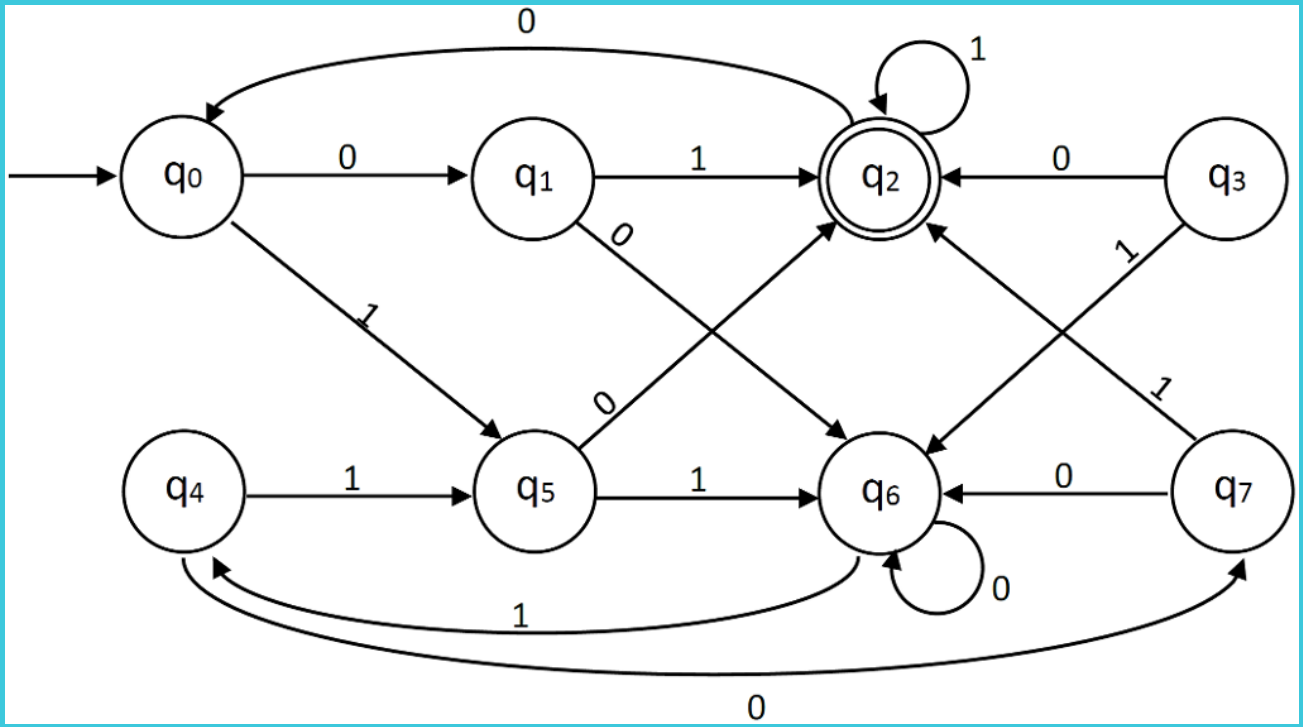


# Automata Theory
## Exploring the outer limits of computing

Pupil Name

Handbook Designed by Dr Felix Weitkämper

# Timetable and Assignment Submission

## Timetable – Tutorials

| Tutorial | Date | Time | Location |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 (Feedback) | | | |

## Timetable – Homework Assignments

| Homework Assignment | Description | Due Date |
|---|---|---|
| Tutorial 1 | | |
| Tutorial 2 | | |
| Tutorial 3 | | |
| Tutorial 4 | | |
| Tutorial 5 | | |
| Tutorial 6 | | |

## Assignment Submission – Lateness and Plagiarism

| Lateness | |
|---|---|
| Submission after the deadline | 10 marks deducted |
| **Plagiarism** | |
| Some plagiarism | 10 marks deducted |
| Moderate plagiarism | 20 marks deducted |
| Extreme plagiarism | Automatic fail |

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Computers have transformed the way we live our lives. From simple calculators to sophisticated systems, from a tiny smartwatch to a room-filling super-computer, they come in all shapes and sizes. They have taken over tasks that seem tedious to us (such as calculating a long division) and do things we could never have dreamt of twenty years ago (like connecting us all to the internet).
Moreover, it seems as if they become more powerful every year, which begs the question: Is there a limit to what they are capable of?

That is precisely the question we will consider in this course. By looking at computers from a different angle, we will be able to see a structure common to every single one there is, or ever will be: They are all *automata*. We will also be looking at a specific model of a task that a computer will often want to perform: Answering a Yes/No question. We will explore what automata can do and what they cannot do, and how we can use our logical thinking to decide whether a problem is in the first or second category.

The focus here is very much on *We*. This is not a class in which I will stand at the front and tell you all sorts of things about automata before giving you a list of questions in which you do what I just did on the board. Instead, after we have set the scene, we will *prove* the big results of the field collaboratively, and then you will be able to think about how those results might help you to solve problems about automata.

Finally, you will bring your understanding of the topic together and write a mathematical essay with the title:

## What does it mean for a language to be recognised by an automaton? Discuss giving fully reasoned examples and counterexamples.

# Final Assignment Mark Scheme

| Skills | 1st (70-100) | 2:1 (60-69) | 2:2 (50-59) |
|---|---|---|---|
| Knowledge and Understanding | • All content included is relevant to the general topic and to the specific question/title<br>• Good understanding of all the relevant topics.<br>• Scientific terms are defined and used accurately throughout<br>• Clear justification on how the content included is related to the specific issues that are the focus of the assignment | • Most of the content included is relevant to the general topic and to the specific question/ title<br>• Good understanding of most the relevant topics<br>• Scientific terms are used accurately but not always clearly defined.<br>• Adequate justification on how the content included is related to the specific issues that are the focus of the assignment | • Some of the content included is relevant to the general topic and to the specific question/ title<br>• Good understanding on some of the relevant topics but occasional confusion on others.<br>• Scientific terms are used mostly accurately with occasional confusion and often not defined.<br>• Some justification on how the content included is related to the specific issues that are the focus of the |
| Critical Evaluation/ Problem solving | • Moved beyond description to an assessment of the value or significance of what is described<br>• Evaluative points are consistently explicit/ systematic/reasoned/ justified<br>• Effective critiques on the reliability of sources provided<br>• Consistently demonstrate clear, analytical and logical steps to solving problems | • Mostly description but some assessment of the value or significance of what is described<br>• Evaluative points are mostly explicit/ systematic/reasoned/ justified<br>• Some evidence of critiques on the reliability of sources provided<br>• Some examples of solving problems but not consistently clear, analytical and logical | • Only description with minimal assessment of the value or significance of what is described<br>• Evaluative points are at times explicit/ systematic/reasoned/ justified<br>• Limited evidence of critiques on the reliability of sources provided<br>• Demonstrate clear steps to solving problems but not consistently analytical or logical |

| | | | |
|---|---|---|---|
| Structure and Presentation | • Ideas are presented in paragraphs and arranged in a logical structure that is appropriate for the assignment<br><br>• The introduction clearly outlines how the essay/report will deal with the issues<br><br>• The conclusion summarises all the main points clearly and concisely<br><br>• All calculations, formulas and methods are clearly structured, clear to follow and correct<br><br>• Tables and graphs are effectively constructed including appropriate headings, units and scales.<br><br>• All sources are referenced correctly in | • Ideas are presented in paragraphs and arranged in a structure that is mostly appropriate for the assignment<br><br>• The introduction adequately describes how the essay/report will deal with the issues<br><br>• The conclusion summarises most of the main points clearly<br><br>• Calculations, formulas and methods are mostly structured, clear to follow and correct<br><br>• Most tables and graphs are well constructed<br><br>• Most sources are referenced correctly in an agreed format | • Ideas are presented in paragraphs and arranged in a structure<br><br>• The introduction mentions how the essay/report will deal with the issues<br><br>• The conclusion summarises some of the main points clearly<br><br>• Calculations, formulas and methods are not always structured, clear to follow and correct.<br><br>• Some tables and graphs are well constructed but contains some errors<br><br>• Some sources are referenced correctly in the agreed format with occasional errors |

# Subject Vocabulary

| Word | Definition | Example |
|---|---|---|
| Automaton | | |
| Language | | |
| Recognised by an automaton | | |
| Closure Properties | | |
| Example | | |
| Counterexample | | |

# Sample Automata

**Tutorial 1 – Automata**

Objectives: To understand what an automaton is, how it is presented and how the concept is used in computer science, as well as to be able to read and write automata diagrams.

What is an *automaton*?

_____
_____
_____

When is a string *accepted* by an automaton?

_____
_____
_____

Homework (Baseline Assessment):

Describe, in a paragraph, what automata are, and, on a separate sheet, draw at least 3 different and new examples of automata to illustrate your points.

Include the main points we covered today, in particular how you imagine automata, the way we present automata and their role in computer science.

## Tutorial 2 – Languages

Objectives: To understand what a language is in this context and what it means for a language to be recognisable.

What to we mean by a *language*?

_____
_____
_____

What does it mean for a language to be *recognised* by a given automaton? When do we call a language *recognisable*?

_____
_____
_____
_____
_____
_____

Which languages do the automata from your last homework assignment recognise?

## Recognisable languages

On a separate A3 sheet, draw automata that recognise the following languages:

All {0,1}-strings ending in 0
All {0,1}-strings starting with 0
All {0,1}-strings containing at least one 0
Only your first name (among all alphabetical strings)
All {0,1}-strings with an even number of letters

Homework:

Finish the exercises on this page and the one before.

# Tutorial 3 – Boolean Algebra

Objectives: To understand the importance of closure properties in general and referring to recognisable languages, and to know de Morgan's laws.

List the *Boolean connectives* here with their definitions:

_____

_____

_____

List *De Morgan's Laws* here:

_____

_____

_____

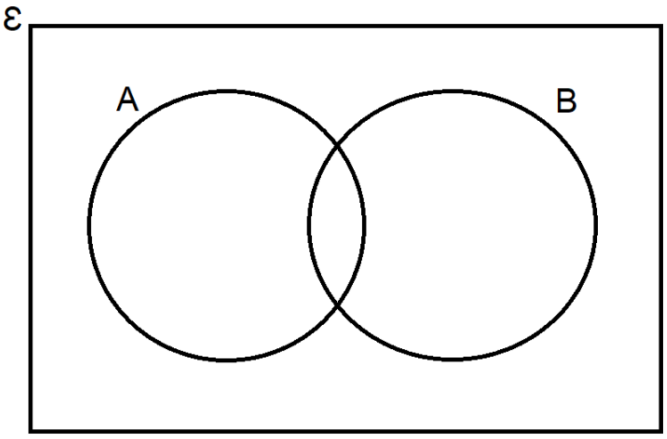
_____

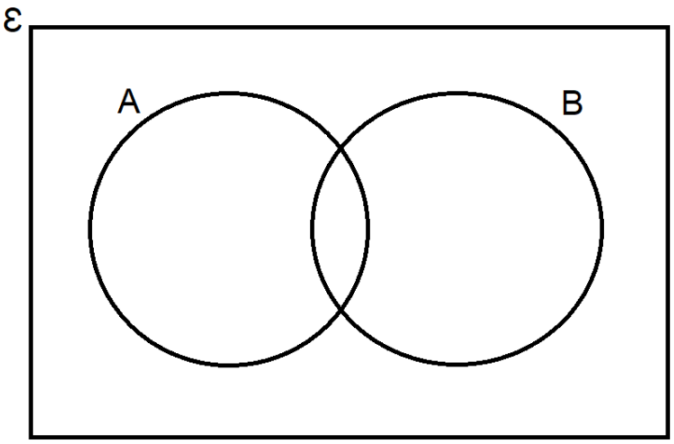
On the next page, shade the regions corresponding to the following Boolean combinations of sets:

1. (A u B) n ¬(A n B)
2. ¬(A u B)
3. ¬(A n B)
4. (¬A) u (¬B)
5. (¬A) n (¬B)
6. (A n B) u C
7. (A u C) n (B n C)
8. (A u B) n C

Homework:

Write a paragraph to justify the validity of one of the other propositional tautologies mentioned in class


ε
A
B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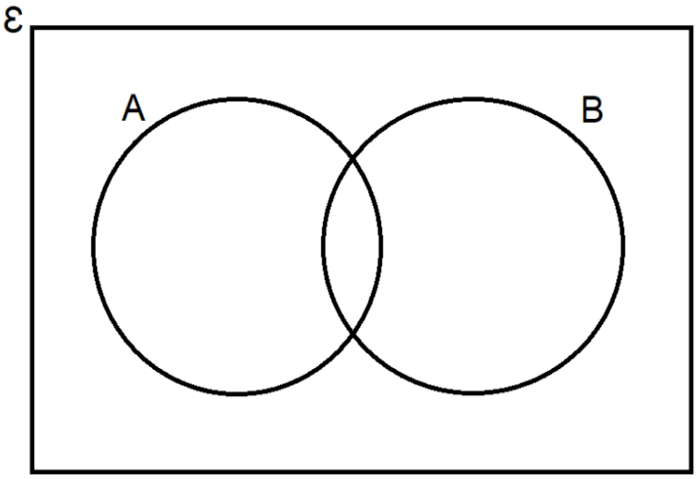
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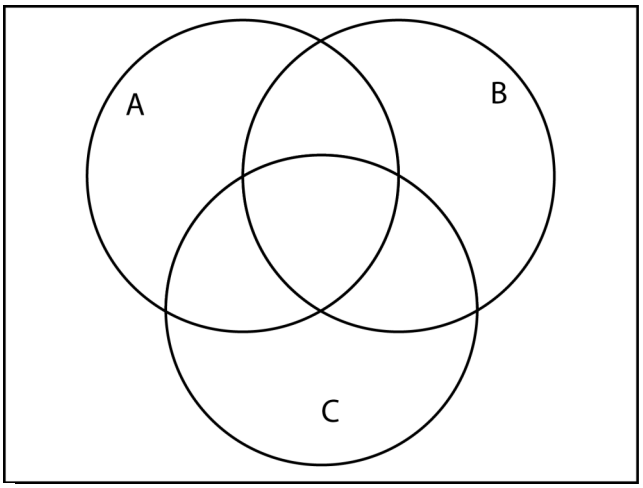
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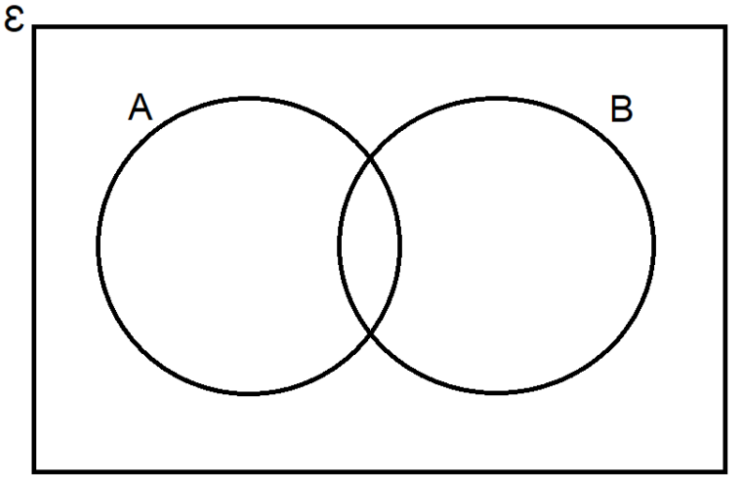
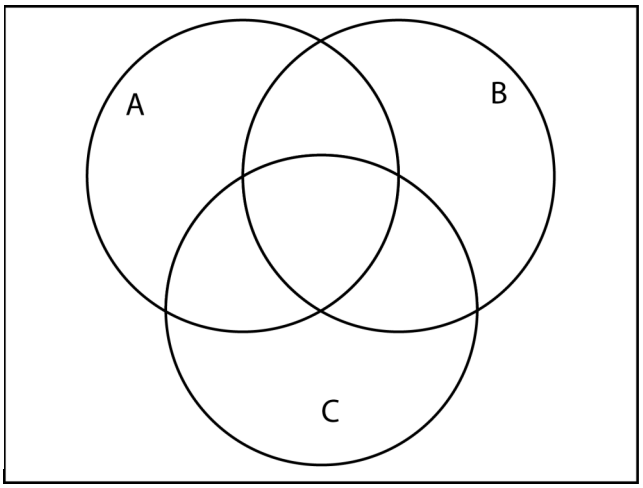

ε
A
B


A
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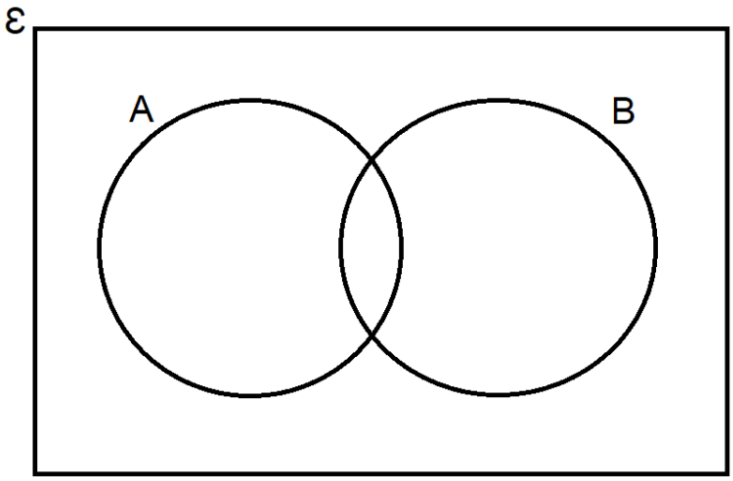
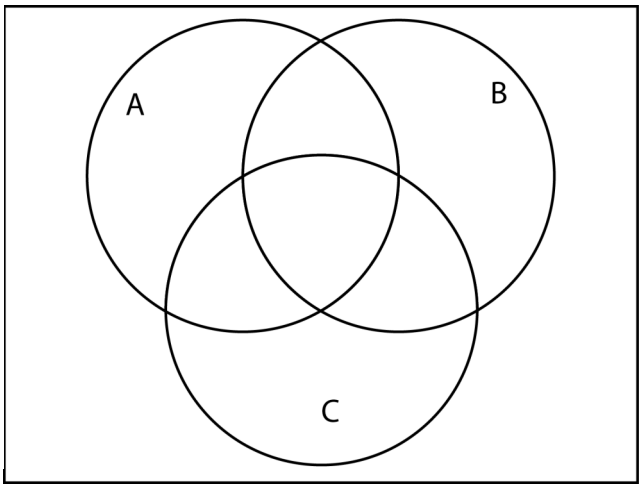

ε
A
B


A
B
C

# Tutorial 4 – Closure Properties

Space for drawing your own automata illustrating the closure under complement and intersection as well as to make notes on the proofs (You can also use a separate A3 sheet and staple it in):

More space for drawing your own automata illustrating the closure under complement and intersection as well as to make notes on the proofs:

•

Exercise:

Use the languages from your baseline assessment to construct new recognisable languages. Show that they are recognisable using the theorems we have just proven.

Then use the closure properties to show that the following languages are recognisable:

1) All {0,1}-strings that start with 0 and contain at least one 1
2) All alphabetical strings that are a first name of one of the students at the UTC
3) All {0,1}-strings with an odd number of letters
4) All {0,1,2,3}-strings with an odd number of zeros and an even number of 1s

Homework:

Can you draw automata recognising one or more of the languages you have established to be recognisable above? How do you think this question relates to the proof of recognisability you have provided above?

# Tutorial 5 – The Pumping Lemma

Objectives: To gain an appreciation of the role of negative results in mathematics, to understand the proof of the pumping lemma, and to be able to apply the pumping lemma to show that certain languages are not recognisable.

State the *Pumping Lemma* below:

_____

_____

_____

_____

Make notes on the proof here:

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

## Congratulations!

You are now equipped to work with automata not just in little problems that I have thought up, but in the exercises below, that are taken from a book aimed at *undergraduates in mathematics and computer science!*

Armed with the examples from there and from the preceding tutorials, you should be confident to tackle the Final Assignment:

### 4.1.3 Exercises for Section 4.1

**Exercise 4.1.1:** Prove that the following are not regular languages.

a) $\{0^n1^n \mid n \geq 1\}$. This language, consisting of a string of 0's followed by an equal-length string of 1's, is the language $L_{01}$ we considered informally at the beginning of the section. Here, you should apply the pumping lemma in the proof.

b) The set of strings of balanced parentheses. These are the strings of characters "(" and ")" that can appear in a well-formed arithmetic expression.

* c) $\{0^n10^n \mid n \geq 1\}$.

d) $\{0^n1^m2^n \mid n$ and $m$ are arbitrary integers$\}$.

e) $\{0^n1^m \mid n \leq m\}$.

f) $\{0^n1^{2n} \mid n \geq 1\}$.



! **Exercise 4.1.2:** Prove that the following are not regular languages.

* a) $\{0^n \mid n$ *is a perfect square*$\}$.

b) $\{0^n \mid n$ is a perfect cube$\}$.

c) $\{0^n \mid n$ is a power of 2$\}$.

d) The set of strings of 0's and 1's whose length is a perfect square.

e) The set of strings of 0's and 1's that are of the form $ww$, that is, some string repeated.

f) The set of strings of 0's and 1's that are of the form $ww^R$, that is, some string followed by its reverse. (See Section 4.2.2 for a formal definition of the reversal of a string.)

g) The set of strings of 0's and 1's of the form $w\overline{w}$, where $\overline{w}$ is formed from $w$ by replacing all 0's by 1's, and vice-versa; e.g., $\overline{011} = 100$, and 011100 is an example of a string in the language.

h) The set of strings of the form $w1^n$, where $w$ is a string of 0's and 1's of length $n$.

# Tutorial 6 – Feedback tutorial

## Final assignment feedback

| Strengths of the Assignment– Don't change! | Possible Areas for Improvement |
| --- | --- |
| | |

| Specific Changes to be made |
| --- |
| |

# Tutorial 7 – Final Pupil Feedback Report

| Grade | Marks | What this means |
|---|---|---|
| 1st | 70+ | Performing to an excellent standard at A-level |
| 2:1 | 60-69 | Performing to a good standard at A-level |
| 2:2 | 50-59 | Performing to an excellent standard at GCSE |
| 3rd | 40-49 | Performing to a good standard at GCSE |
| Working towards a pass | 0-39 | Performing below a good standard at GCSE |
| Did not submit | DNS | No assignment received by The Brilliant Club |

| Lateness | |
|---|---|
| Any lateness | 10 marks deducted |
| **Plagiarism** | |
| Some plagiarism | 10 marks deducted |
| Moderate plagiarism | 20 marks deducted |
| Extreme plagiarism | Automatic fail |

| | | | |
|---|---|---|---|
| Name of RIS teacher | | | |
| Title of Assignment | | | |
| Name of Pupil | | | |
| Name of School | | | |
| ORIGINAL MARK / 100 | | FINAL MARK / 100 | |
| DEDUCTED MARKS | | FINAL GRADE | |

| |
|---|
| |
| Knowledge and Understanding |
| |
| Critical Evaluation/Problem Solving |
| |
| Structure and Presentation |
| |
| Resilience Comment |
| |

# Reflecting on Uni Pathways

| What did you most enjoy about Uni Pathways? |
| --- |
| |

| What challenged you the most? | How did you overcome these challenges? |
| --- | --- |
| | |

# Appendix – Referencing Guide

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

## What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

## Why should I reference?

Referencing is important in your work for the following reasons:

It gives credit to the authors of any sources you have referred to or been influenced by.
It supports the arguments you make in your assignments.
It demonstrates the variety of sources you have used.
It helps to prevent you losing marks, or failing, due to plagiarism.

## When should I use a reference?

You should use a reference when you:

Quote directly from another source.
Summarise or rephrase another piece of work.
Include a specific statistic or fact from a source.

## How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. **A marker in your assignment:** After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].

2. **Bibliography:** This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

**Websites** – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

**Books** – Author, date published, title of book (in italics), pages where the information came from.

E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

**Articles** – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

# Notes

# Notes

# Notes

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