2020 cohort

Henry Allen – Maths Teacher

How does teaching metacognitive problem-solving strategies to Year 11 HPAs impact upon their ability to answer A03 exam questions?

Context

Metacognition, simply defined along the lines of "thinking about thinking" has been recognised by the Education Endowment Federation's Teaching and Learning Toolkit as a "High impact for very low cost, based on extensive evidence" strategy for improving pupils' outcomes, [1]. While all pupils will develop metacognition to some extent as they mature, the majority will not spontaneously develop the wide array of metacognitive strategies that they will need, e.g. [2]. Overall, research suggests that pupils should be explicitly taught metacognitive strategies, however, to be effective this should be done only after they have gained sufficient subject knowledge.

Is there a set of generic metacognitive skills that are particularly useful in mathematics? Successful problem-solvers display several shared metacognitive strategies: reading the problem more than once, checking what the problem was asking, showing the problem schematically, developing a strategy for solving the problem, thinking about how the solving process was going, asking whether answers were meaningful, checking calculations, and checking the information stated in the problem for anything requiring particular attention, [3]. Interventions that do not explicitly focus on teaching metacognition are not enough for pupils to develop their metacognitive skills, [2, 4]. Altogether, this suggests that there are generic metacognitive strategies that are useful in mathematics, however it does not necessarily mean that these strategies can be taught outside of mathematics classes.

The importance of problem solving for mathematics education has been appreciated since the 1980s, [5]. The seminal text on problem solving in mathematics describes the following framework: understand the problem, plan how you will solve it, carry out the solution plan, and reflect on the process, [6]. Understanding a problem requires sufficient subject knowledge, for example novices presented with problems with the same deep structure get distracted by surface features, [7,8].

The Research

Pupils took a three-week course and were instructed in metacognitive strategies for tackling A03 (problem-solving) mathematics questions. The instruction was based around the three steps of problem solving as discussed by Polya: understanding the problem, devising a solution, carrying out the solution, [6]. Each intervention session was based around one of the three steps to allow pupils to practise their metacognitive skills. Before the intervention began a baseline test was administered, and at the end of the intervention another test was administered to assess progress. The primary focus was to examine the impact on pupils academic attainment and ability to reason mathematically.

The first intervention focused around understanding the problem, and in order to promote understanding and recognition of I employed SSDD problems to help pupils see different deep mathematical structures from similar surface level problems, [9]. The second intervention focused around devising a solution to a problem. To aid devising of a solution I employed open problems

where a scenario from an exam-style question is presented, and pupils are invited to work out everything they can from the scenario. The importance of open problems in mathematics education has been highlighted as a tool to increase understanding and motivation, [10,11]. The third intervention focused on pupils implementing solutions. This session involved pupils being presented a goal-free problem instructing them simply to "explore".

Pupils' progress was measured using pre- and post-assessments. In order to ensure that problem solving was the focus, AO3 questions (questions identified by AQA as having significant problem-solving elements) were selected. The pre data was taken from pupils' recent mock examinations, completed shortly prior to starting the project. In order to ensure that the data was comparable, the post-assessment was taken from AQA shadow papers – papers that have similar questions to the mock papers, [12]. For the post assessments AO3 questions in which pupils had performed relatively poorly were chosen, and a bespoke assessment containing those questions was created.

Participants were selected based upon their ability in mathematics, i.e. potential to achieve grades 8/9 at GCSE. Part of the purpose of the intervention is to expose pupils to grade 8/9 topics that they have not been able to access during their regular lessons, as well as to better prepare them for the potential of continuing to study mathematics at A-level.

Discussion

Table 1 shows the percentage change for each pupil for each question from their pre-assessment to their post-assessment. Results show a significant percentage change of between 17% and 27% for five of the six participants. Results also show a good improvement over most questions, between 11% and 50%. The exceptions are Question 6 which shows a 25% decrease in average score, Question 9 which shows no improvement for part a and only 8% improvement for part b, and Question 10 which showed only a 4% improvement.

Question	1	2	3	4a	4b	5	6	7	8	9a	9b	10	
Marks	3	3	4	3	2	4	4	5	3	2	4	4	41
P1	0%	0%	-25%	33%	0%	50%	0%	0%	-33%	0%	0%	0%	2%
P2	0%	67%	50%	67%	0%	0%	0%	80%	33%	0%	0%	0%	27%
P3	67%	33%	0%	0%	0%	75%	-50%	40%	0%	0%	0%	25%	17%
P4	-33%	0%	-25%	67%	100%	75%	-25%	100%	33%	0%	0%	0%	24%
P5	0%	33%	50%	0%	100%	0%	-50%	20%	67%	0%	50%	0%	20%
P6	33%	0%	50%	0%	100%	-25%	-25%	40%	67%	0%	0%	0%	17%
	11%	22%	17%	28%	50%	29%	-25%	47%	28%	0%	8%	4%	18%

Table 2 shows which question belongs to which topic strand, the total marks scored across all participants for each strand, and percentage change from pre- to post-assessment. This shows the significant improvement of between 22% and 37% for the Algebra, Ratio/Proportion, and Probability strands, with the largest improvement being Probability. Comparatively the change for the geometry strand is much smaller at only 5%, which includes the three questions shown to have the smallest percentage change in the previous slide.

Strand	Questions					Pre	Post	% Change
Algebra	Q1	Q5	Q7			30	53	32%
Ratio/Proportion	Q2					9	13	22%
Geometry	Q3	Q6	Q8	Q9	Q10	34	40	5%
Probability	Q4					8	19	37%

Although some promising results have been suggested, the small number of participants makes it difficult to draw conclusions from the data, further, due to the relatively small scope of this project two important strands of subject knowledge were not tested: Number and Statistics. One way to develop this project therefore would be to roll it out to a whole class. This study was restricted to pupils aiming to achieve grade 8s or 9s at GCSE, extending the intervention to pupils with targets of grades 5, 6, and 7 would therefore be interesting. Another interesting development would be to

change the focus from being an intervention to part of normal class teaching. Refining the intervention to teach specific strategies for each topic could be useful. Finally, the intervention should be extended to allow pupils to reflect upon problem-solving endeavours.

References

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