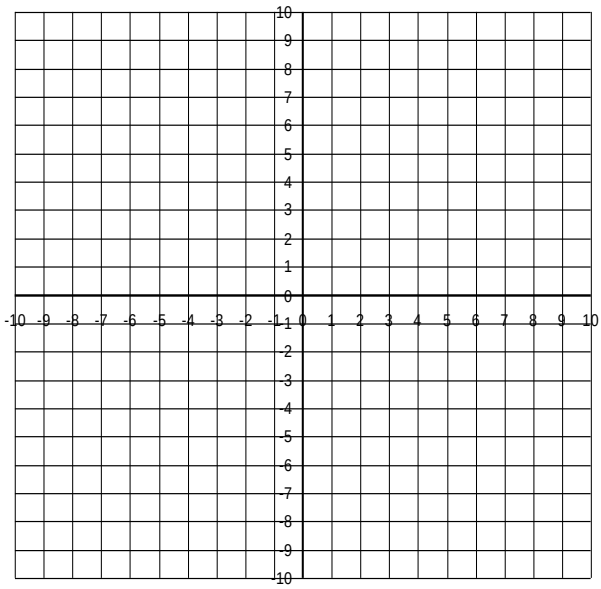
1) $2x^2 + 2x - 3 > 0$



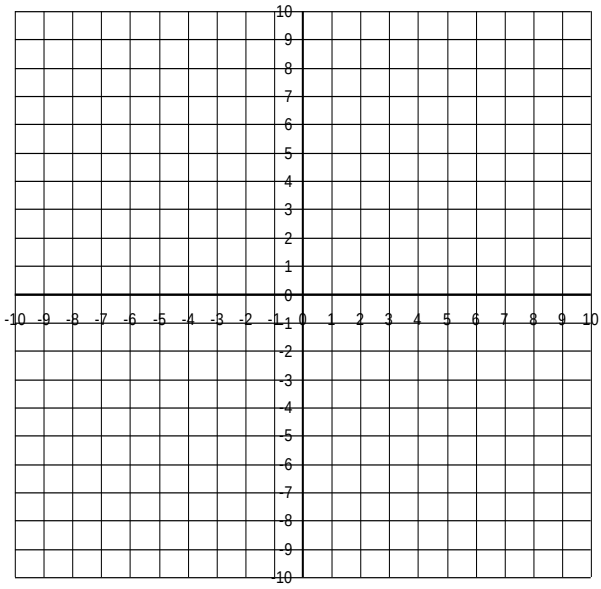
2) $2x^2 - 5x - 3 > 1$



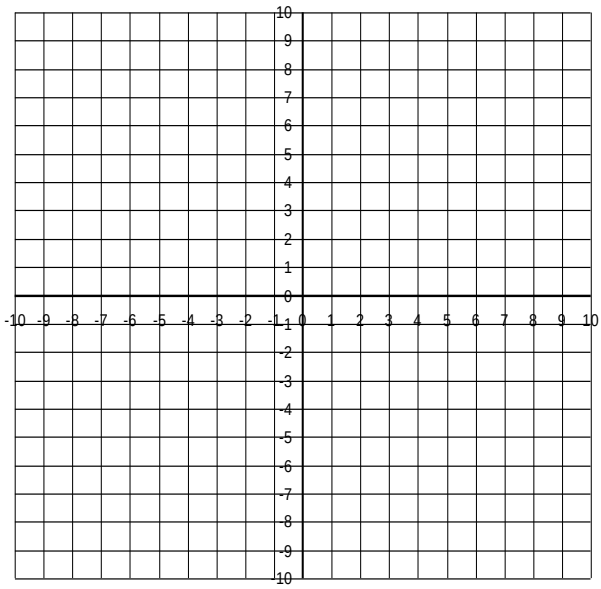
3) $2x^2 - 9x + 4 < x$



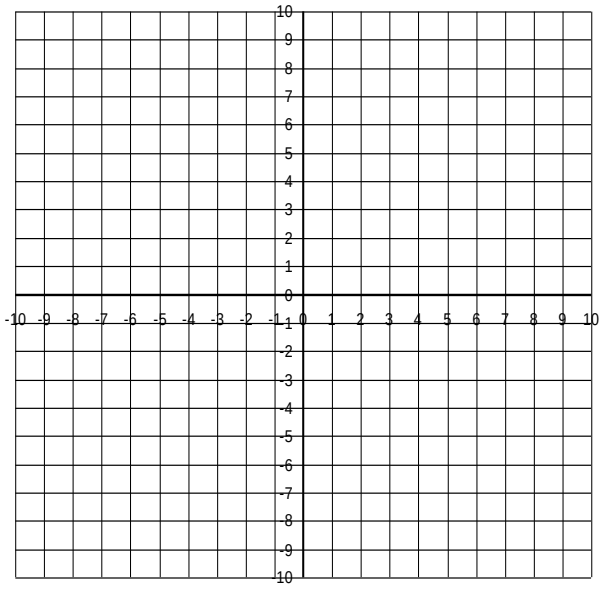
4) $-x^2 - 2x + 3 \geq -1$



5) $2x^2 - 3x - 2 < x - 1$

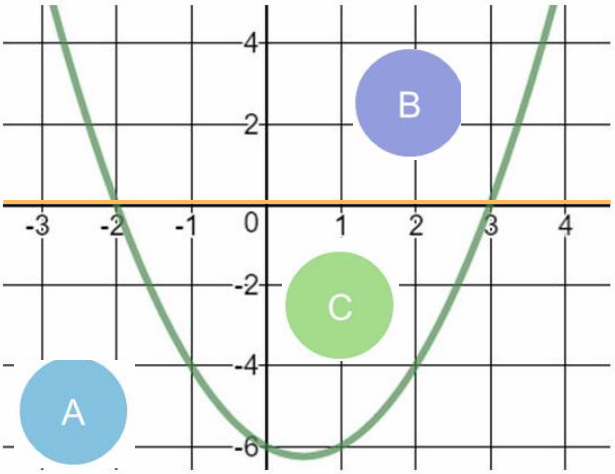


6) $2x^2 - 5x - 3 < 2x$



Assessment C: Solve linear and quadratic simultaneous inequalities graphically

Using the graph $y = x^2 - x - 6$, which region/s represents the solution to the inequality $0 \leq x^2 - x - 6$.



Feedback

What is the purpose of the feedback tutorial?

- To receive feedback on the final test.
- Create a revision plan based on the feedback.

What I did well	What I can improve

Revision plan

What am I going to do?	How will I do this?	When will I do this?

Notes

Simultaneous Equations

Key Stage 4 Programme

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