Can Fish Solve Equations?

From the small to the large
Key Stage 4 Programme

Pupil Name

Handbook Designed by Dr Stone

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after midnight on	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

The Brilliant Club KS4 Programme – Pupil Feedback Report

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at AS-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Name of PhD Tutor	Dr Stone		
Title of Assignment			
Name of Pupil			
Name of School	Therfield School		
ORIGINAL MARK / 100		FINAL MARK / 100	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the PhD tutor should give an explanation in this section:

Learning Feedback Comment 1 –	
What you did in relation to this Key Learning Priority	How you could improve in the future
Learning Feedback Comment 2 –	
What you did in relation to this Key Learning Priority	How you could improve in the future
Learning Feedback Comment 3 –	
What you did in relation to this Key Learning Priority	How you could improve in the future
Resilience Comment	
How you showed learning resilience during the course	How you could build learning resilience in the future

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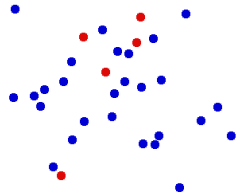
Course Rationale



$$\begin{cases} \partial_t f + v \cdot \nabla_x f - \nabla_x F \cdot \nabla_v f &= Q(f,f), \\ f_{t=0} &= f_0, \end{cases}$$

How do fish in large schools know where to move? How does a flock of a thousand birds behave? How can we model this behaviour to understand it better? What happens when there are so many fish or birds that we can no longer track each individual? These are some of the questions we will be looking at in this course.

A big area of mathematical and science research is multi-scale modelling, where a system behaves on at least two scales: the small scale where each individual item evolves in some way and the large scale where we look at the system as a whole. Moving between these two scales is a difficult problem that is still being researched. For example we might have an equation that describes the large scale behaviour but be unable to show the small scale system leads to the same behaviour, or we might understand the small scale behaviour but be unable to find the large scale behaviour from this.

We are going to look at number of different multi-scale problems and begin to get an understanding of the advantages and issues with modelling. We will be focusing on particle models of a gas and the derivation of large scale equations from this. This will include looking at areas such as mechanics, probability and Brownian motion. Towards the end of this course we will be reading extracts from real researchers papers.

By the end of this course you will be able to describe multi-scale modelling, understand how this relates to Hilbert's Sixth problem and have an idea of the kind of problems in this area currently being worked on by mathematicians.

In your final assignment you will write and evaluate your own mathematical model, apply the topics we have covered to more involved questions and write a brief summary of a research paper in this area.

Mark Scheme Table

	Skills	1st (70-100)	2:1 (60-69)	2:2 (50-59)
Subject knowledge	Knowledge and Understanding	• All content included is relevant to the general topic and to the specific question/title • Good understanding of all the relevant topics. • Scientific terms are defined and used accurately throughout • Clear justification on how the content included is related to the specific issues that are the focus of the assignment	• Most of the content included is relevant to the general topic and to the specific question/title • Good understanding of most the relevant topics • Scientific terms are used accurately but not always clearly defined. • Adequate justification on how the content included is related to the specific issues that are the focus of the assignment	• Some of the content included is relevant to the general topic and to the specific question/title • Good understanding on some of the relevant topics but occasional confusion on others. • Scientific terms are used mostly accurately with occasional confusion and often not defined. • Some justification on how the content included is related to the specific issues that are the focus of
Critical Thinking	Research and Evidence	• Inclusion of rich sources of research findings, data, quotations or other sourced material as evidence for the claims/ ideas • Use evidence/ calculations to support claims/ assertions/ideas, consistently clearly and convincingly • Evidence of further reading beyond materials provided which were used in an appropriate context • Data is effectively analysed and appropriate assumptions/ conclusions are reached	• Inclusion of adequate sources of research findings, data, quotations or other sourced material as evidence for the claims/ ideas • Use evidence/calculations to support claims/assertions/ ideas, mostly clearly and convincingly • Evidence of further reading beyond materials provided • Data is analysed and the assumptions/conclusions that are reached are mostly appropriate	• Inclusion of some sources of research findings, data, quotations or other sourced material as evidence for the claims/ ideas • Use evidence/ calculations to support claims/ assertions/ideas, at times clearly and convincingly • Limited evidence of further reading beyond materials provided • There is an attempt to analyse data is and draw assumptions/ conclusions

Critical Thinking	Developing an Argument	• A point of view or position in relation to the title or question is consistently clear • Argument/proof exceptionally well-developed and well-justified • A position is clearly established in relation to the question, and is developed effectively and consistently throughout the essay • Makes links effectively between subjects that have not previously been associated • Uses concepts from the tutorials in an unfamiliar context, and does so accurately and confidently. • Content is analysed effectively to support the argument	• A point of view or position in relation to the title or question is adequately clear • Argument/proof clear and well-developed and position justified • A position is established in relation to the question, and is well-developed in most of the essay • Some evidence of linking subjects that have not previously been associated • Use some concepts from the tutorials in an unfamiliar context, but not always accurate • Analysis of content to support the argument	• A point of view or position in relation to the title or question is somewhat clear • Argument/proof clear but not well-developed • A position is established in relation to the question, and is well-developed in parts of the essay • Limited evidence of linking subjects that have not previously been associated • Limited use of concepts from the tutorials in other contexts • Some analysis of content to support the argument
	Critical Evaluation	• Moved beyond description to an assessment of the value or significance of what is described • Evaluative points are consistently explicit/ systematic/ reasoned/justified • Effective critiques on the reliability of sources provided	• Mostly description but some assessment of the value or significance of what is described • Evaluative points are mostly explicit/ systematic/ reasoned/justified • Some evidence of critiques on the reliability of sources provided	• Only description with minimal assessment of the value or significance of what is described • Evaluative points are at times explicit/ systematic/ reasoned/justified • Limited evidence of critiques on the reliability of sources provided

Written Communication	Structure and Presentation	• Ideas are presented in paragraphs and arranged in a logical structure that is appropriate for the assignment • The introduction clearly outlines how the essay/report will deal with the issues • The conclusion summarises all the main points clearly and concisely • All calculations, formulas and methods are clearly structured, clear to follow and correct • Tables and graphs are effectively constructed including appropriate headings, units and scales. • All sources are referenced correctly in an agreed format	• Ideas are presented in paragraphs and arranged in a structure that is mostly appropriate for the assignment • The introduction adequately describes how the essay/report will deal with the issues • The conclusion summarises most of the main points clearly • Calculations, formulas and methods are mostly structured, clear to follow and correct • Most tables and graphs are well constructed • Most sources are referenced correctly in an agreed format	• Ideas are presented in paragraphs and arranged in a structure • The introduction mentions how the essay/report will deal with the issues • The conclusion summarises some of the main points clearly • Calculations, formulas and methods are not always structured, clear to follow and correct. • Some tables and graphs are well constructed but contains some errors • Some sources are referenced correctly in the agreed format with occasional errors
	Language and Style	• No spelling, grammar or punctuation errors • Units and significant figures are presented accurately throughout • Writing style consistently clear, appropriate for scientific documents and easy to follow • Accurate and consistent use of technical language and vocabulary	• Minimal spelling, grammar or punctuation errors • Units and significant figures are presented accurately throughout • Writing style mostly clear, appropriate for scientific documents and easy to follow • Some attempts of using technical language and vocab alary, but not always accurate	• Some spelling, grammar or punctuation errors • Units and significant figures are presented accurately throughout with occasional errors • Writing style moderately clear, appropriate for scientific documents and easy to follow • Use of simple language and vocabulary effectively but struggles to use technical language

Glossary of Keywords

Word	Definition	In a sentence
Mathematical Model		
Axiom		
Discrete		
Continuous		
Velocity		
Momentum		
Conservation of Momentum		
Hard Sphere		
Short Range Potential		
Simple Random Walk		
Markov Property		
Markov Process		
Symmetric Simple Random Walk		

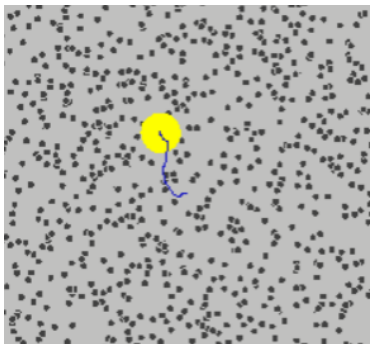
Glossary of Keywords (ctd)

Word	Definition	In a sentence
Poyla's Recurrence Theorem		
Brownian Motion		
Hilbert's Sixth Problem		
Navier-Stokes Equation		
Boltzmann Equation		

Tutorial 1 – Overview and Small Scale Models

What is the Purpose of Tutorial 1?

- To understand the process of mathematical modelling
- To be able to describe and evaluate mathematical models
- To understand the difference between small scale and large scale models



Read the course rationale on page 5. Write down any words or phrases you don't understand.

_____ _____

_____ _____

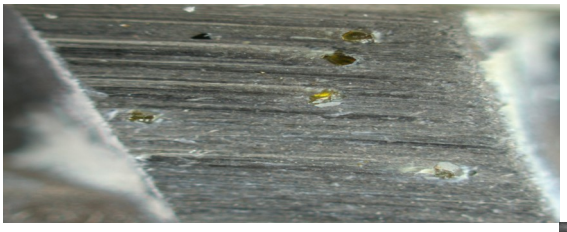
_____ _____

Write down any questions you have at this point.

1.1 – The Small World

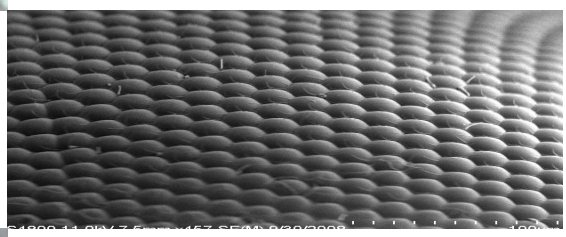
Many things we see and interact with look and behave differently on the small scale when you zoom into them.

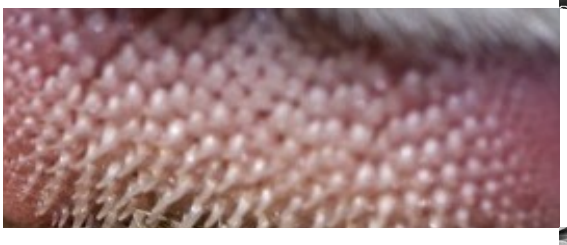
Can you guess what the following are pictures of?



1.

2.





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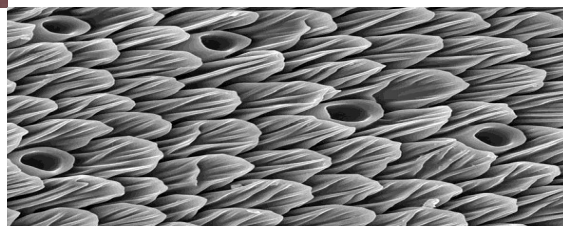
4.



1.2 – Mathematical Models

Throughout this course we are going to be looking at mathematical models of real world phenomena.

What is a mathematical model?

My definition:

Our definition:

Order the following statements about a mathematical model:

Evaluate the prediction against the real behaviour

Change the assumptions about the behaviour and repeat the process

Use the mathematical model to predict behaviour

Write a mathematical model to describe the behaviour

Observe a phenomena

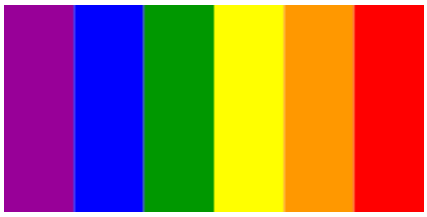
Make a number of assumptions on the behaviour

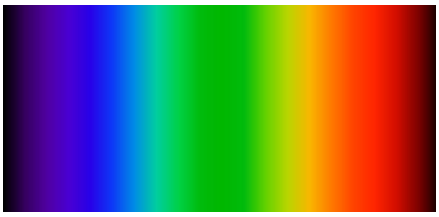
Let's model a number of behaviours. For each system think of ah model or a number of rules that describes how each individual behaves.

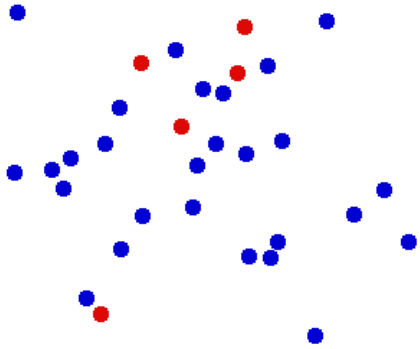
System	Model	Good points	Improvements
Fish			
Wilde-beest			
Exit			
Particles			

1.3 From Discrete to Continuous

What is the difference between the following?







Discrete _____

_____

_____

Continuous _____

_____

_____

The aim of this area of research in mathematical physics is to derive equations that describe gas and fluids as continuous systems from discrete (or atomistic) particle models. Discrete models are the kinds of models we have been looking at - ones that describe the behaviour of each individual, like fish or particles. These are known as multi-scale problems because we are looking at the system at two different scales!

Baseline Assignment

These are some questions for you to do as homework so that I can have an idea of what you already know in this general area. It's not a test so don't worry if you're not sure - we will be covering this in future, but I'd really like you to give each question a good try! These questions are also good starting points for you to think about for the rest of the course!

1. In your own words, what is a multi-scale problem? Give an example.

2. Consider flipping a fair coin. Let H denote heads and T tails.
   A) What is the probability of flipping the coin twice and getting HH?
   B) I flip the coin four times and get HTTT. What is the probability of getting H on the fifth flip?
   C) What is the probability of getting HHT or HHH after three flips?

3. What is the difference between speed and velocity?

4. What is the momentum of an object with mass 5kg and travelling with velocity 4m/s? What is its momentum if it is travelling with the same speed in the opposite direction?

5. Approximately how many particles are in an average room? Show how you got to that number.

6. How do liquids behave differently to gases? What is different between a liquid like water and one like honey?

7. What are Hilbert's 23 problems (you don't need to list them - instead can you understand any of them)?

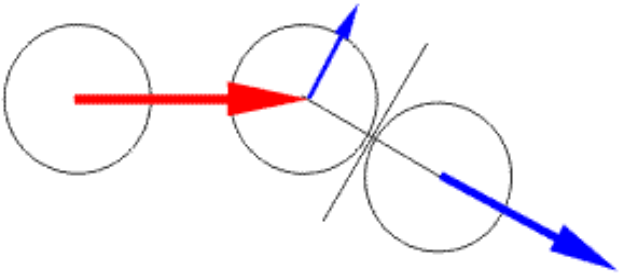
8. What are the Millennium Prize Problems? (again you don't need to list them but instead are there any you can understand?)

9. What is mathematical research? What might it involve?

Notes

Notes

Tutorial 2– Hard Spheres



What is the Purpose of Tutorial 2?

- To be able to define and describe elastic collisions
- To be able to calculate the change in velocity after a collision
- To be able to evaluate the hard sphere particle model

What happens when two marbles collide?

Start by restricting the marbles so that they can travel in only one dimension.

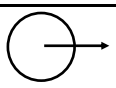
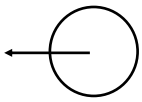
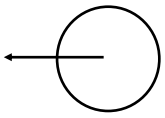
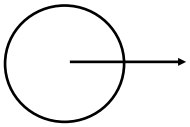
Do some experiments and see what rules you can come up with.

Can you make predictions about how the marbles will collide?

2.1 Momentum in 1d

What is the difference between velocity and speed?

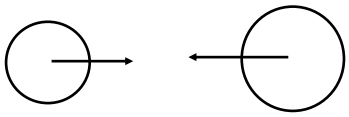
The momentum of an object is its **velocity multiplied by** its **mass**. Momentum has units **kg m/s**. Momentum is a vector.

Object	Velocity	Mass	Momentum
	3m/s	2kg	
	-6m/s	9kg	
		10kg	-50kg m/s
	9m/s		108kg m/s

The **conservation of momentum** says that momentum cannot be created or destroyed, only transferred (as long as there is no external force). This means that the total momentum before the collision is equal to the total momentum after the collision.

2kg

3m/s



5kg

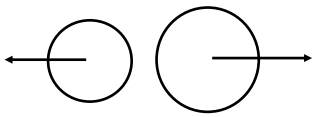
-4m/s

Before collision:

Total momentum =

2kg

-8m/s



5kg

0.4m/s

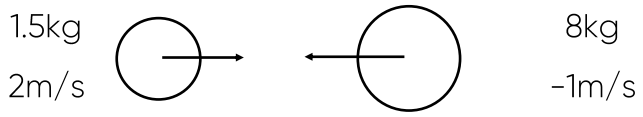
After collision:

Total momentum=

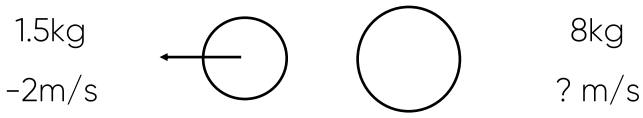
Using the conservation of momentum we can find unknown velocities after a collision or even find the unknown mass of an object.

1. Find the unknown velocity

Before collision:

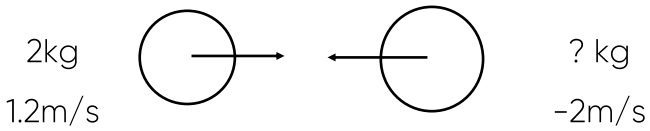


After collision:

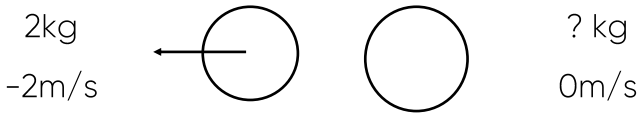


2. Find the unknown mass

Before collision:



After collision:



3. Particle 1 with mass 2kg travels with speed 3m/s towards particle 2 with mass 1kg at rest. They experience an elastic collision. After the collision both particles are travelling in the same direction and the particle 2 is now travelling twice the speed of the particle 1. Find the new velocity of particles 1 and 2.

2.2 Hard Spheres

A hard sphere particle model is one way to model the particles of a gas. It models the particles of the gas like billiard balls and uses the conservation of momentum we have been studying (as well as the conservation of kinetic energy). A hard sphere particle model assumes the following:

- Every particle is treated as a solid ball
- Every particle has equal mass and radius
- Every particle moves in a straight line with constant velocity as long as it avoids all other particles
- When two particles collide they instantly change velocity as given by the conservation of momentum (and conservation of kinetic energy).

Evaluate this model:

Homework 2

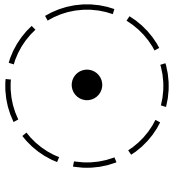
1a) Particle 1 with mass 5kg moves with velocity 10m/s. It collides with particle 2 of mass 12kg which is moving with velocity -2m/s. After the collision particle 1 has velocity of -3m/s. Draw a labelled diagram of this collision. Hence find the velocity of particle 2 after the collision.

1b) A cannon of mass 1,000kg fires a cannonball of mass 5kg. Both the cannon and the cannonball are initially at rest. The cannon fires the cannonball with velocity 50m/s. Assuming the conservation of momentum find the velocity of the cannon after firing the cannonball.

2) The kinetic energy of a particle is given by the formula $\frac{1}{2}mv^2$. Find the kinetic energy of each particle in the table on page 19. What are the units of kinetic energy and why?

3) So far we have looked at particles travelling in 1d - they move either forwards or backwards in one direction. Think about what happens when we allow particles to move in 2d, that is when they can move forwards and backwards and left and right. How might we do similar calculations for collisions in 2d? Write down any ideas you may have. If you want, look through an A-level maths textbook and see if you can find the relevant information.

4) Instead of a hard-sphere particle model we can use what's called a short-range potential particle model. In this model again all particle are treated as identical. Instead of being modelled as solid balls the particles are modelled as points with spherical areas of influence.



If another particle enters this particles are of influence it is pushed away, in the same way that if you pushed the north poles of two magnets together they would push each other away.

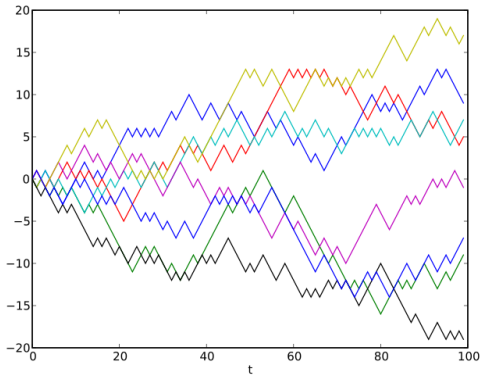
The particles still move in a straight line with constant velocity while they remain outside the influence of the other particles.

Evaluate this particle model. You should included good points about this model and ways in which the model could be improved. When discussing these points you should compare this model to the hard-sphere particle model.

Notes

Notes

Tutorial 3 – Simple Random Walks + Brownian Motion



What is the Purpose of Tutorial 3?

- To be able to describe simple random walks
- To be able to calculate common properties of a random walk
- To understand Brownian motion and how it relates to random walks

Put a counter on 0. Flip a coin. If heads move up 1 and if tails move down 1.

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

A **simple random walk (srw)** is a process where an object moves one step in a random direction at each given time interval.

The **Markov property** is a property of some random processes, sometimes called 'memorylessness', where the future state of the process depends only on the present state and not on the past states.

A **Markov process** is a random process that satisfies the **Markov property.**

Is a simple random walk a Markov process? Why or why not?

3.1 Probability

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

A **symmetric simple random walk** is a simple random walk where the object moves with equal probability to either of its immediate neighbours. In one dimension this means that there is equal probability 1/2 of moving either up or down.

Let a_n denote the nth step of a srw. So if the first coin flip is heads then $a_1 = +1$ and if the second flip is tails then $a_2 = -1$. Define S_n to be the sum of all the steps up to step n, $S_n = a_1 + \ldots + a_n$. That is S_n is the current position of the object after n steps. We can now use this to do some mathematics and ask questions like how long does it take to… ? Or what is the probability of… ?

Recap: P(A) means the probability of event A. For example $P(a_1 = 1)$ = P(first flip heads) = 1/2.

1. Find $P(a_1 = 1 \text{ and } a_2 = 1)$

2. Find $P(S_2 = 2)$

3. Find $P(a_1 = 1 \text{ or } a_2 = 1)$

4. Find $P(S_2 = 0)$

5. Find $P(S_3 = 1)$

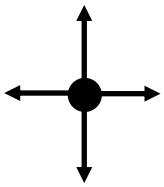
3.2 Gambler's Ruin

We can use a srw to answer interesting probability questions.

Suppose a gambler walks into a casino with £10. They pay £1 a turn to play a perfectly fair game with probability 1/2 of winning an additional £1 and probability 1/2 of losing their £1. So at the end of each game their balance either goes up by £1 or down by £1. What is the probability that the gambler gets to £12 before they go broke and run out of money?

What is the probability that the gambler gets to £10+m for any m before they go broke?

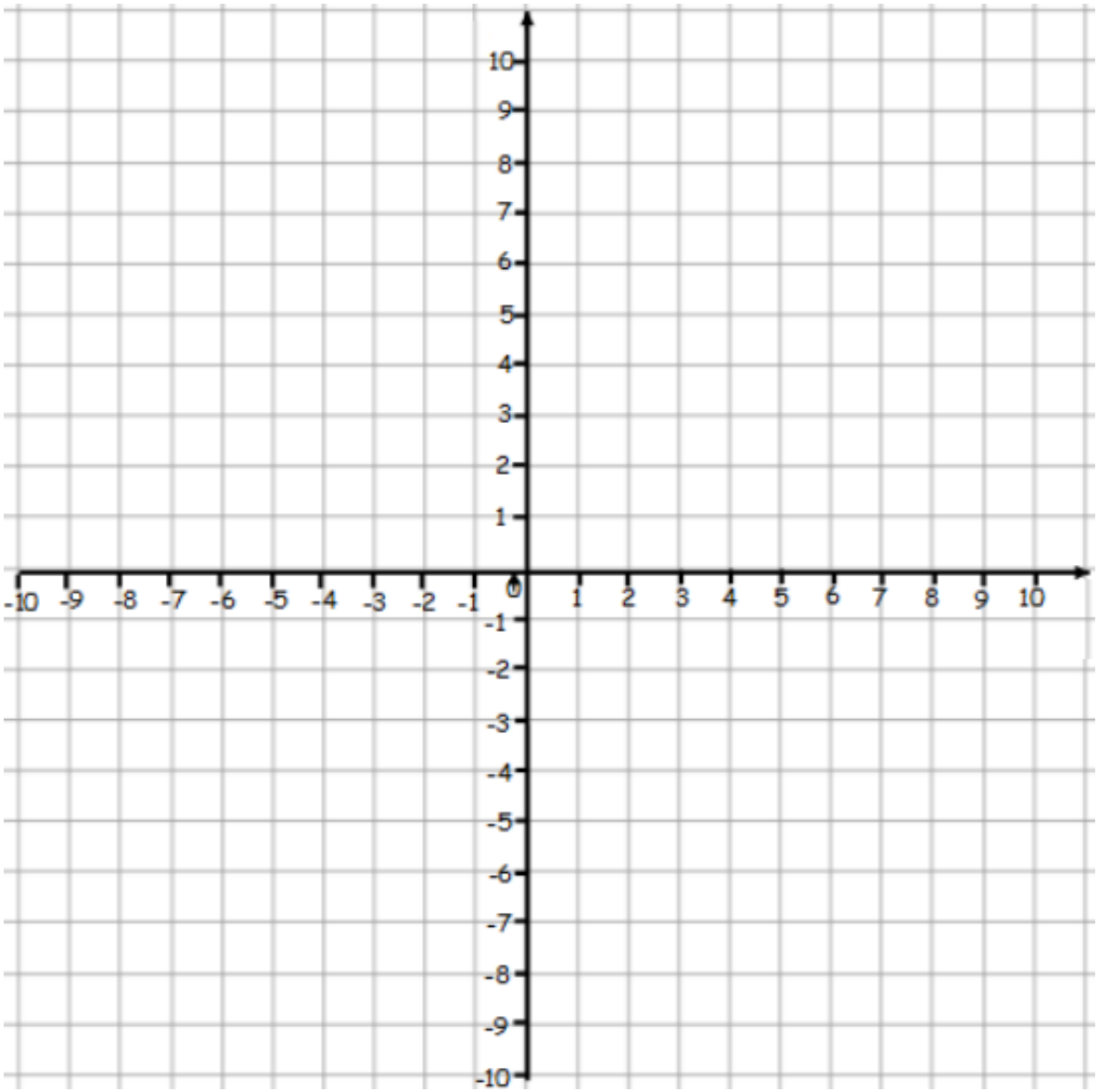
3.3 In 2 Dimensions

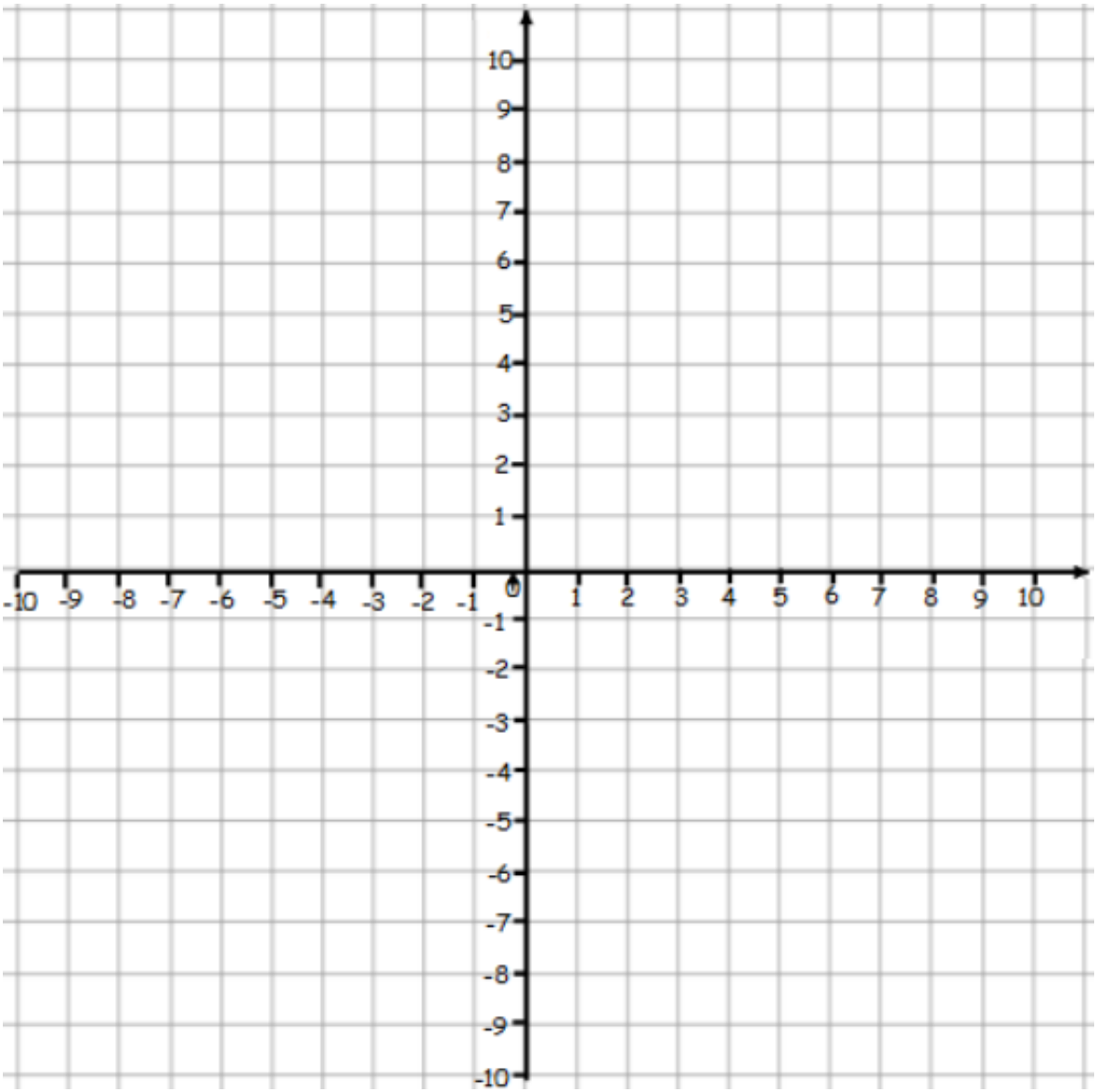


Let's now consider what happens in 2 dimensions. This time at any point the object has four possible directions to travel in - it can move up/down and left/right.

With your two coins choose coin 1 to be the **up/down direction** and coin 2 to be the **left/right direction.** Start from the origin, the point (0,0), and move one step in each direction as given by your coin flips. Draw a line in pencil to keep track of your path.

		Coin 2	
		Heads	Tails
Coin 1	Heads	Up + Right	Up + Left
	Tails	Down + Right	Down + Left





The Drunken Walk

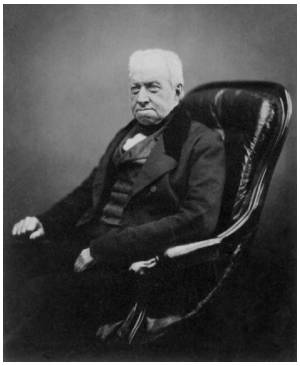
A drunken person stumbles out of the pub and starts walking home following the city grid lines. When they get to an intersection they randomly choose a direction and walk that way (including possibly walking back the way they came from). This is a 2d random walk.

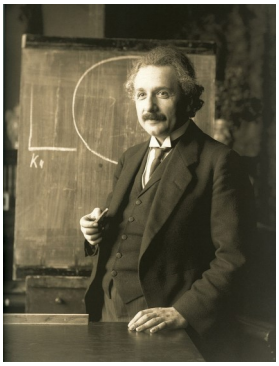
The probability they find their way home is 1. This means that they will definitely get back home again. But the expected time it takes to get back home is....

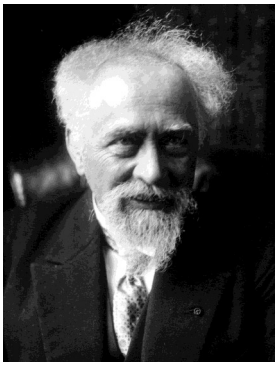
This is known as **Pólya's Recurrence Theorem** or Pólya random walk theorem and this probability can be computed for any dimension!

3.4 Brownian Motion

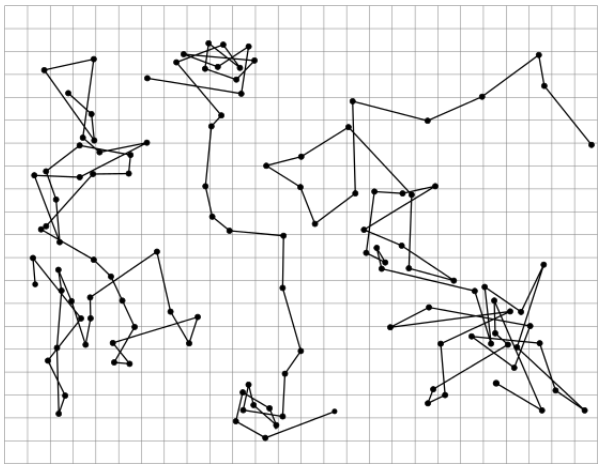
A simple random walk takes a step in a random direction at each time interval. What happens if we let the time interval be any size rather than the same size? What happens if the object can move in any direction and any distance?







In 1827 Robert Brown noticed that a pollen grain in water would randomly move around but could not explain why. In 1905 Albert Einstein explained that this was the result of the pollen being hit by water molecules. In 1926 Jean Perrin received the Nobel prize in Physics for experimentally confirming Einstein's explanation. This work was fundamental to proving that matter is indeed made of small molecules!



In mathematics to model Brownian motion we consider the particle to move randomly at random times in random directions!

Homework 3

1. Consider a symmetric simple random walk in 1d.
   A) Find $P(a_1 = a_2)$.
   B) Find $P(S_3 = 0)$.
   C) Find $P(S_2 = S_3)$.
   D) Find $P(S_n = 0)$ for any odd number n.

2. Consider a symmetric simple random walk in 1d. What do you expect to happen over a large time, say after 10,000 steps? Why?

3. Consider following one particle from the hard-sphere particle model. Does its path look like Brownian motion? How are they similar? How are they different?

4. Consider particle 1 which evolves in the hard-sphere particle model amongst 9,999 other particles. Also consider particle 2 which evolves via Brownian motion.

   How would you write instructions for a computer to follow the motion of particle 1 and particle 2? Write down any ideas you have.

   Which of these instructions would be easier for the computer to follow? Why?

Notes

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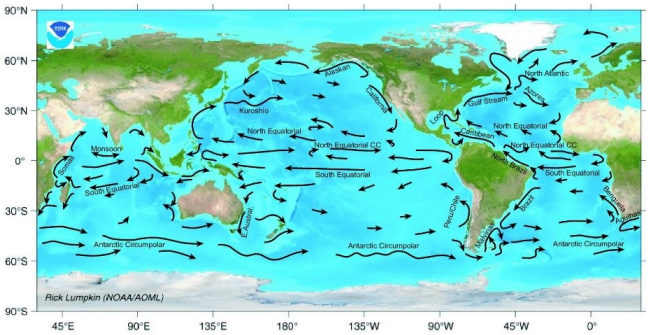
Tutorial 4 – Continua Equations



What is the Purpose of Tutorial 4?

- To see an example of a fluid equation.
- To understand a number of concepts related to equations describing continua
- To recap over the material we have covered in the tutorials so far

So far we have been looking at small scale models that describe how a particle evolves. But what happens when there are so many particles that it becomes impractical to keep track of them all? For example there might be 10^{26} molecules of gas in this room. The gas in this room does not act like it is made of discrete particles - it acts as one continuous body!

David Hilbert (1862 - 1943) was a German mathematician.

One of his most influential contributions to mathematics was his list of 23 unsolved problems in 1900 - of which 3 are still unsolved and 7 are only partially solved.

We are going to look at the sixth problem.

Hilbert's sixth problem is about how can we move from discrete particle models of a gas to continua equations that describe fluid and gas behaviour.

We have looked at discrete particle models - such as the hard-sphere model and the short-range potential model.

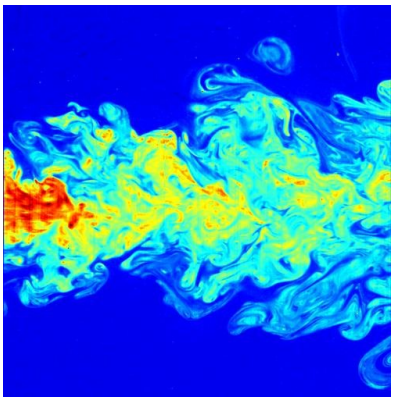
4.1 Navier-Stokes Equations

Think about pouring a glass of water and then think about pouring a glass full of honey. In what ways are they different?

The Navier-Stokes equation describes the motion of a fluid (a gas or a liquid).

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f}(\boldsymbol{x}, t)$$

This equation looks confusing and involves a lot of high level maths - all I want you to take away is that the left hand side says that the velocity **v** of the fluid is changing and the right hand side says that this change depends on *p* the pressure, **v** the velocity itself, **ν** the viscosity and **f** a force acting on the fluid.



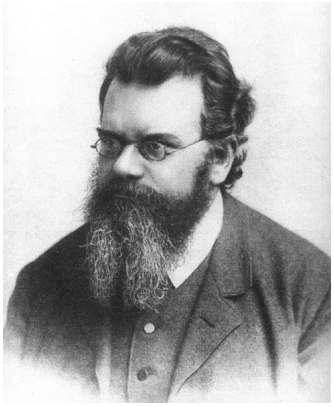
The image to the left shows an example of turbulent flow. This is very complex behaviour.

These equations are used by mathematicians and physicists to model complex behaviour of gas and liquids.

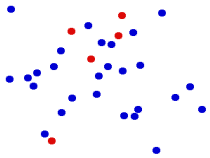
The Navier-Stokes equations are still being worked on by mathematicians and physicists - they are very difficult to solve. In the year 2000 a list called the Millennium Prize Problems gave seven of biggest problems in mathematics. A solution to any of the seven problems earns a prize of $1,000,000. Only one of the problems has been solved so far (Poincaré's conjecture).

4.2 The Boltzmann Equation

To solve Hilbert's Sixth problem we want to go from a particle model of a gas to an equation like the Navier-Stokes equation. We do this by using the Boltzmann equation as a stepping stone.



Ludwig Eduard Boltzmann (1844 - 1906) was an Austrian mathematician who studied statistical mechanics (the kind of work we have been looking at) - how the properties of the particles that make up a say a gas can determine the behaviour of the gas as a whole.



$$\longrightarrow \quad \begin{cases} \partial_t f + v \cdot \nabla_x f - \nabla_x F \cdot \nabla_v f &= Q(f,f), \\ f_{t=0} &= f_0, \end{cases} \quad \longrightarrow \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f}(\boldsymbol{x}, t)$$

Particle Model | Boltzmann Equation | Navier-Stokes equation

The Boltzmann equation supposes that the gas is made up of infinitely many particles that are infinitely small. It considers where the particles are on average rather than tracking every individual particle.

Next tutorial we are going to look at some research papers trying to prove this first step of going from a particle model to the Boltzmann equation.

Homework 4

Questions 3 and 4 are very difficult. Lots of mathematicians around the world are working on these problems and they are very complex - even I only understand the very basics of most of the problems. But I'd like you as a challenge to see what you can find out about the problems.

1. Describe in your own words viscosity. Gave an example of two substances, one with a higher viscosity and one with a lower viscosity.

2. In your own words and using what we have covered in the tutorials, summarise Hilbert's sixth problem.

3. Research Hilbert's 22 other problems (we have been looking at the sixth one!). Are there any that you can understand and summarise in 50 words?

4. Research the six other Millennium Prize Problems. Are there any that you can understand and summarise in 50 words?

5. Who is Grigori Perelman and what is his relation to the Millennium Prize Problems? What did he do?

Notes

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Tutorial 5 – Journal Club

What is the Purpose of Tutorial 5?

- To see genuine research papers in mathematical physics and understand the processes involved in writing an article.
- To understand how the concepts we have learnt in this course can be linked together
- To begin to understand extracts of journals and books on the area we have been studying.

What do you know about a research article or paper? What is the purpose of research articles and papers? What might an article include? What steps are involved along the way? Who might be involved at the different stages?

Thierry Bodineau, Isabelle Gallagher, and Laure Saint-Raymond. The Brownian motion as the limit of a deterministic system of hard-spheres. *Inventiones mathematicae*, 203(2):493–553, 2015.

Download online here: https://arxiv.org/abs/1305.3397

Invent. math.
DOI 10.1007/s00222-015-0593-9

The Brownian motion as the limit of a deterministic system of hard-spheres

Thierry Bodineau[1] · Isabelle Gallagher[2] · Laure Saint-Raymond[3]

Received: 14 May 2013 / Accepted: 18 March 2015
© Springer-Verlag Berlin Heidelberg 2015

Abstract We provide a rigorous derivation of the Brownian motion as the limit of a deterministic system of hard-spheres as the number of particles N goes to infinity and their diameter ε simultaneously goes to 0, in the fast relaxation limit $\alpha = N\varepsilon^{d-1} \to \infty$ (with a suitable diffusive scaling of the observation time). As suggested by Hilbert in his sixth problem, we rely on a kinetic formulation as an intermediate level of description between the microscopic and the fluid descriptions: we use indeed the linear Boltzmann equation to describe one tagged particle in a gas close to global equilibrium. Our proof is based on the fundamental ideas of Lanford. The main novelty here is the detailed study of the branching process, leading to explicit estimates on pathological collision trees.

Karsten Matthies and Florian Theil. A semigroup approach to the justification of kinetic theory. *SIAM Journal on Mathematical Analysis*, 44(6):4345–4379, 2012.

Download online here: http://opus.bath.ac.uk/32273/1/MT_SIMA2012.pdf

SIAM J. MATH. ANAL.
Vol. 44, No. 6, pp. 4345–4379

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A SEMIGROUP APPROACH TO THE JUSTIFICATION OF KINETIC THEORY*

KARSTEN MATTHIES† AND FLORIAN THEIL‡

Abstract. This paper develops a method to rigorously show the validity of continuum description for the deterministic dynamics of many interacting particles with random initial data. We consider a hard sphere flow where particles are removed after the first collision. A fixed number of particles is drawn randomly according to an initial density $f_0(u, v)$ depending on d-dimensional position u and velocity v. In the Boltzmann–Grad scaling, we derive the validity of a Boltzmann equation without gain term for arbitrary long times, when we assume finiteness of moments up to order two and initial data that are L^∞ in space. We characterize the many-particle flow by collision trees which encode possible collisions. The convergence of the many-particle dynamics to the Boltzmann dynamics is achieved via the convergence of associated probability measures on collision trees. These probability measures satisfy nonlinear Kolmogorov equations, which are shown to be well-posed by semigroup methods.

Karsten Matthies, George Stone, Florian Theil . The Derivation of the Linear Boltzmann Equation from a Rayleigh Gas Particle Model. *Kinetic and Related Models* 2018

Download online here: https://arxiv.org/abs/1603.07574

Kinetic and Related Models
doi:10.3934/krm.2018008
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THE DERIVATION OF THE LINEAR BOLTZMANN EQUATION FROM A RAYLEIGH GAS PARTICLE MODEL

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(Communicated by Laure Saint-Raymond)

ABSTRACT. A linear Boltzmann equation is derived in the Boltzmann-Grad scaling for the deterministic dynamics of many interacting particles with random initial data. We study a Rayleigh gas where a tagged particle is undergoing hard-sphere collisions with background particles, which do not interact among each other. In the Boltzmann-Grad scaling, we derive the validity of a linear Boltzmann equation for arbitrary long times under moderate assumptions on higher moments of the initial distributions of the tagged particle and the possibly non-equilibrium distribution of the background. The convergence of the empiric dynamics to the Boltzmann dynamics is shown using Kolmogorov equations for associated probability measures on collision histories.

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Final Assignment

1. Write a mathematical model for one of the given systems. Include a detailed description of how each object moves in relation to its goal and relative to the other objects, include whether there is an external force effecting the objects, whether collisions are possible, and if so what happens at a collision. Think about your model as being instructions for a computer to simulate the situation - so you should include all the detail you can think of.

   Evaluate your model. State three good aspects of your model. State at least three improvements that could be made to the model to make it more realistic.

2. A railway truck P, of mass *m* kg, is moving along a straight horizontal track with speed 15 m/s. Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is 3 m/s and the speed of Q is 9 m/s. The direction of motion of P is reversed by the collision. Find *m*.

3. Three particles A,B and C of mass 2, 4 and 5 kg respectively are at rest in a straight line. Particle A is projected with velocity 4m/s towards particle B. After the collision particle A has velocity -1m/s. Particle B then collides with particle C and after this collision particle C has velocity 3.6m/s. Show that particle B will collide with particle A a second time.

4. An object of mass *m* travelling with velocity v has kinetic energy $1/2\ mv^2$.

   Particle 1 of mass 5kg has velocity 2m/s. It collides with particle 2 of mass 20kg which is at rest. Assume that the total kinetic energy before the collision is equal to the total kinetic energy after the collision and that the conservation of momentum holds. Given that particle 1 changes direction after the collision find the velocity of particle 1 and 2 after the collision.

5. Consider a 1d simple random walk. It starts at 0 and at each step it moves either one step right with probability p or one step left with probability (1-p).
   A) Find $P(a_2 = 1)$.
   B) Find $P(S_2 = 0)$.
   C) Describe what you expect to happen over a long time for different values of p.

6. Suppose a gambler walks into a casino with £10. They pay £1 a turn to play a bias game with probability *p* of winning an additional £1 and probability *1-p* of losing their £1. What is the probability that the gambler gets to £12 before they go broke and run out of money? What if they started with £n? Comment on what this result means when $p<1/2$ regardless of the value of n.

7. Write a short summary (200 - 250 words) of one of the papers from tutorial 5. This should include a summary of the area (this can come from the things we have spoken about during the tutorials), the main results of the paper and any other comments you can make.

Tutorial 6 – Feedback tutorial

What is the Purpose of Tutorial 6?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.

To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well...	What I could have improved on...

My target for future work is...

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?

What did you find challenging about the programme?	How did you overcome these challenges?

Appendix 1 – Referencing correctly

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

Why should I reference?

Referencing is important in your work for the following reasons:

It gives credit to the authors of any sources you have referred to or been influenced by.
It supports the arguments you make in your assignments.
It demonstrates the variety of sources you have used.
It helps to prevent you losing marks, or failing, due to plagiarism.

When should I use a reference?

You should use a reference when you:

Quote directly from another source.
Summarise or rephrase another piece of work.
Include a specific statistic or fact from a source.

How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. **A marker in your assignment:** After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].

2. **Bibliography:** This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

Websites – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

Books – Author, date published, title of book (in italics), pages where the information came from.

E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

Articles – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

Appendix 2 – Using the VLE

VLE username	
VLE password	

Please remember the following key details:

You are able log into the VLE either through the link on our website (www.thebrilliantclub.org) or going directly to the VLE site at (https://portal.thebrilliantclub.org/sign-in).

- Please update your profile with your full name and email address- this will allow you to retrieve forgotten passwords or usernames

If you forget your log-in details you can request them to be emailed to you by clicking the link on the VLE home page. (If you are still having problems you can email: schools@thebrilliantclub.org)

What is the VLE?

The VLE is a virtual learning environment for all pupils on Uni Pathways it is used for:

messaging your tutor

submitting homework

submitting your final assignment

accessing resources for your tutorials

finding out more information about university and careers

How should I use the VLE?

The VLE is a professional academic environment in which pupils are able to message their PhD Tutor. Here are a few things to consider:

Ensure you keep a professional tone in the messages you send to your tutors.

Ensure you always reply to your tutors in a timely manner.

Thank your tutor for the effort they are putting in to give you your feedback etc.

Submit all homework to your tutor on time.

IMPORTANT: Final assignment

- When you submit your final assignment, please remember that you need to do so through the 'My Activities' tab and not as an attachment to a message.

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