2016 cohort

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How do we teach Mathematical problem solving?

The new GCSE specification in maths (9-1) has a greater focus on problem solving (Edexcel, 2015) which includes the need for creativity, expert thinking and metacognition in solving questions. Reports from the Pearson (Pearson, 2017), Ofqual (2015) and the Advisory Committee for Mathematics Education (ACME, 2016) identified a key shift towards an emphasis on problem-solving.

Mathematical problem solving which involves applying mathematical concept in practical questions is a key area of development for Beckfoot Thornton school. Analysing the results of students at Beckfoot Thornton school, in 2017, 49% of students gained grade 4+, in Mathematics (Nationally 70.7%), 26% gained Grade 5+. This similar compared to 2018 results where 49% still gained grade 4+ with an increase to 35% for grade 5+.

More recent research shows that problem solving enhances brain activity and that the semantic network in the brain sub-serves mathematical problem solving. Zhou et al., (2018) and Boaler, (1998) postulate that building a culture of problem solving into the curriculum for students helps better mathematical understanding and promotes progress.

	2017 Edexcel GCSE paper	
	Non-problem Solving (%): [Total Marks 80]	Problem solving (%):[Total marks 80
Paper 1	63% [50]	37% [30]
Paper 2	60% [49]	40% [31]
Paper 3	65% [51]	35% [29]

Table 1: Percentage analysis of problem-solving content in GCSE Mathematics 2017

Table 1 shows an analysis of higher Maths Edexcel 2017 GCSE papers considering problem solving and non-problem-solving questions. Problem solving questions makes up about 38% of the total questions for all three papers and considering a total mark of 240, problem solving questions covers more than one-third of the total marks. The criteria for questions to be considered problem solving question include: inter-topic related questions, questions requiring reverse engineering of mathematics ideas and heavily worded questions that require unpacking the necessary mathematical details and formulating equations.

The research/intervention is carried out on a focus group of middle set year 8 students in Beckfoot Thornton. These middle set year 8 students are not high ability students (like top set students) and are not those with high levels of disengagement, thus implying that the outcome of the research

will help clarify whether the strategies used for the intervention are effective and not affected either by the students' abilities or disengagement issues.

The intervention schedule is given below: -

1. First data collection: This involves collecting data from previous class problem solving sessions over 3 months.
2. Introduction to problem solving and define the nitty-gritty of mathematical problem solving
3. First questionnaire on problem-solving. This questionnaire will seek to understand student perspective on problem solving, difficulties and queries.
4. First class intervention session on problem solving.
5. Interviews on problem solving.
6. Second class intervention session on problem solving.
7. Final questionnaire on problem solving to determine student progress.

The research methodology given by Mathshub (2019) on problem solving is used in schools and training institutions for teaching and introducing mathematical problem solving to both trainee maths teachers and students. The model highlights how problem solving questions should be framed and presented to students. The model framework for problem solving questions is given below:-

Generating opportunities to develop learning and deeper understanding	
Strategies The strategies/activities outlined below offer opportunities for varying degrees of reasoning. They invite pupils to reason and justify including: making sense of, representing and explaining approaches and calculations	*The stimulus question/focus:* *This would be the focus to which one (or more) of the activities below might be chosen and applied to generate rich reasoning opportunities for pupils in the lesson.* *The stimulus or focus might be a particualr question, context or any mathematical object which might be at the start or at any point in the teaching of the topic.*
The Strategy	**The stimulus question/focus**
Hide and Reveal	
Sketch a diagram/picture to represent this information	
What do you know?	
What questions could you ask?	
Here are some calculations – what could be the question?	
Classification	
As many ways you can	
Sequencing solutions	
Change the numbers to create your own question	
Change the context	
How do you know you are right?	
Using different models	
Questions to promote reasoning	
Sometimes, always, never	

Testing conjectures and investigating rich tasks	
Living graphs	

The template model given above will be used to formulate every question to be used in the problem-solving sessions. Each category will be used to create a methodological approach to help students access the questions and guide them in the problem solving task.

Preliminary results from the questionnaire showed that lots of students in the focus group find problem solving hard because they lack clear starting points to solving the questions. This has led to helping students understand key words in every question and encouraging students to underline these key words.

One other student said, "I work better when discussing mathematical problem in a group". The comment justifies the work of Julie & Mancy (2018) which encourages mathematical discussions in classrooms and highlights the impact of group work on metacognition.

These two strategies of emphasis on key mathematical and facilitating cognition through group tasks was well highlighted by Marjorie & Mary (1997). Both strategies will be used in the second wave of classroom intervention before the final results are collected.

The research is still in progress, however, key findings from research that build confidence and encourage cognitive thinking in Mathematics include building strong mathematical vocabulary and facilitating discussions in groups of not more than three students.

More results will be collected from the second wave of intervention and one-to-one interview with the students.

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