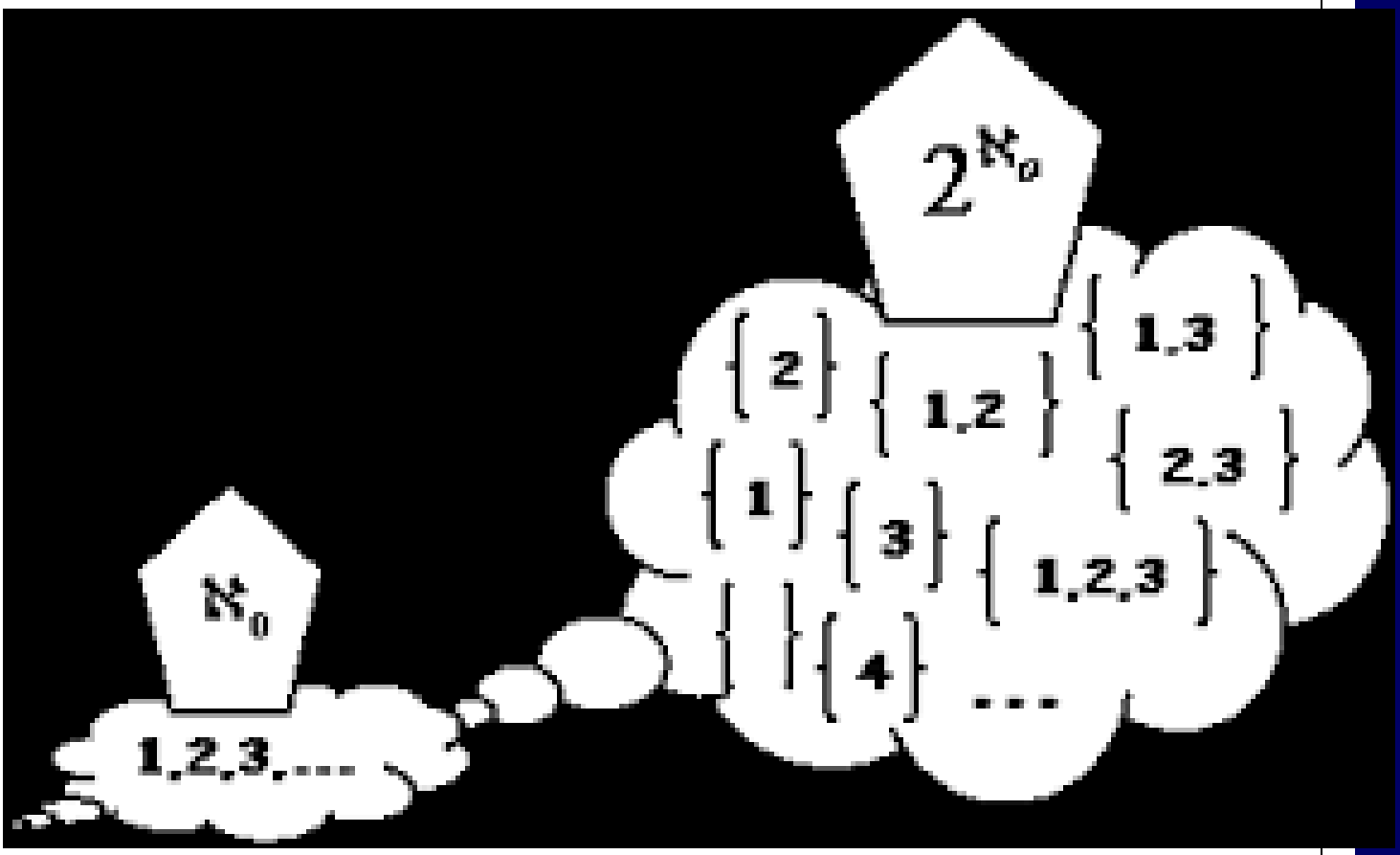


THE "WHOLE" IS NOT NECESSARILY LARGER THAN THE "PART"! AN INTRODUCTION TO TRANSFINITE CARDINALS.

Key Stage 4 Programme – 2016-2017

Teacher Name: Dr Carmelo Di Natale

Pupil Name: _____

Fill this in with your teacher.

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1	Baseline assessment: differentiated questions about sets and functions.	
Tutorial 2	A list of exercises about relations.	
Tutorial 3	A list of open questions to answer.	
Tutorial 4	A list of exercises about cardinality and countable sets	
Tutorial 5	Final assignment: write a short report about _____	

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after 4.00pm on _____	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

KS4 Programme 2016-17 – Pupil Feedback Report

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-Level
2:1	60-69	Performing to a good standard at A-Level
2:2	50-59	Performing to an excellent standard at current key stage
3rd	40-49	Performing to a good standard at current key stage
Working towards a pass	0-39	Performing below a good standard at current key stage
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Name of RIS Teacher	Dr Carmelo Di Natale		
Title of Assignment			
Name of Pupil			
Name of School	**Cleeve Park School**, Bexley Lane, Sidcup, Kent, DA14 4JN		
ORIGINAL MARK / 100		**FINAL MARK / 100**	
DEDUCTED MARKS		**FINAL GRADE**	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Learning Feedback Comment 1 – *Contents comprehension and mastery*	
What you did in relation to this Key Learning Priority	**How you could improve in the future**
Learning Feedback Comment 2 – *Logical Reasoning, Application and Problem Solving*	
What you did in relation to this Key Learning Priority	**How you could improve in the future**
Learning Feedback Comment 3 – *Language and Communication*	
What you did in relation to this Key Learning Priority	**How you could improve in the future**
Resilience Comment	
How you showed learning resilience during the course	**How you could build learning resilience in the future**

Contents

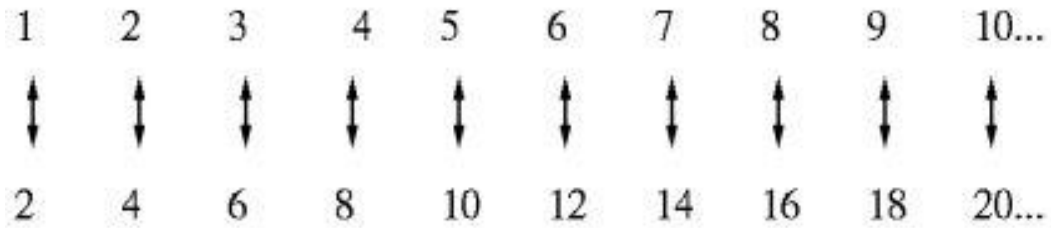
Course Rationale	Page 5
Course Map	Page 6
Mark Scheme	Page 7
Glossary of Keywords	Page 8
Tutorial 1	Page 10
Baseline Assessment	Page 11
Tutorial 2	Page 12
Tutorial 3	Page 13
Tutorial 4	Page 14
Tutorial 5	Page 15
Tutorial 6	Page 16
Appendix: University Application Guidance	Page 17

Course Rationale

We all have an intuitive idea of the concepts of finite set and infinite set: the set of pens in my pencil case is finite (I have exactly 1 pen in my pencil case), the set of tables in this classroom is finite (there are exactly 9 tables in this classroom), the set of students studying at Cleeve Park School is finite (there are exactly 1025 students in this school). Even the set of blades of grass in the school playground field is finite: there may be loads and loads of blades, but if we were patient enough we would be able to count down all of them. In other words, all those sets come with an attached whole number identifying clearly their size: these sets are called finite. However, we are also aware that infinite sets do exist: the set of whole numbers is infinite, because we cannot list all of its elements (whatever number I can think, if I add 1 I obtain a larger number). Similarly, the set of points on a line is infinite: if I pick any two points, there is always one in between. For sets like those, there is no number capturing their size: this is why they are called infinite.

$$\textbf{Whole Numbers} = \{0,1,2,3,\ldots,1000,1001,\ldots,1000000,\ldots,10^{100},10^{100}+1,10^{100}+2,\ldots\}$$

This course is an opportunity for you to reflect formally and systematically on the idea of "size" of a set, both in the finite and the finite case. As a matter of fact quantitative common sense is seriously challenged by infinite sets and it is rather easy to run into paradoxes and wrong statements. As an example, it is largely believed that "the whole is always larger than the part", i.e. – more formally – that a proper subset of a given set should always be made of "fewer" elements than the ambient set. This is certainly true in the finite case: the set of female Cleeve Park students is certainly smaller in size than the set of all students of this school. Similarly, the set of tables by the window wall is certainly made of fewer elements than the set of all tables in our classroom. Any part (or better subset) of a finite set is genuinely smaller in size than the whole set. However, this fact – to which we are all so used – fails to be true in the context of infinite sets. Indeed, in these series of tutorials we will discover that there are as many even numbers as whole numbers: more formally we will show that the set of even numbers – which is a proper subset of the set of whole numbers – has the same size of the entire set of whole numbers. Even more surprisingly, we will prove that the set of fractions has the same size of the set of whole numbers. On the other hand, we will show that the line is genuinely bigger than the set of whole numbers: there are actually "more" points on a line than whole numbers, whereas there are exactly "as many fractions as whole numbers". In formal mathematical terms, you will learn that there is a "hierarchy of infinities" – called transfinite cardinals – made in such a way that a set and a proper subset may share the same "level of infiniteness".



We will begin this course by reviewing and deepening the concepts of set, set operation, function and relation. After that we will formalize the intuitive notion of size of a set, by introducing the relation of equipotence and the notion of cardinality. As a major application of such concepts, we will discuss Cantor's approach to the foundation of arithmetic and appreciate the paradoxes to which it leads (Russell's antinomy). At this point, we will finally introduce the "hierarchy of infinities" and deeply analyse the "infinity of whole numbers" (known as countable infinity) and the "infinity of the line" (known as uncountable infinity"). At the end of the course we will prove in full detail that the set of fractions is countable (i.e. it has the same size of the set of "whole numbers") and that the set of all decimal numbers is uncountable (i.e. it is genuinely larger than the set of "whole numbers"

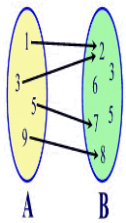
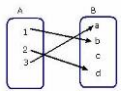
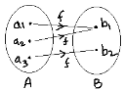
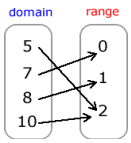
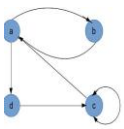
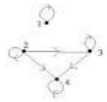
Course Map

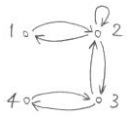
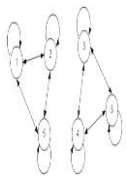
Tutorial 2
RELATIONS
The Cartesian product;
Relations and their properties;
Equivalence relations and quotient sets.
Tutorial 1
BASIC SET THEORY
Sets and set operations;
Subsets and power sets;
Functions and their properties.
Tutorial 3
CANTOR ARITHMETIC
The relation of equipotence;
Natural numbers according to Cantor;
Russell's Antinomy.
THE "WHOLE" IS NOT NECESSARILY LARGER THAN THE "PART".
An introduction to transfinite cardinals
Tutorial 5
TWO KEY EXAMPLES: ℚ AND ℝ
The set of rational numbers is countable.
The set of real numbers is uncountable.
Tutorial 4
CARDINALITY
Cardinals;
Countable vs uncountable sets;
The Continuum Hypothesis.

Mark Scheme

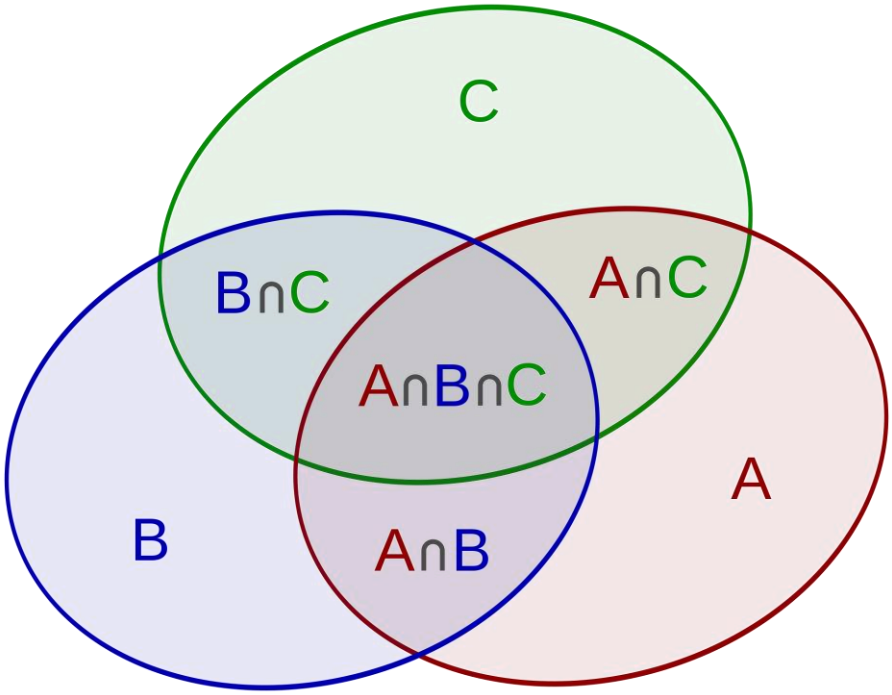
Key Skill	1st	2.2
Contents comprehension and mastery	☐ Clear, complete and detailed understanding of all notions and definitions. ☐ Full mastery of all techniques, methods and proof strategies.	☐ Clear understanding of major definitions and constructions. ☐ Decent mastery of the main proof strategies and arguments.
Logical Reasoning, Application and Problem Solving	☐ Excellent capabilities related to applying abstract concepts from the course in non-trivial examples. ☐ Good capabilities related to applying notions, methods and strategies developed within the course to prove unknown statements.	☐ Good capabilities related to construct simple examples attached to related abstract notions developed in lectures. ☐ Decent abilities related to adapting proofs and arguments developed in lectures to demonstrate slightly modified statements
Language and Communication	☐ Writing is coherent and organised following a logical sequence and an appropriate mathematical style. ☐ Correct, complete, informed and varied use of mathematical vocabulary.	☐ Mathematical writing is readable and understandable even if not fully coherent and structured. ☐ Correct and informed use of main mathematical vocabulary.

Glossary of Keywords

Word	Definition	Brief explanation	Example
Intersection	Given two sets A and B, the intersection $A\cap B$ is the set made of elements who belong to both A and B. In formula: $A\cap B:=\{x\in A \text{ and } x\in B\}$ Two sets are said to be disjoint if their intersection is empty.	The intersection of two sets is the set of common elements.	$A=\{1,2,3,4\}$ $B=\{3,4,5\}$ $A\cap B=\{3,4\}$
Union	Given two sets A and B, the union $A\cup B$ is the set made of elements who belong to A or B (or both). In formula: $A\cup B:=\{x\in A \text{ or } x\in B\}$	The union is of two sets is the set of all elements.	$A=\{1,2\}$ $B=\{2,3\}$ $A\cup B=\{1,2,3\}$
Complement	Given two sets A and B, the complement $A-B$ of B in A is the set made of elements who belong to A but do not belong to B. In formula: $A-B:=\{x\in A \text{ and } x\notin B\}$	The complement is made of the elements of a set which do not belong to the other set.	$A=\{a,b,c,d\}$ $B=\{d,e,f\}$ $A-B=\{a,b,c\}$ $B-A=\{e,f\}$
Subset	Given a set A, we say that a set S is a subset of A (and we write $S\subset A$) if all elements of S are elements of A. In formula: $S\subset A$ if and only if $x\in S \Rightarrow x\in A$	A subset is a set completely contained in another ambient set, i.e. a set whose elements are some of the elements of the ambient set.	$A=\{a,b,c,d\}$ $B=\{b,c\}$ then $B\subset A$
Power set	Given a set A, the power set $P(A)$ of A is the set whose elements are the subsets of A. In formula: $P(A):=\{S \text{ such that } S\subset A\}$	The power set is the set of all subsets of a given set.	$A=\{2,3\}$ $P(A)=\{\emptyset,\{2\},\{3\},A\}$
Function	Given two sets A and B, a function $f:A\to B$ is a law which to every element $x\in A$ associates exactly one element $f(x)\in B$. We have that: • Set A is said to be the domain of the function f. • Set B is said to be the codomain of the function f. • For every element $x\in A$, the element $f(x)\in B$ is said to be the image of x under f. • The set R of all images of f is called range: this is the subset of the codomain B defined as $R:=\{y\in B \text{ such that } y=f(x) \text{ for some } x\in A\}$	A function is a machine which picks an input element in the source set and returns one and only one output element in the target set.	
Injective function	A function $f:A\to B$ is said to be injective if different elements in the domain have different images under f in the codomain. In formula: $f \text{ injective}: \quad \forall x_1,x_2\in A \quad f(x_1)=f(x_2) \Rightarrow x_1=x_2$	An injective function is a one-to-one machine.	
Surjective function	A function $f:A\to B$ is said to be surjective if all elements in the codomain are images of some element in the domain, i.e. if the range of f is equal to the entire codomain. In formula: $f \text{ surjective}: \quad \forall y\in B \ \exists x\in A \text{ such that } y=f(x)$	A surjective function is a machine in which all elements in the target set are the output of some element in the source set.	
Bijective function	A function $f:A\to B$ is said to be bijective if it is both injective and surjective.	A bijective function is a one-to-one machine I which all element in the target set are the output of some element in the source set.	
Cartesian product	Given two sets A and B, the Cartesian product $A\times B$ is the set made of pairs (x,y) where x belongs to A and y belongs to B. In formula: $A\times B:=\{(x,y) \text{ such that } x\in A \text{ and } y\in B\}$	The Cartesian product of two sets is the set of ordered pairs of elements in those two sets.	$A=\{1,2\}$ $B=\{a,b\}$ $A\times B=\{(1,a),(1,b),(2,a),(2,b)\}$
Relation	Given two sets A and B, a binary relation $\mathcal{R}$ between A and B is a subset of the Cartesian product $A\times B$. Given a set A, a relation in A is a subset of $A\times A$.	A relation in a set is a collection of ordered pairs of elements belonging to such a set. Relations are a way to connect elements of a set without the stricter requirements of functions.	
Reflexive property	Given a set A, a relation $\mathcal{R}$ in A is said to be reflexive if all elements in A are in relation with themselves. In formula: $\mathcal{R} \text{ reflexive}: \quad \forall x\in A \quad x\,\mathcal{R}\,x$	A reflexive relation is a relation in which any element is self-related.	

Symmetric property	Given a set A, a relation $\mathcal{R}$ in A is said to be <u>symmetric</u> if for any two elements $x_1,x_2 \in A$, x_1 is related to x_2 if and only if x_2 is related to x_1. In formula: $\mathcal{R}$ *symmetric :* $\forall x_1,x_2 \in A \quad x_1 \mathcal{R} x_2 \Leftrightarrow x_2 \mathcal{R} x_1$	A symmetric relation is a relation in which all couples come in pairs, i.e. if any two elements happen to be related, they must be related in both directions, not just in one.	
Transitive property	Given a set A, a relation $\mathcal{R}$ in A is said to be <u>transitive</u> if for any three elements $x_1,x_2,x_3 \in A$ such that x_1 is related to x_2 and x_2 is related to x_3, then x_1 is related to x_3. In formula: $\mathcal{R}$ *transitive :* $\forall x_1,x_2,x_3 \in A \ (x_1 \mathcal{R} x_2$ *and* $x_2 \mathcal{R} x_1) \Rightarrow x_1 \mathcal{R} x_3$	A transitive relation is a relation in which "being related is transmitted through elements", i.e. if an element is related to another element and this element is related to a third one, than the first element is related to the third one.	
Equivalence relation	Given a set A, a relation $\mathcal{R}$ in A is said to be an <u>equivalence relation</u> if it is simultaneously reflexive, symmetric and transitive. An <u>equivalence class in A under $\mathcal{R}$</u> is the subset of elements in A which are related to one another under $\mathcal{R}$. Two elements of A belong to the same equivalence class if and only if they are related to one another: this means that equivalence classes are disjoint and their union is the entire set A. In formal mathematical terms, we say that an equivalence relation determines a <u>partition</u> of A.	An equivalence relation is a relation slicing out a set into equivalence classes, i.e. into a number of subsets without any common element which give back the entire original set when joined together. For this to happen, the relation needs to be simultaneously reflexive, symmetric and transitive.	
Quotient set	Given a set A equipped with an equivalence relation $\mathcal{R}$, the <u>quotient set $A/\mathcal{R}$</u> is the set of all equivalence classes in A under $\mathcal{R}$.	Given a set with an equivalence relation, we can choose a representative for each equivalence class: the set of such representatives is the quotient set.	$A/\mathcal{R}$ = {{1,2,5}, {3,4,6}}
Equipotence	Two sets A and B are said to be <u>equipotent</u> (or <u>equinumerous</u>) if there exists a bijective function between A and B. In formula: *A is equipotent to B if and only if* $\exists f{:}A \to B$ *with f bijective* We can prove that equipotence is an equivalence relation in the collection of all sets.	Two sets are equipotent if we can write down a one-to-one function whose domain is the first set and whose range is the second set. If the two sets are finite, saying that they are equivalent is the same as saying that they have the same number of elements.	A={1,2,3} B={a,b,c} A and B are equipotent
Cardinality	Given a set A, the cardinality $\lvert A \rvert$ of A is the equivalence class of A under the relation of equipotence.	The cardinality of a set is just a measure of the size of the set. • If the set is finite, the cadinality is just the number of elements of the set. • If the set is infinite, the cardinality is not a number, but a symbol indicating "the level of infiniteness" of such a set. Two infinite sets may sit at different stages of the "hierarchy of infinities" determined by cardinality, so even if they are both infinite one may be "bigger" than the other.	A={a,b,c,d} $\lvert A \rvert=4$ $\mathbb{N}$={0,1,2,..} $\lvert \mathbb{N} \rvert=\aleph_0$ $\mathbb{R}$={decimals} $\lvert \mathbb{R} \rvert=\aleph_1$
Countable set	An infinite set A is said to be <u>countable</u> if it is equipotent to the set $\mathbb{N}$ of natural numbers (aka whole numbers). The cardinality of a countable set is denoted with $\aleph_0$. In formula: *A countable if and only if* $\lvert A \rvert=\lvert \mathbb{N} \rvert=\aleph_0$ An <u>uncountable</u> set is an infinite set which is not countable.	A countable set is a set which has the same size of the set of whole numbers, i.e. a set which can be put in bijection with the set of whole numbers. In other words, an infinite set is countable if and only if we can rearrange its elements and list them in a sequence, just like we can do for whole numbers. The cardinality of countable sets is denoted by the symbol $\aleph_0$ ($\aleph$ stands for letter A in the Hebrew alphabet). This is the smallest infinite: the size of an infinite set needs to be at least $\aleph_0$.	Examples of countable sets are: $\mathbb{N}$={0,1,2,..} {odd numbers} {even numbers} {exact squares} {exact cubes} $\mathbb{Q}$={fractions}
Cardinality of the Contnuum	An infinite set A is said to have the <u>cardinality of the continuum</u> if it is equipotent to the line (where we see the line as a set of points The cardinality of the continuum is denoted with $\aleph_1$. In formula: *A has the card. of the continuum if and only if* $\lvert A \rvert=\lvert \backslash \rvert=\aleph_1$	A set has the cardinality of the continuum if it can be put in bijection with a line, i.e. if we can associate to any point of the set exactly one point on the line and by doing so we label all points on the line. The cardinality the continuum is denoted by the symbol $\aleph_1$: according to the Continuum Hypothesis there are no possible infinities between $\aleph_0$ and $\aleph_1$.	Examples of sets with continuum card. are $\mathbb{R}$={decimals} {decimals between 0 and 1}

Tutorial 1 – Basic Set Theory



Today's Key Questions:

- What is a set and how do we represent it?
- What sort of operations can we perform with sets?
- How do we compare to different sets?

What is the Purpose of Tutorial 1?

- Review the language of sets and their basic operations (union, intersection, complement)
- Introduce the notion of subset and power set.
- Review and deepen the notion of function.
- Introduce the notions of injective function, surjective function and bijective function.

Today's Homework is:

- Baseline Assessment: see next page.

Homework 1 – Baseline Test

The homework assignment for the first tutorial is a baseline test to see your initial level of attainment in this subject area. The assignment will test for some or all of the subject specific skills that are required later in the final assignment. However, it is shorter than the final assignment and is will be an introduction to the subject as well as a challenge!

Do not worry too much about doing 'well' or 'badly' on the baseline test, it takes into account the fact that you may not be familiar with the subject area. It is designed to help you and your PhD tutor identify where you are at the start of the programme and to help you measure your progress along the way.

Question 1

Consider the sets

$$A:=\{1,3,4,5,6,7\}$$
$$B:=\{2,3,4,5,7\}$$
$$C:=\{x \text{ whole number such that } 4\leq x\leq 9\}$$

a) Represent the above sets through Venn diagrams;
b) Work out (by listing all elements) the sets

$$A\cap B,\ A\cap C,\ B\cap C,\ A\cap B\cap C$$
$$A\cup B,\ A\cup C,\ B\cup C,\ A\cup B\cup C$$
$$A-B,\ B-A,\ A-C,\ C-A,\ B-C,\ C-B$$
$$(A\cap B)-(A\cap B\cap C),\ (A\cap C)-(A\cap B\cap C),\ (B\cap C)-(A\cap B\cap C)$$

c) Work out (by listing all elements) the sets

$$P(A\cap B\cap C)$$
$$P(C-(A\cup B))$$
$$P((B\cap C)-(A\cap B\cap C))$$

Question 2

A *number set* is a set whose elements are numbers. Standard number sets (which we will constantly use during this course) are the following:

$$\mathbb{N}:=\{\text{natural numbers}\}=\{0,1,2,3,4,\ldots,1000,1001,\ldots\}$$
$$\mathbb{Z}:=\{\text{(relative) integer numbers}\}=\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$$
$$\mathbb{Q}:=\{\text{rational numbers}\}=\{\text{all fractions}\}$$
$$\mathbb{R}:=\{\text{real numbers}\}=\{\text{all decimals}\}$$

Explain in your own words why $\mathbb{N}\subset\mathbb{Z}\subset\mathbb{Q}\subset\mathbb{R}$ and all inclusions are *strict* (meaning that each set is a proper subset of the preceding one).

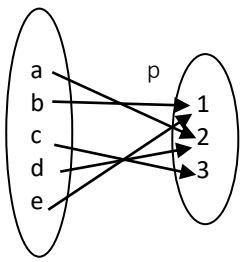
Question 3

Consider the following functions

$f: \mathbb{R}\to \mathbb{R}$, $x\mapsto 2x+1$

$g:\mathbb{R}\to\mathbb{R}$, $x\mapsto x^2+1$

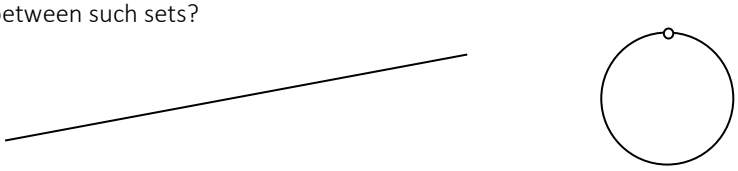
$h:\mathbb{N}\to\mathbb{N}$, $n\mapsto n^2+1$



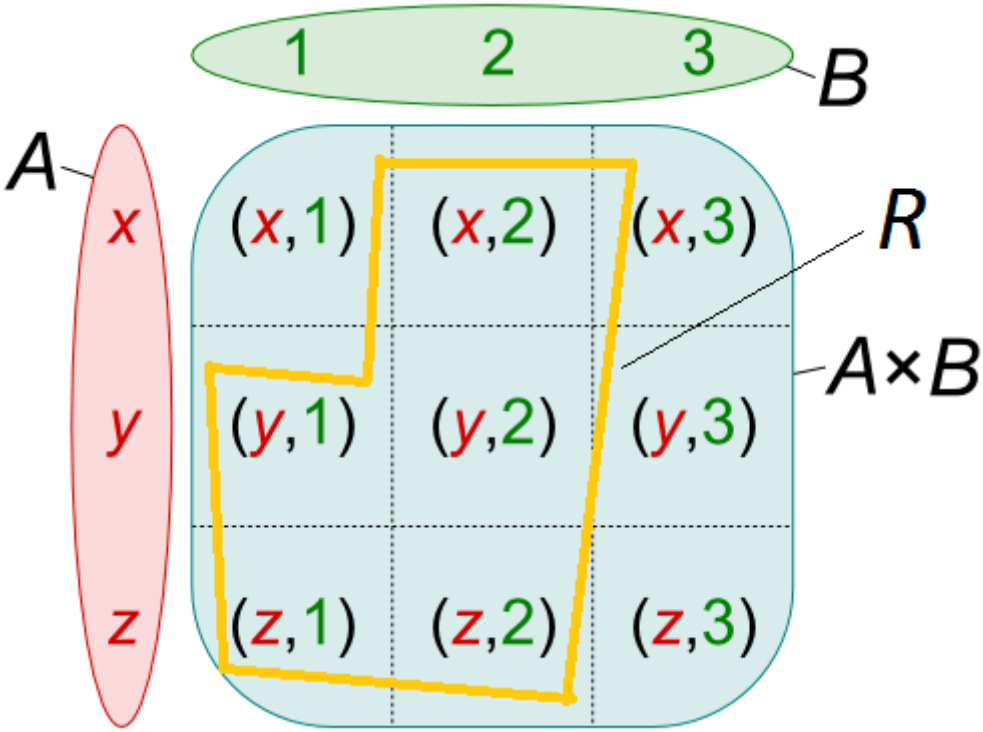
Which function is injective? Which one surjetive? Which bijective? Explain your answers.

Question 4 (Extension question)

Consider a line and a circumference and view them as sets of points. Now take a point out of the circumference. Can you find a bijective function between such sets?



Tutorial 2 – Relations



Today's Key Questions:

- What is a (set-theoretic) relation?
- What is an equivalence relation?
- What is a quotient set?

What is the Purpose of Tutorial 2?

- Introduce the notion of Cartesian product of two sets and the notion of relation; realise functions as special kinds of relations.
- Understand and describe the main properties of relations, i.e.: reflexive property, symmetric property and transitive property.
- Identify equivalence relations among all relations.
- Understand that any equivalence relation determines a partition of a set and work out such a partition; understand the idea of quotient set.

Today's Homework is:

- See handout with list of problems.

Tutorial 3 – Cantor Arithmetic

Today's Key Questions:

- What is the meaning of equipotence?
- What is the Cantor's definition of natural number?
- How did Russell criticize the logical validity of Cantor's approach?

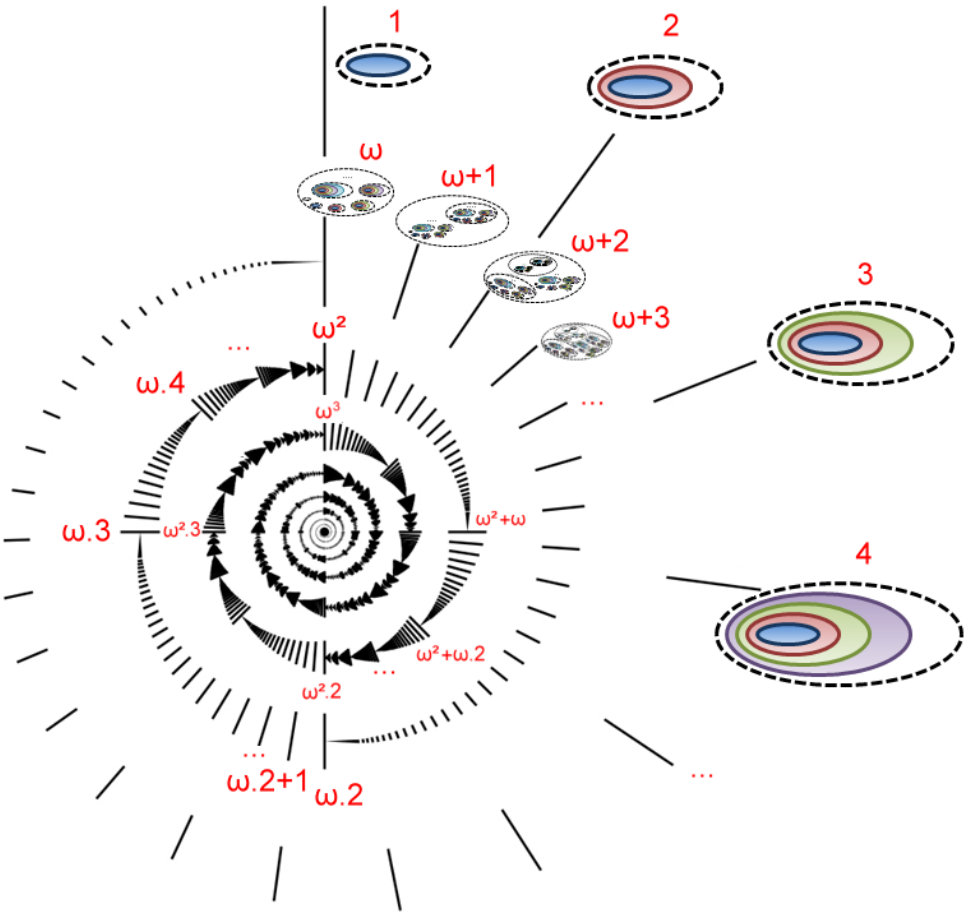
What is the Purpose of Tutorial 3?

- Understand the relation of equipotence as an equivalence relation.
- Describe Cantor's way to construct the set of natural numbers.
- Understand Russell's Antinomy.
- Assess strengths and weaknesses of Cantor's arithmetic.

Today's Homework is:

- See handout with list of problems.

Tutorial 4 – Cardinality



Today's Key Questions:

- How can we characterize the "size" of a (possibly infinite) set?
- What is the meaning of cardinality and cardinal? What is a transfinite cardinal?
- What are the basic operations between cardinals?
- How do we compare two transfinite cardinals ("hierarchy of infinities")?

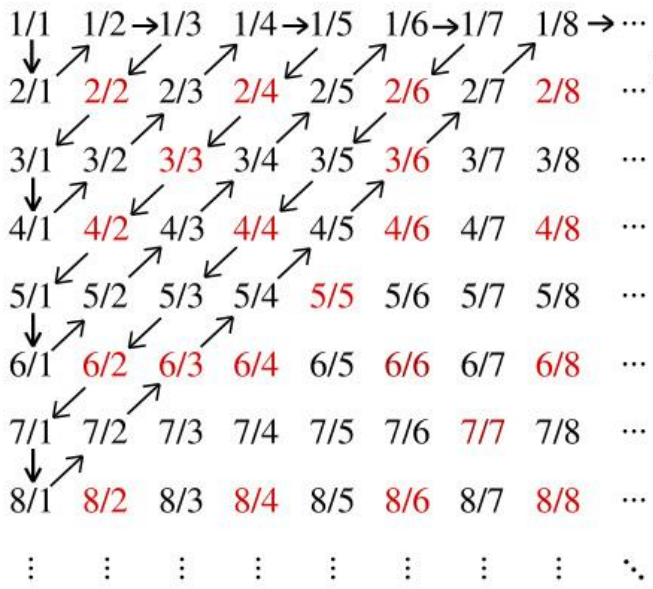
What is the Purpose of Tutorial 4?

- Understand the notion of cardinality as "the quotient set" of the equipotence relation;
- Understand that a set can have either finite or transfinite cardinality.
- Distinguish between countable sets and uncountable sets.
- Understand that the set $\mathbb{N}$ of natural numbers and the set $\mathbb{Z}$ of relative integer numbers are countable.
- Prove through specific examples that "the Whole is not necessarily larger than the part".
- Introduce the Continuum Hypothesis and discuss its relevance in the history of modern Mathematics

Today's Homework is:

- See handout with list of problems.

Tutorial 5 – Two Key Examples: $\mathbb{Q}$ and $\mathbb{R}$



Natural	Real
0	0.236436775676...
1	0.098473294543...
2	0.193214042202...
3	0.843279242093...
4	0.012934812343...
5	0.639423412934...
6	0.017773923845...
7	0.238920090909...
8	0.123984732999...
9	0.646329878122...
10	0.000123943437...
11	0.981298312892...
⋮	⋮
	0.293233992132...
	0.746894310875...

Today's Key Question(s):

- What is the cardinality of the set of the set $\mathbb{Q}$ of rational numbers?
- What is the cardinality of the set of the set $\mathbb{R}$ of real numbers?

What is the Purpose of Tutorial 5?

- Understand and practice Cantor diagonal argument.
- Prove – by means of Cantor diagonal method – that $\mathbb{Q}$ is countable.
- Further discuss the fact that $\mathbb{Q}$ is countable and realise how surprising/unexpected such a result is.
- Prove – by means of Cantor diagonal method – that $\mathbb{R}$ is uncountable.

Today's Homework is:

- Final Assignment: write a short report about _____.
- Specifications:_____
  _____
  _____
  _____
  _____
  _____
  _____
  _____

The Deadline is: _____

Tutorial 6 – Feedback

What is the Purpose of Tutorial 6?

- Reflect on skills learned on the programme with a view to encouraging resilience
- Receive feedback on final assignment

What are the key questions?

- What strengths have I demonstrated in my work and what areas of development are there left for me to address?
- What steps do I need to take to improve my academic output?

Activity 1 – Marking your own

Using the mark scheme, identify three things you did well and three things you could improve.

What went well?

1.

2.

3.

What could be improved?

1.

2.

3.

Activity 2 – Responding to feedback

Read through your feedback sheet and highlight on your assignment where you received positive feedback (one colour) and areas for improvement (different colour). You might find it helpful to annotate this with a brief comment or two.

Activity 3 – Looking Forwards: Think, Pair Share

1. Think about how you can use this feedback in your future school work.
2. Talk to your partner about how you both plan to make changes to your school work in the future as result of this feedback
3. Share your ideas as a group.

Appendix – University Applications Guidance

For course choices and careers information:	www.brightknowledge.org.uk
For information and statistics on universities and course:	www.unistats.direct.gov.uk
For comprehensive information on universities, including rankings:	www.thecompleteuniversityguide.co.uk

PLEASE NOTE – The Researchers in Schools programme is designed to support the university applications process at your school. The work you complete for the programme should add to your schoolwork rather than detract from it. Please be aware of deadlines and try to manage your workload appropriately. If you are worried about the programme interfering with your schoolwork then please speak to your teacher.

Subject Specific Top Tips for Strengthening an application?

- Study Mathematics (and where possible Further Mathematics) at A-levels.
- Try to attend as many Maths clubs as you possibly can.
- Tackle as many exercises and problems as you can, on all possible topics you encounter. Ask your teachers for more exercises if you run out.
- Never compromise with your understanding: dig into a concept/definition/statement/theorem as deep as you can;
- Ask questions: Maths teachers are there to try and answer those (or at least, give you good references to look at!)
- Do not get frustrated when you can't solve an exercise: Maths is cleverer than anybody tries to deal with it.
- Most important of all: enjoy what you study! You can't possibly study Maths at university if you don't enjoy it.