Surds

Key Stage 4 Revision Programme

Pupil Name	
Handbook Designed by	Dr. D. Jethwa

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1. Assessment			**E138**
2			**E138**
3			**E138**
4			**E138**
5			**E138**
6. Assessment 2			**E138**
7. Feedback			**E138**

Timetable – Homework Assignments

Homework assignments are due in every week at the start of the lesson.

All questions must be completed within the space provided and showing full workings out.

If you ever have any issues with homework, please come and see me at least 48 hours in advance of the next tutorial.

Homework Assignment	Description	Due Date
Tutorial 1	No Homework	n/a
Tutorial 2	Basic Operations (1)	
Tutorial 3	Basic Operations (2)	
Tutorial 4	Rationalising the denominator	
Tutorial 5	Revision for final assessment using past papers	

Uni Pathways – Pupil Weekly Progress Report

As each tutorial and related homework assignment are completed, feedback will be given here on your progress.

Tutorial	Feedback	
	What went well:	**Even better if:**
<u>Tutorial 1</u> Confidence with basic surd operations		
<u>Tutorial 2</u> Confidence with intermediate level surd operations		
<u>Tutorial 3</u> Confidence with higher level surd operations		
<u>Tutorial 4</u> Confidence in application of surd knowledge in proof questions		
<u>Tutorial 5</u> Confidence in application of surd knowledge in worded questions		

Contents

Course Rationale

Tutorial Format

Tutorial 1: Baseline Assessment 7

Tutorial 2: Basic Operations (1) 10

Tutorial 3: Basic Operations (2) 15

Tutorial 4: Rationalising the denominator 21

Tutorial 5: Surds in Proof and Worded Questions 31

Tutorial 6: Final Assessment and Feedback 42

Notes

Course Rationale

Pre-2017, it was considered “easy” to achieve the top A*mathematics grade through a combination of hard work and past papers due to the repetition of questions year after year. Although this meant many students were achieving A* grades, it was determined difficult to differentiate between students who had mastered the understanding behind difficult mathematical concepts, and those who were replying on the repetition of exams.

The 2017+ exams have sort to solve this problem with the introduction of the grade 9. Students who can obtain a grade 9 are truly considered to be gifted in that subject and mastered topics that many other students will fail to access.

Mastery of a subject comes from not only knowing the basic and advanced principles behind a topic, but also being able to apply them to a question/format they may not be familiar with. By practising mastery of a topic, it will allow students to access questions and have the best chance in achieving a level 9 mathematics grade.

This course focuses on the mastery of Surds. Statistically, only 5% of students obtain full marks in questions involving surds and therefore it can be assumed that the topic is difficult to master. To improve student confidence in this topic we will be extensively looking at the principles of surds and how they are used within a range of questions and other topics. Simultaneously, we will be looking at key study skills to allow you to make the most of your time and how you apply these techniques to not only other topics in maths, but other subjects as well.

Tutorial Format

Each tutorial will follow a very similar style. At the start of every tutorial, the tutorial objectives will be stated (up to 3). Following this each objective will be broken down into three parts:

1. **Objective explanation**

   This section will explain any theory knowledge which is required to complete that objective. For example, general rules and principles.

2. **Practice**

   This section will focus on practice in a group format. We will go through several worked examples showing the variety of questions which can come up in exams. This is your opportunity to ask questions and make notes which you can refer to later.

3. **Assessment**

   This section will focus on your practice of questions. Using the notes from the practice questions, you will get to attempt similar exam style questions. This section will try to combat any misconceptions you may have through multiple choice and practice questions.

Following each tutorial, a homework task will be given out to further practice and solidify what has been covered within class. Each homework is due in the following week to assess learning.

1. Baseline Assessment

This objective of a baseline assessment is to determine your current knowledge with surds. From this, I can identify what your strengths and weaknesses are and how we can best improve the knowledge that you have.

Attempt as many questions as you can showing full working out in the spaces provided.

You have 40 minutes to complete this assessment

Baseline Assessment - Student Feedback

1. How would you rate this assessment?

Very easy | Easy | Neutral | Difficult | Very Difficult

2. How confident are you with surds?

Can do no questions | Can do some questions | Can do all questions

3. How do you think you best learn?

Group discussions | Worked examples | Paired explanations

Practice | Homework

4. What other topics do you find challenging in maths?

Vectors | Circle Theorems | Pythagoras

Trigonometry | Data Handling | Other (please specify)

5. What would you like to get out of these tutorials?

Mastery with surds | Tips on revision and planning

Exam question practice | Theory practice

Other (please specify)

Baseline Assessment -Teacher Feedback

What went well:

1.

2.

3.

How could you improve:

1.

2.

3.

2. Basic Operations (1)

Key words: rational, irrational, simplifying, binomial, monomial, factor, square, simplify

Objectives this tutorial:

A) Identifying surds
B) Simplifying surds
C) Addition and subtraction of surds

Starter:

What is an irrational number?

Can you think of any examples of numbers in maths with are irrational?

Objective A: Identifying Surds

Surds are numbers left in root form to express its **exact value**. It has an infinite number of non-recurring decimals. Therefore, surds are **irrational** numbers.

There are two types of surds in maths:

1) Monomial – Surds with only one expression (i.e, $\sqrt{8}$ or $3\sqrt{8}$)
2) Binomial – Surds with two parts (i.e, 3+ $\sqrt{8}$)

Practice A: Identifying Surds

Is this a surd?

1. $\sqrt{1}$ Yes / No
2. $\sqrt{4}$ Yes / No
3. $\sqrt{9}$ Yes / No
4. $\sqrt{25}$ Yes / No
5. $\sqrt{20}$ Yes / No
6. $\sqrt{2}$ Yes / No
7. $2\sqrt{2}$ Yes / No
8. $2\sqrt{1}$ Yes / No
9. $2\sqrt{9}$ Yes / No
10. $3\sqrt{3}$ Yes / No

Assessment A: Identifying Surds

1. $\sqrt{49}$ Yes / No
2. $\sqrt{40}$ Yes / No
3. $4\sqrt{9}$ Yes / No
4. $7\sqrt{25}$ Yes / No
5. $9\sqrt{2}$ Yes / No
6. $3\sqrt{16}$ Yes / No

Objective B: Simplifying Surds

An entire surd is a surd which the entire number is under the root sign. For example, $\sqrt{12}$. However, this surd can be simplified to make the simplest form of a surd.

We can split a surd up by writing it as two surds being multiplied together. As a rule we can say:

$$\sqrt{a} \text{ x } \sqrt{b} = \sqrt{ab}$$

Practice B: Simplifying Surds

Worked example 1:
Write $\sqrt{12}$ in its simplest form.

$\sqrt{12} = \sqrt{4} \text{ x } \sqrt{3}$
$= 2 \text{ x } \sqrt{3}$
$= 2\sqrt{3}$

Note: Although 12 also has the factors
6 x 2 , 12 x 1

We want to use the factor pair with the **highest square number**. Hence, we separate 12 into the factors 3 and 4.

Worked example 2:
Write $2\sqrt{12}$ in its simplest form.

$2\sqrt{12} = 2 \text{ x } \sqrt{12}$
$= 2 \text{ x } \sqrt{4} \text{ x } \sqrt{3}$
$= 2 \text{ x } 2 \text{ x } \sqrt{3}$
$= 4\sqrt{3}$

Note: Simplify the surd first and then multiply by the integer first

Worked example 3:
Simplify $5\sqrt{144}$

$\mathbf{5}\sqrt{144} = 5 \text{ x } \sqrt{144}$
$= 5 \text{ x } 12$
$= 60$

Assessment B: Simplifying Surds

1. How would you simplify $\sqrt{72}$ a) $\sqrt{3} \text{ x } \sqrt{24}$ b) $\sqrt{18} \text{ x } \sqrt{4}$ c) $\sqrt{36} \text{ x } \sqrt{2}$	2. How would you simplify $7\sqrt{147}$ a) $7 \text{ x } \sqrt{49} \text{ x } \sqrt{3}$ b) $\sqrt{18} \text{ x } \sqrt{4}$ c) $\sqrt{36} \text{ x } \sqrt{2}$	3. Why would you not simplify $5\sqrt{36}$ like this: $\mathbf{5} \text{ x } \sqrt{18} \text{ x } \sqrt{2}$
4. Write $\sqrt{80}$ in its simplest form:	5. Simplify $14\sqrt{147}$	6. Simplify $8\sqrt{36}$

Objective C: Adding and Subtracting Surds

There is just one rule for adding and subtracting surds: **the square root number must be the same.**

Once the square root number is the same, you can find the sum or different of the rational coefficient of like surds.

Practice C: Addition and Subtraction of Surds

Worked example 1a

$$5\sqrt{2} + 3\sqrt{2} = (5+3)\sqrt{2}$$
$$= 8\sqrt{2}$$

Worked example 1b

$$9\sqrt{6} - 4\sqrt{6} = (9-4)\sqrt{6}$$
$$= 5\sqrt{6}$$

Worked example 2a

$$\sqrt{27} + \sqrt{12}$$

Note: Simplify both surds to have the same square root number

$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

$\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3}$
$= 5\sqrt{3}$

Worked example 2b

$$\sqrt{27} - \sqrt{12}$$

Note: Simplify both surds to have the same square root number

$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

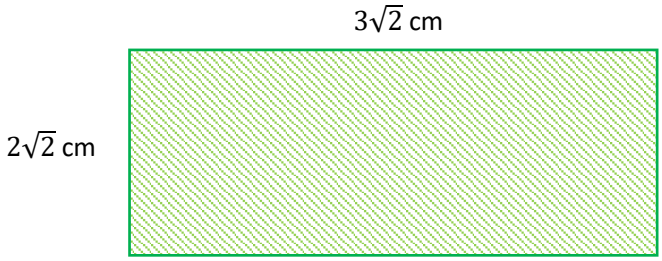
$\sqrt{27} - \sqrt{12} = 3\sqrt{3} - 2\sqrt{3}$
$= 1\sqrt{3}$

Worked example 3

Calculate the perimeter of the field shown in the diagram.

$2\sqrt{2} + 2\sqrt{2} + 3\sqrt{3} + 3\sqrt{3} = 4\sqrt{2} + 6\sqrt{3}$ cm

Note: You **cannot** add the two surds together as they do not have the same root.



Assessment C: Addition and Subtraction of Surds

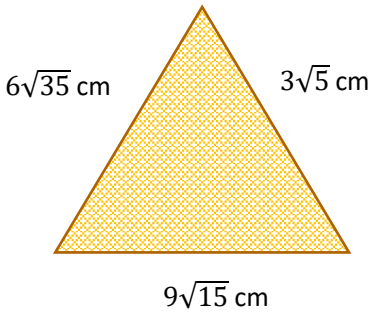
1. Which of these surds can be added/subtracted?

1. $6\sqrt{2} + 2\sqrt{3}$
2. $8\sqrt{2} + 7\sqrt{2}$
3. $9\sqrt{6} - 5\sqrt{3}$
4. $8\sqrt{6} - \sqrt{6}$

2. $8\sqrt{27} - 3\sqrt{75}$

3. $\sqrt{48} + \sqrt{27}$

4. Find the perimeter of the shape below:



5. The equation $2\sqrt{45}$ - $4\sqrt{20}$ can not be simplified. Is this true?

Homework 1 – Basic Operations

1. Circle all the correct statements:

a) All irrational numbers must be surds

b) An irrational number is one which is never ending

c) Reciprocal numbers are types of surds

d) Surds are a specific type of irrational number

2. Assign the values below to groups a, b or c:

$2\sqrt{3}$ $6 - 3\sqrt{5}$ $3 + \sqrt{9}$ $2 - \sqrt{8}$ $\sqrt{36}$ 8 $\sqrt{25} + \sqrt{4}$ $0.\dot{4}$ $2\sqrt{4}$

a) Irrational number which is s surd

b) Rational number

c) Binomial surd

3. Simplify the following:

a) $\sqrt{8}$	b) $2\sqrt{8}$	c) $3\sqrt{8}$
d) $\sqrt{72}$	e) $5\sqrt{72}$	f) $5\sqrt{144}$
g) $5\sqrt{1440}$	h) $5\sqrt{720}$	i) $5\sqrt{360}$
j) $5\sqrt{36}$	k) $6\sqrt{36}$	l) $7\sqrt{49}$

4. Simplify the following:

a) $4\sqrt{2} + 3\sqrt{2}$	b) $5\sqrt{2} - 3\sqrt{2}$	c) $6\sqrt{3} + 2\sqrt{3}$
d) $\sqrt{8} + \sqrt{2}$	e) $\sqrt{18} - \sqrt{2}$	f) $\sqrt{125} - 5\sqrt{5}$
g) $\sqrt{48} - \sqrt{12}$	h) $\sqrt{32} + \sqrt{18}$	i) $\sqrt{75} - \sqrt{12}$
j) $\sqrt{45} - \sqrt{20}$	k) $\sqrt{63} - \sqrt{28}$	l) $\sqrt{7} - \sqrt{42}$

3. Basic Operations (2)

Key words: numerator, denominator, root

Objectives this tutorial:

A) Multiplying monomial surds
B) Surds with powers
C) Simplifying fractional surds

Starter: Simplify these fractions

1.	2.	3.	4.
$\frac{6}{24}$	$\frac{12a}{4a}$	$\frac{4ab}{20b}$	$\frac{49axc}{12bx^2c}$

Objective A: Multiplying Surds

Simplifying and multiplying surds goes hand in hand. Where in simplifying we are separating a root into two roots, we are combining the roots when we multiply them. In general:

$$\sqrt{a} \text{ x } \sqrt{b} = \sqrt{ab}$$

If there are coefficients in the surd, we multiply the numbers and multiply the surd parts of the equations.

$$y\sqrt{a} \text{ x } z\sqrt{b} = (\text{y x z})\sqrt{ab}$$
$$= \text{yz}\sqrt{ab}$$

Practice A: Multiplying Surds

Worked Example 1 $\sqrt{3} \text{ x } \sqrt{4} = \sqrt{3} \text{ x } 2$ $= 2\sqrt{3}$	**Your Turn 1** $\sqrt{9} \text{ x } \sqrt{7}$
Worked Example 2 $4\sqrt{3} \text{ x } 6\sqrt{2} = 4 \text{ x } 6 \text{ x } \sqrt{3} \text{ x } \sqrt{2}$ $= 24 \text{ x } \sqrt{6}$ $= 24\sqrt{6}$	**Your Turn 2** $5\sqrt{3} \text{ x } 6\sqrt{5}$
Worked Example 3 $6\sqrt{4} \text{ x } 2\sqrt{4} = 6 \text{ x } 2 \text{ x } \sqrt{4} \text{ x } \sqrt{4}$ $= 12 \text{ x } 4$ $= 48$	**Your Turn 3** $5\sqrt{2} \text{ x } 6\sqrt{2}$
Worked Example 4 $7\sqrt{3} \text{ x } 9\sqrt{4} = 7 \text{ x } 9 \text{ x } \sqrt{3} \text{ x } \sqrt{4}$ $= 63 \text{ x } \sqrt{3} \text{ x } 2$ $= 126 \text{ x } \sqrt{3}$ $= 126\sqrt{3}$	**Your Turn 4** $8\sqrt{5} \text{ x } 4\sqrt{9}$
Worked Example 5 $4\sqrt{32} \text{ x } 6\sqrt{2} = 24 \text{ x } \sqrt{64}$ $= 24 \text{ x } 8$ $= 192$	**Your Turn 5** $7\sqrt{3} \text{ x } 4\sqrt{27}$

Assessment A: Multiplying Surds

1. Simplify fully: $2\sqrt{2} \times 3\sqrt{8}$

a) 96
b) $\sqrt{96}$
c) 24
d) $6\sqrt{16}$

2. Simplify fully: $2\sqrt{10} \times 4\sqrt{10}$

a) $6\sqrt{10}$
b) $8\sqrt{100}$
c) $8\sqrt{10}$
d) 80

3. Identify the mistake with this answer:

$4\sqrt{2} \times 6\sqrt{9} = 24 \times \sqrt{18}$
$= 24\sqrt{18}$

Objective B: Surds with Powers

When dealing with surds you should know:

$$(\sqrt{x})^2 = x$$

This statement is correct as $\sqrt{x}$ can be written as $x^{1/2}$. Therefore:

$$(x^{1/2})^2 = x^{1/2} \times x^{1/2}$$
$$= x$$

Practice B: Surds with Powers

Worked Example 1

$(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2}$
$= \sqrt{4}$
$= 2$

Worked Example 2

$(\sqrt{3})^2 = \sqrt{3} \times \sqrt{3}$
$= \sqrt{3}$
$= 3$

Worked Example 3

$(2\sqrt{10})^2 = 2\sqrt{10} \times 2\sqrt{10}$
$= 2 \times 2 \times \sqrt{10} \times \sqrt{10}$
$= 4 \times \sqrt{10}$
$= 4 \times 10$
$= 40$

Worked Example 4

$(2\sqrt{10})^3 = 2\sqrt{10} \times 2\sqrt{10} \times 2\sqrt{10}$
$= 2 \times 2 \times 2 \times \sqrt{10} \times \sqrt{10} \times \sqrt{10}$
$= 8 \times 10\sqrt{10}$
$= 80\sqrt{10}$

Worked Example 5

$10(2\sqrt{2})^2 = 10 \times 2\sqrt{2} \times 2\sqrt{2}$
$= 10 \times 2 \times 2 \times \sqrt{2} \times \sqrt{2}$
$= 40 \times \sqrt{4}$
$= 40 \times 2$
$= 80$

Worked Example 6

$(\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2}$
$= \sqrt{4} \times \sqrt{2}$
$= 2\sqrt{2}$

Assessment B: Surds with Powers

1. Simplify $(\sqrt{2})^2$

A) $\sqrt{4}$
B) 2
C) 4
D) $\sqrt{2}$

2. $(5\sqrt{8})^2$

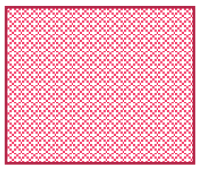
3. Select all the statements which are true when simplifying $(4\sqrt{3})^3$

A) 64 x $3\sqrt{3}$)
B) 64 x 9 x $\sqrt{3}$
C) 4^3 x 3 x $\sqrt{9}$
D) $(4\sqrt{3})^2$ x 4 x $\sqrt{3}$

4. $7(3\sqrt{5})^3$

5. Find the area of the square shown in the diagram.

$6\sqrt{4}$ cm



Objective C: Simplifying Fractional Surds

In this objective we will look at fractions with surds in the numerator.

We can simplify these fractions the same way in which we simplify integer fractions and/or algebraic fractions. We can cancel out common rational factors between the numerator and denominator.

Note: This is different to rationalising which we will come to in the next tutorial.

Practice C: Simplifying Fractional Surds

Worked Example 1

$$\frac{6\sqrt{5}}{2} = 3\sqrt{5}$$

Worked Example 2

$$\frac{6\sqrt{12}}{2} = \frac{6 \text{ x } \sqrt{4} \text{ x } \sqrt{3}}{2} = \frac{6 \text{ x } 2 \text{ x } \sqrt{3}}{2}$$

$$= \frac{12\sqrt{3}}{2} = 6\sqrt{3}$$

Worked Example 3

$$\frac{\sqrt{20}}{4} = \frac{\sqrt{4} \text{ x } \sqrt{5}}{2} = \frac{2 \text{ x } \sqrt{5}}{2} = \sqrt{5}$$

Assessment C: Simplifying Fractional Surds		
1. $\frac{12\sqrt{5}}{2}$	2. $\frac{4\sqrt{2}}{6}$	3. $\frac{4\sqrt{24}}{8}$
4. $\frac{6\sqrt{40}}{3}$	5. $\frac{\sqrt{48}}{20}$	6. $\frac{\sqrt{50}}{25}$

Homework 2 – Basic Operations (2)

1. Simplify:

a) $\sqrt{2} \times \sqrt{2}$	b) $\sqrt{6} \times \sqrt{6}$	c) $\sqrt{3} \times \sqrt{12}$
d) $4\sqrt{10} \times \sqrt{2}$	e) $6\sqrt{3} \times 7\sqrt{45}$	f) $3\sqrt{5} \times 9\sqrt{10}$

2. Lily wrote in her maths exam:

$$\sqrt{48} = \sqrt{16} \times 3 = 4 \times 3 = 12$$

Lily is incorrect.

Explain what Lily has done wrong. Correct her answer.

.....

.....

.....

.....

3. Simplify:

a) $(7\sqrt{2})^2$	b) $3(5\sqrt{2})^2$	c) $\frac{1}{2}(6\sqrt{9})^3$

4. Expand and simplify the following:

a) $(2 + \sqrt{2})(3 + \sqrt{2})$	b) $(2 + \sqrt{2})(3 - \sqrt{2})$	c) $(5 - \sqrt{3})(5 + \sqrt{3})$
d) $(\sqrt{3} - \sqrt{2})^2$	e) $(\frac{1}{\sqrt{2}} + \sqrt{2})(\frac{1}{\sqrt{2}} - \sqrt{2})$	f) $(\frac{1}{\sqrt{2}} + \sqrt{2})$

5. Simplify the following:

a) $\frac{20\sqrt{5}}{5}$	b) $\frac{8\sqrt{8}}{20}$	c) $\frac{5\sqrt{72}}{40}$
d) $\frac{7\sqrt{63}}{42}$	e). $\frac{\sqrt{75}}{35}$	f). $\frac{\sqrt{98}}{77}$

6. State whether the following statements are <u>always true</u>, <u>sometimes true</u> or <u>never true.</u> Justify you answer in each case:

a) $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$

b) $\sqrt{a \text{ x } b} = \sqrt{a} \text{ x } \sqrt{b}$

c) $\sqrt{a^2 + b^2} = a + b$

d) $(a - b^2) = a^2 - b$

4. Rationalising the denominator

Key words: monomial, binomial, rationalising, numerator, denominator

Objectives this tutorial:

A) Rationalising with a monomial surd in the denominator
B) Rationalising with binomial surds in the denominator
C) Rationalising with binomial surds in the numerator and denominator

Starter:

Expand:	Expand:	Expand:	Expand:
$x(x + 3)$	$(x - 3)(x + 7)$	$(x^2 + 3)(x - 4)$	$(x^2 - 4x)(x^2 + 2x)$

Objective A: Rationalising with a monomial surd in the denominator

Rationalising an expression is where we get rid of any surds from the denominator of a fraction.

When a monomial surd is seen in the denominator, we rationalise the expression to remove the surd from the denominator. This is done by multiplying the top and bottom of the fraction by the surd in the denominator

The general rule for rationalising the denominator is:

$$\frac{x}{\sqrt{a}} \xrightarrow{\text{rationalise}} \frac{x}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$$

Rationalising the denominator is often seen as a precursor to simplifying fractions.

Practice A: Rationalising with a monomial surd in the denominator

Worked Example 1	**Solution**
Rationalise $\frac{6}{\sqrt{2}}$	$\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$ $= 3\sqrt{2}$ Note: Remember to simplify your answer!
Worked Example 2	**Solution**
Rationalise $\frac{6}{3\sqrt{2}}$	$\frac{6}{3\sqrt{2}} \times \frac{3\sqrt{2}}{3\sqrt{2}} = \frac{6 \times 3 \times \sqrt{2}}{3 \times \sqrt{2} \times 3 \times \sqrt{2}} = \frac{18\sqrt{2}}{9 \times \sqrt{4}}$ $= \frac{18\sqrt{2}}{9 \times 2} = \frac{18\sqrt{2}}{18}$ $= \sqrt{2}$

Worked Example 3 Rationalise $\frac{3\sqrt{5}}{3\sqrt{2}}$	**Solution** $\frac{3\sqrt{5}}{3\sqrt{2}} \text{ x } \frac{3\sqrt{2}}{3\sqrt{2}} = \frac{3 \text{ x } \sqrt{5} \text{ x } 3 \text{ x } \sqrt{2}}{3 \text{ x } \sqrt{2} \text{ x } 3 \text{ x } \sqrt{2}}$ $= \frac{9 \text{ x } \sqrt{10}}{9 \text{ x } \sqrt{4}}$ $= \frac{9\sqrt{10}}{9 \text{ x } 2}$ $= \frac{9\sqrt{10}}{18}$ $= \frac{\sqrt{10}}{2}$
Worked Example 4 Rationalise $\frac{3+\sqrt{5}}{\sqrt{2}}$	**Solution** $\frac{3+\sqrt{5}}{\sqrt{2}} \text{ x } \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(3+\sqrt{5})}{\sqrt{2} \text{ x } \sqrt{2}}$ **Note**: Expand the numerator like normal $= \frac{(\sqrt{2} \text{ x } 3)+(\sqrt{2} \text{ x } \sqrt{5})}{\sqrt{4}}$ $= \frac{3\sqrt{2}+\sqrt{10}}{2}$
Worked Example 5 Rationalise $\frac{3-\sqrt{5}}{3\sqrt{2}}$	**Solution** $\frac{3-\sqrt{5}}{3\sqrt{2}} \text{ x } \frac{3\sqrt{2}}{3\sqrt{2}} = \frac{3\sqrt{2}(3-\sqrt{5})}{3\sqrt{2} \text{ x } 3\sqrt{2}}$ $= \frac{(3\sqrt{2} \text{ x } 3)-(3\sqrt{2} \text{ x } \sqrt{5})}{3 \text{ x } 3 \text{ x } \sqrt{2} \text{ x } \sqrt{2}}$ $= \frac{9\sqrt{2}-3\sqrt{10}}{3 \text{ x } 3 \text{ x } \sqrt{2} \text{ x } \sqrt{2}}$ $= \frac{9\sqrt{2}-3\sqrt{10}}{9 \text{ x } 2}$ $= \frac{9\sqrt{2}-3\sqrt{10}}{18}$

Assessment A: Rationalising with a monomial surd in the denominator

1. Select the right format to rationalise $\frac{9}{\sqrt{2}}$

A) $\frac{9}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

B) $\frac{9}{\sqrt{2}} \times \frac{9}{\sqrt{2}}$

C) $\frac{9}{\sqrt{2}} \times \frac{9}{9}$

D) $\frac{9}{\sqrt{2}} \times \frac{2}{\sqrt{2}}$

2. Select the right format to rationalise $\frac{9}{3\sqrt{2}}$

A) $\frac{9}{3\sqrt{2}} \times \frac{3\sqrt{2}}{\sqrt{2}}$

B) $\frac{9}{3\sqrt{2}} \times \frac{3\sqrt{2}}{3\sqrt{2}}$

C) $\frac{9}{3\sqrt{2}} \times \frac{3\sqrt{2}}{3 \text{ x } 4}$

D) $\frac{9}{3\sqrt{2}} \times \frac{9}{3\sqrt{2}}$

3. Rationalise $\frac{7-\sqrt{6}}{\sqrt{5}}$

4. Rationalise $\frac{4-\sqrt{6}}{4\sqrt{8}}$

Objective B: Rationalising with binomial surds in the denominator

As we mentioned in tutorial 1, binomial surds are those with two parts (i.e, $3+\sqrt{8}$)

We cannot rationalise fractions with these types of denominators the same way as before.

If we multiplied the numerator and denominator by the original denominator, there would still be a surd left within the denominator (as shown below).

$$\frac{x}{a+\sqrt{b}} = \frac{x}{a+\sqrt{b}} \times \frac{a+\sqrt{b}}{a+\sqrt{b}} = \frac{x(a+\sqrt{b})}{(a+\sqrt{b})(a+\sqrt{b})} = \frac{ax+x\sqrt{b}}{a^2+b+2a\sqrt{b}}$$

Still a surd left in the denominator!

Therefore, to get rid of the surd, we must use "the differences of two squares" law to remove the surds (as shown below).

$$\frac{x}{a+\sqrt{b}} = \frac{x}{a+\sqrt{b}} \times \frac{a-\sqrt{b}}{a-\sqrt{b}}$$

$$= \frac{ax-x\sqrt{b}}{(a+\sqrt{b})(a-\sqrt{b})}$$

$$= \frac{ax-x\sqrt{b}}{a^2-b+a\sqrt{b}-a\sqrt{b}}$$

Note: The surds cancel out

$$= \frac{ax-x\sqrt{b}}{a^2-b}$$

Practice B: Rationalising with binomial surds in the denominator

Worked Example 1

Rationalise the denominator: $\frac{7}{\sqrt{7}-3}$

Solution

$$\frac{7}{\sqrt{7}-3} \times \frac{\sqrt{7}+3}{\sqrt{7}+3} = \frac{7(\sqrt{7}+3)}{(\sqrt{7}-3)(\sqrt{7}+3)}$$

$$= \frac{7\sqrt{7}+21}{7-9-3\sqrt{7}+3\sqrt{7}}$$

$$= \frac{7\sqrt{7}+21}{-2}$$

Worked Example 2

Rationalise the denominator: $\frac{\sqrt{7}}{\sqrt{7}-3}$

Solution

$$\frac{\sqrt{7}}{\sqrt{7}-3} \times \frac{\sqrt{7}+3}{\sqrt{7}+3} = \frac{\sqrt{7}(\sqrt{7}+3)}{(\sqrt{7}-3)(\sqrt{7}+3)}$$

$$= \frac{7+\sqrt{21}}{7-9-3\sqrt{7}+3\sqrt{7}}$$

$$= \frac{7+\sqrt{21}}{-2}$$

Worked Example 3

Rationalise the denominator: $\frac{2\sqrt{5}}{5-\sqrt{3}}$

Solution

$$\frac{2\sqrt{5}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{2\sqrt{5}(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$$

$$= \frac{10\sqrt{5}+2\sqrt{15}}{25-3-3\sqrt{5}+3\sqrt{5}}$$

$$= \frac{10\sqrt{5}+2\sqrt{15}}{22}$$

$$= \frac{5\sqrt{5}+\sqrt{15}}{11}$$

Note: 10, 2 and 22 all have common factor of 2 therefore we can further simplify our answer

Worked Example 4

Rationalise the denominator: $\frac{4\sqrt{5}}{5+\sqrt{3}}$

Solution

$$\frac{4\sqrt{5}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{4\sqrt{5}(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$$

$$= \frac{20\sqrt{5}+4\sqrt{15}}{25-3-3\sqrt{5}+3\sqrt{5}}$$

$$= \frac{20\sqrt{5}+4\sqrt{15}}{22}$$

$$= \frac{10\sqrt{5}+2\sqrt{15}}{11}$$

Worked Example 5

Rationalise the denominator: $\frac{4\sqrt{5}}{5+2\sqrt{3}}$

Solution

$$\frac{4\sqrt{5}}{5+2\sqrt{3}} \times \frac{5-2\sqrt{3}}{5-2\sqrt{3}} = \frac{4\sqrt{5}(5-2\sqrt{3})}{(5+2\sqrt{3})(5-2\sqrt{3})}$$

$$= \frac{20\sqrt{5}-8\sqrt{15}}{25-3-2\sqrt{3}+2\sqrt{3}}$$

$$= \frac{20\sqrt{5}-8\sqrt{15}}{22}$$

$$= \frac{10\sqrt{5}-4\sqrt{15}}{11}$$

Assessment B: Rationalising with binomial surds in the denominator

1. Rationalise the denominator: $\frac{1}{\sqrt{3}+1}$

A) $\frac{\sqrt{3}-1}{2}$ C) $\frac{\sqrt{3}+1}{2}$

B) $\frac{\sqrt{3}-1}{4}$ D) $\frac{\sqrt{3}+1}{4}$

2. Rationalise the denominator: $\frac{\sqrt{3}}{3-\sqrt{3}}$

A) $\frac{1}{2}+3\sqrt{3}$ C) $\frac{\sqrt{3}}{3}$ - 1

B) $\frac{1+\sqrt{3}}{2}$ D) $\frac{1}{2}$

3. Rationalise the denominator: $\frac{2}{3+\sqrt{5}}$

A) $\frac{3-\sqrt{5}}{2}$ C) $\frac{2}{3}+\frac{2\sqrt{5}}{5}$

B) $\frac{4}{14}$ D) $\frac{2\sqrt{5}}{8}$

4. Rationalise the denominator of $\frac{4}{2+\sqrt{3}}$

A) $\frac{4\sqrt{3}}{5}$ C) 2 + $\frac{4\sqrt{3}}{3}$

B) $\frac{16}{7}$ D) 8 - $4\sqrt{3}$

Objective C: Rationalising with binomial surds in the numerator and denominator

Rationalising with binomial surds follows the same principles as those covered in objective B in this tutorial (difference of two squares).

However, these questions often require simplification which is where many students lose marks.

Practice C: Rationalising with binomial surds in the numerator and denominator

Worked Example 1

Rationalise the denominator: $\frac{3+\sqrt{2}}{\sqrt{7}-3}$

Solution

$$\frac{3+\sqrt{2}}{\sqrt{7}-3} \times \frac{\sqrt{7}+3}{\sqrt{7}+3} = \frac{(3+\sqrt{2})(\sqrt{7}+3)}{(\sqrt{7}+3)(\sqrt{7}-3)}$$

$$= \frac{3\sqrt{7}+3\sqrt{2}+9+\sqrt{14}}{7-9+3\sqrt{7}-3\sqrt{7}}$$

$$= \frac{9+3\sqrt{7}+3\sqrt{2}+\sqrt{14}}{-2}$$

Note: The numerator cannot be simplified as there are no common surds

Worked Example 2

Rationalise the denominator: $\frac{3+\sqrt{2}}{\sqrt{8}-3}$

Solution

$$\frac{3+\sqrt{2}}{\sqrt{8}-3} \times \frac{\sqrt{8}+3}{\sqrt{8}+3} = \frac{(3+\sqrt{2})(\sqrt{8}+3)}{(\sqrt{8}+3)(\sqrt{8}-3)}$$

$$= \frac{3\sqrt{8}+3\sqrt{2}+9+\sqrt{16}}{8-9+3\sqrt{8}-3\sqrt{8}}$$

$$= \frac{3\sqrt{8}+3\sqrt{2}+9+4}{-1}$$

$$= \frac{3\sqrt{8}+3\sqrt{2}+9+4}{-1}$$

Note: $3\sqrt{8}$ can be further simplified to
$= 3 \times \sqrt{4} \times \sqrt{2}$
$= 3 \times 2 \times \sqrt{2}$
$= 6\sqrt{2}$

$$= \frac{6\sqrt{2}+3\sqrt{2}+13}{-1}$$

$$= \frac{9\sqrt{2}+13}{-1}$$

Worked Example 3

Rationalise the denominator: $\frac{4\sqrt{12}-7\sqrt{6}}{5\sqrt{6}-2\sqrt{12}}$

Solution

$$\frac{4\sqrt{12}-7\sqrt{6}}{5\sqrt{6}-2\sqrt{12}} \times \frac{4\sqrt{12}-7\sqrt{6}}{5\sqrt{6}-2\sqrt{12}} = \frac{20\sqrt{72}+8\sqrt{144}-35\sqrt{36}-14\sqrt{72}}{25\sqrt{36}+10\sqrt{72}-10\sqrt{72}-4\sqrt{144}}$$

$$= \frac{8\sqrt{144}+6\sqrt{72}-35\sqrt{36}}{25\sqrt{36}-4\sqrt{144}}$$

$$= \frac{8(12)+6(6\sqrt{2})-35(6)}{25(6)-4(12)}$$

$$= \frac{-114+36\sqrt{2}}{102}$$

$$= \frac{-19+6\sqrt{2}}{17}$$

Worked Example 4

Rationalise the denominator: $\frac{5\sqrt{3}+2\sqrt{6}}{3\sqrt{3}-8\sqrt{6}}$

Solution

$$\frac{5\sqrt{3}+2\sqrt{6}}{3\sqrt{3}-8\sqrt{6}} \times \frac{5\sqrt{3}+2\sqrt{6}}{3\sqrt{3}-8\sqrt{6}} = \frac{15\sqrt{9}+40\sqrt{18}+6\sqrt{18}+16\sqrt{36}}{9\sqrt{9}+24\sqrt{18}-24\sqrt{18}-64\sqrt{36}}$$

$$= \frac{15\sqrt{9}+46\sqrt{18}+16\sqrt{36}}{9\sqrt{9}-64\sqrt{36}}$$

$$= \frac{15(3)+46(3\sqrt{2})+16(6)}{9(3)-64(6)}$$

$$= \frac{45+138\sqrt{2}+96}{27-384}$$

$$= \frac{141+138\sqrt{2}}{-357}$$

$$= \frac{141+46\sqrt{2}}{-119}$$

Assessment C: Rationalising with binomial surds in the numerator and denominator

1. Expand the following: $(3+\sqrt{3})(5-\sqrt{7})$

2. Expand the following: $(9+4\sqrt{6})(5-2\sqrt{3})$

3. Rationalise the denominator: $\frac{3\sqrt{6}-7\sqrt{2}}{5\sqrt{6}+2\sqrt{2}}$	4. Rationalise the denominator: $\frac{3\sqrt{8}-5\sqrt{6}}{2\sqrt{8}+3\sqrt{6}}$

Homework 3 – Rationalising the denominator

1. Rationalise the denominator:

a) $\frac{5}{10\sqrt{5}}$	b) $\frac{5}{\sqrt{5}}$	c) $\frac{8}{\sqrt{8}}$
d) $\frac{10}{\sqrt{8}}$	e) $\frac{10}{\sqrt{12}}$	f) $\frac{10}{2\sqrt{3}}$

2. Rationalise the denominator:

a) $\frac{\sqrt{5}}{5-\sqrt{3}}$	b) $\frac{2\sqrt{5}}{5-\sqrt{3}}$	c) $\frac{4\sqrt{5}}{5-\sqrt{3}}$
d) $\frac{4\sqrt{5}}{5-2\sqrt{3}}$	e) $\frac{4\sqrt{5}}{5-3\sqrt{3}}$	f) $\frac{4\sqrt{5}}{5-3\sqrt{3}}$

3. Rationalise the denominator:

a) $\frac{2+\sqrt{5}}{5+\sqrt{3}}$	b) $\frac{3+\sqrt{6}}{\sqrt{6}-2}$	c) $\frac{3+\sqrt{2}}{2+\sqrt{3}}$

d) $\frac{6\sqrt{3}+2\sqrt{8}}{4\sqrt{7}-8\sqrt{2}}$	e) $\frac{6\sqrt{3}+4\sqrt{6}}{3\sqrt{3}-8\sqrt{6}}$	f) $\frac{8\sqrt{3}+4\sqrt{6}}{2\sqrt{3}-8\sqrt{6}}$

5. Surds in Questions
Key Words: Proof, application, geometry

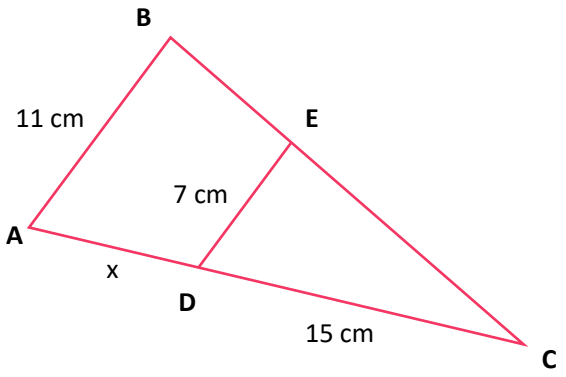
Objectives this tutorial:

A) Application of surds in proof questions
B) Solving equations with surds
C) Application in geometry problems

Starter:

Triangles ABC and DEC are similar.

Calculate the length of AD.



.....

.....

.....

.....

.....

Objective A: Application of surds in proof questions

Surd proof questions are those which ask you to simplify an expression and leave the answer in a certain format.

Generally, these questions will ask you to leave your answer in a format such as $k\sqrt{3}$ or $a + b\sqrt{3}$ (where k/a/b are integers).

Note: the number in the root or the letters representing the constant will vary from question to question.

The best way to answer these questions is to ignore what the final format of the answer is to begin with, and work through simplifying the answer first. Once simplified, the answer shouldn't be too far away from the final format and will only require minor rearrangements.

Many questions will incorporate the use of BIDMAS and rationalising.

Practice A: Application of surds in proof questions

Worked Example 1

Write the following expression in the form $k\sqrt{3}$, where k in an integer.

$$\frac{90}{\sqrt{3}} - \sqrt{6} \times \sqrt{8} - (2\sqrt{3})^3$$

Solution

Step 1 to simplifying this expression is to separate the question into its individual elements.

$\frac{90}{\sqrt{3}}$ -> This needs to be rationalised.

$$= \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{90\sqrt{3}}{\sqrt{3}} = 30\sqrt{3}$$

$\sqrt{6} \times \sqrt{8}$ -> Multiply to simplify

$$= \sqrt{6} \times \sqrt{8} = \sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

$(2\sqrt{3})^3$ -> Expand out the brackets

$$= 2\sqrt{3} \times 2\sqrt{3} \times 2\sqrt{3}$$
$$= 2\sqrt{3} \times 4 \times 3$$
$$= 24\sqrt{3}$$

Step 2 involve putting all the elements back together again

$$= 30\sqrt{3} - 4\sqrt{3} - 24\sqrt{3}$$

$$= 2\sqrt{3}$$

Note: Check the sign (positive/negative) of each part of the expression!

Worked Example 2

Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where a and b are integers.

$$\frac{3-\sqrt{3}}{3+\sqrt{3}}$$

Solution

$$\frac{3-\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{(3-\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$

$$= \frac{9-3\sqrt{3}-3\sqrt{3}+3}{9-3\sqrt{3}+3\sqrt{3}-3}$$

$$= \frac{12-6\sqrt{3}}{6}$$

$$= 2 - \sqrt{3}$$

Worked Example 3

Simplify the following expression, writing the final answer in the form $a + b\sqrt{3}$, where a and b are integers.

$$\frac{2\sqrt{3}-1}{2-\sqrt{3}}$$

Solution

$$\frac{2\sqrt{3}-1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2\sqrt{3}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{4\sqrt{3}+6-2-\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3}$$

$$= \frac{3\sqrt{3}+4}{1}$$

$$= 4 + 3\sqrt{3}$$

Assessment A: Application of surds in proof questions

1. Show that $\frac{5 + 2\sqrt{3}}{2 + \sqrt{3}}$ can be written as 4 - $\sqrt{3}$

2. Simplify the following expression, writing the final answer in the form a + b$\sqrt{2}$, where a and b are integers.

$$\frac{10 - \sqrt{32}}{\sqrt{2}}$$

3. A shed has dimensions, in metres of:

Height = $\sqrt{5}$

Width = $\sqrt{6}$

Length = $\frac{9}{\sqrt{2}}$

Find the volume of the shed.

Give your answer in the form a$\sqrt{15}$ where a is an integer.

Objective B: Solving equations with surds

Solving equations with surds follows the same principles as solving equations with algebra.

There are a few tricks we must remember which include:

a) $(\sqrt{x})^2 = x$

b) $x^{\frac{1}{2}} = \sqrt{x}$

Many of these questions involve knowing how to manipulate the questions once again.

Practice B: Solving equations with surds

Worked Example 1

$$z\sqrt{8} - 6 = \frac{2z}{\sqrt{2}}$$

Solve the above equation giving your answer in the form $k\sqrt{2}$, where k in an integer

Solution

Step 1: Multiply both sides by $\sqrt{2}$

-> $z\sqrt{8}\sqrt{2} - 6\sqrt{2} = 2z$

Step 2: Simplify surds and collect like terms

-> $z\sqrt{16} - 6\sqrt{2} = 2z$
-> $4z - 6\sqrt{2} = 2z$
-> $2z = 6\sqrt{2}$
-> $z = 3\sqrt{2}$

Worked Example 2

$$\frac{2+y}{y} = \sqrt{2}$$

Solve the above equation giving the answer in the form $a + b\sqrt{2}$, where a and b are integers

Solution

Step 1: Re-arrange for y

-> $2 + y = \sqrt{2}y$
-> $2 = \sqrt{2}y - y$
-> $2 = y(\sqrt{2} - 1)$
-> $y = \frac{2}{\sqrt{2}-1}$

Step 2: Although we have solved for y, we still need to rationalise our answer to get it in the correct form.

-> $y = \frac{2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

-> $y = \frac{2(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

-> $y = \frac{2\sqrt{2}+2}{2+\sqrt{2}-\sqrt{2}-1}$

-> $y = \frac{2\sqrt{2}+2}{1}$

-> $y = 2 + 2\sqrt{2}$

Worked Example 3

Find the value of x if:

$$\frac{15\sqrt{x}}{\sqrt{5}} = 6\sqrt{5}$$

Solution

Step 1: Remove fraction and simplify surds

-> $15\sqrt{x} = (6\sqrt{5})(\sqrt{5})$
-> $15\sqrt{x} = 6 \times 5$
-> $15\sqrt{x} = 30$

Step 2: Rearrange and solve for x

-> $\sqrt{x} = (30 \div 15)$
-> $\sqrt{x} = 2$

Note: The opposite function of a square root is square. Therefore, if we square both sides:

-> $(\sqrt{x})^2 = 2^2$
-> $x = 4$

Worked Example 4

Find the value of x if:

$$\sqrt{3x + 22} = x + 4$$

Solution

Step 1: To remove the square root, we need to square both sides.

-> $(\sqrt{3x + 22})^2 = (x + 4)^2$
-> $3x + 22 = x^2 + 8x + 16$

Step 2: You should notice that the equation above can be re-arranged to look like a quadratic equation. From here you can solve for x.

-> $x^2 + 5x - 6 = 0$
-> $(x + 6)(x - 1) = 0$
-> $x \neq -6$
-> $x = 1$

Note: You cannot have a negative value in a square root therefore the only valid answer is x = 1.

Assessment B: Solving Equations with Surds

1. Solve the below equation giving your answer in the form $k\sqrt{x}$

$$z\sqrt{27} - 5 = \frac{2z}{\sqrt{3}}$$

2. Solve the equation below giving your answer in the form $\frac{a}{\sqrt{3}}+$ b, where a and b are integers

$$\frac{5+y}{y}=\sqrt{3}$$

3. Find the value of x if:

$$\frac{30\sqrt{x}}{\sqrt{3}}=\mathbf{2}\sqrt{3}$$

4. Find the value of x if:

$$\sqrt{10x+24}=x+4$$

Objective C: Application in geometry questions

So far we have looked at surds in a very mathematical way. The questions have revolved around practising key skills such as simplifying, rationalising and solving equations.

In this objective we will explore surds within questions. These questions will often combine skills such as calculating perimeters, areas, volumes, trigonometry and Pythagoras.

Practice C: Application in geometry questions

Worked Example 1

The area of a rectangle is $\sqrt{125}$ cm^2.

The length of the rectangle is $(2+\sqrt{5})$cm.

Calculate the width of the rectangle.

Express your answer in the form $a + b\sqrt{5}$, where a and b are integers.

Solution

Step 1: Dissect the what skills are needed to answer the question. We can identify that we will need to work with:

-> Calculating area

-> Re-arranging equations

-> Rationalising

Step 2: Set up an equation.

Area = length x width

-> $\sqrt{125} = (2+\sqrt{5})$ x width

-> Width = $\frac{\sqrt{125}}{2+\sqrt{5}}$

Step 3: Solve

-> $\frac{\sqrt{125}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$

-> $\frac{\sqrt{125}(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$

-> $\frac{2\sqrt{125}-\sqrt{625})}{4-5+2\sqrt{5}-2\sqrt{5}}$

-> $\frac{2 \text{ x } 5 \text{ x } \sqrt{5} - 25}{-1}$

-> $\frac{10\sqrt{5} - 25}{-1}$

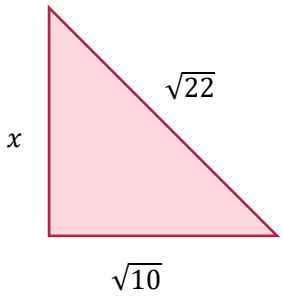
-> $\frac{10\sqrt{5}}{-1} - \frac{25}{-1}$

-> $-10\sqrt{5} + 25$

-> $(25 - 10\sqrt{5})$ cm

Worked Example 2

Find the length of the missing side, giving your answer in as simple form as possible. All sides lengths are in centimetres.



Solution

Step 1: Identify what processes may be involved in the question.

You should be able to identify from the image that the shape is a right angle triangle. Therefore, to find the missing side, Pythagoras is required.

Step 2: Set up equation

-> $a^2 + b^2 = c^2$

-> $x^2 + (\sqrt{10})^2 = (\sqrt{12})^2$

-> $x^2 = 12 - 10$

-> $x^2 = 2$

-> $x = \sqrt{2}$ cm

Note: Don't forget your units!

Worked Example 3

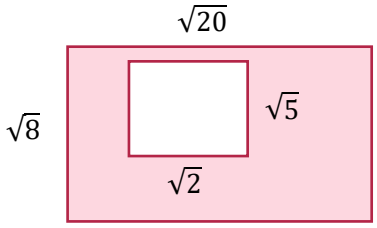
A large rectangle piece of card is $\sqrt{20}$ cm long and $\sqrt{8}$ cm wide.

A small rectangle $\sqrt{2}$ cm long and $\sqrt{5}$ cm wide is cut out of the piece of card.

Express the area of the card that is left as a percentage of the area of the large rectangle.

Solution

Step 1: Draw a diagram! The best way to understand this problem is to draw the diagram and label it using the information in the question.



Step 2: Dissect the information.

The question clearly states that it wants you to find the area therefore this should be your starting point.

Step 3: Calculation:

Area of large card: $\sqrt{20} \times \sqrt{8} = \sqrt{160} = \sqrt{16} \times \sqrt{10} = 4\sqrt{10}$

Area of cut out = $\sqrt{2} \times \sqrt{5} = \sqrt{10}$

Step 4: Percentage

To calculate the percentage cut out you need to divide the cut out by the total.

-> $\frac{\sqrt{10}}{4\sqrt{10}} = 0.25$

-> 0.25 = 25% cut out.

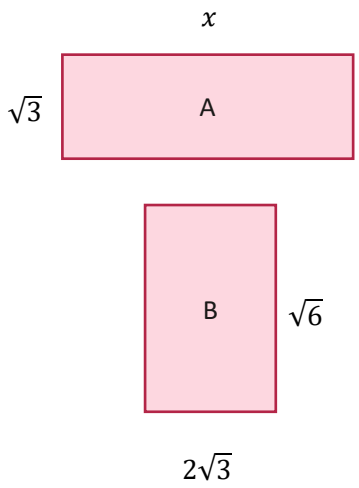
Therefore, percentage left of card:

-> 100% - 25% = 75%

Worked Example 4

The area of two rectangles, A and B, have the same area.

Find the value of x. Give your answer in the form $a\sqrt{6}$.



Solution

Step 1: Determine the tools you need to answer the question.

The question here mentions area. You should also identify that you will need to set up and rearrange an equation to solve for x.

Step 2: Set up equation and solve

We know the area of the two shapes are equal therefore

-> Area A = Area B

-> $\sqrt{3} \times x = \sqrt{6} \times 2\sqrt{3}$

-> $x\sqrt{3} = 2\sqrt{18}$

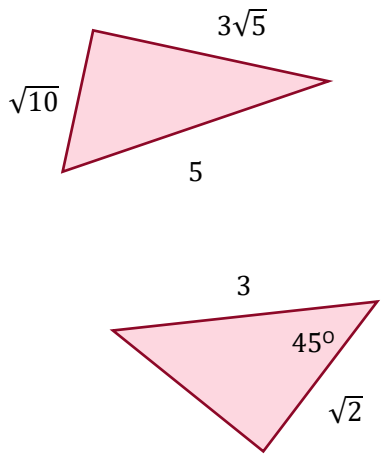
-> $x = \frac{2\sqrt{18}}{\sqrt{3}}$

-> $= \frac{2(\sqrt{3})(\sqrt{6})}{\sqrt{3}}$

-> $=(2\sqrt{6})$ cm

Worked Example 5

Show the two triangles are similar.



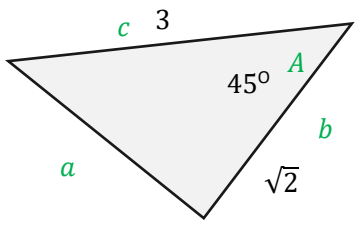
Solution

Step 1: Identify what you know and what we need to find out.

To determine if the triangles are similar, we need to determine the lengths of all three sides of both triangles.

Similar triangles will have scalar sides of each other.

Step 2: Determine the length of the missing side.



You should be able to identify that the cosine rule is needed to determine the length of side a.

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 3^2 + (\sqrt{2})^2 - (2 \times 3 \times \sqrt{2} \times \cos 45)$

$a^2 = 9 + 2 - (6\sqrt{2} \times \frac{\sqrt{2}}{2})$

$\frac{6 \times \sqrt{2} \times \sqrt{2}}{2} = \frac{6 \times 2}{2} = \frac{12}{2} = 6$

$a^2 = 9 + 2 - 6$

Note: Cos 45 = $\frac{\sqrt{2}}{2}$
(Memorise this)

$a^2 = 5$

$a = \sqrt{5}$

<u>Step 3</u>: Determine similarity by comparing all three sides

	Triangle 1	**Triangle 2**
Side 1:	$3\sqrt{5}$	3
Side 2:	5	$\sqrt{5}$
Side 3:	$\sqrt{10}$	$\sqrt{2}$

Each of the lengths of sides of triangle have the same scale factor.

Therefore, the triangles are similar.

Assessment C: Application in geometry questions

1. Calculate the area of the rectangle below. All sides are measured in metres.

$1 + \sqrt{3}$

$2 - \sqrt{3}$

Simplify your answer.

2. A triangle has an area of $2\sqrt{6}$ cm^2 and a base of $\sqrt{12}$ cm.

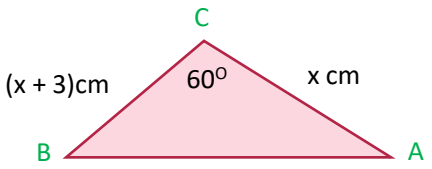
Find the height of the triangle and give your answer as a simplified surd.

3. A large rectangle piece of card is $\sqrt{40}$ cm long and $\sqrt{12}$ cm wide.

A small circle of diameter $\sqrt{5}$ cm is cut out of the piece of card.

Express the area of the card that is left as a percentage of the area of the large rectangle.

4. The area of the triangle below is $\sqrt{300}$ cm^2. Calculate the length of AB.



6. Final Assessment

Baseline Assessment - Student Feedback

1. How would you rate this assessment?

Very easy | Easy | Neutral | Difficult | Very Difficult

2. How confident are you with surds?

Can do no questions | Can do some questions | Can do all questions

3. How do you think you best learn?

Group discussions | Worked examples | Paired explanations

Practice | Homework

4. What other topics do you find challenging in maths?

Vectors | Circle Theorems | Pythagoras

Trigonometry | Data Handling | Other (please specify)

5. What would you like to get out of these tutorials?

Mastery with surds | Tips on revision and planning

Exam question practice | Theory practice

Other (please specify)

Baseline Assessment -Teacher Feedback

What went well:

1.

2.

3.

How could you improve:

1.

2.

3.

Notes

Notes

Notes

It’s not abSURD.

Key Stage 4 Programme

researchersinschools.org