Researchers in Schools

Fun with paper: An introduction to surfaces

Key Stage 4 Programme

Pupil Name	
Coursebook Designed by	Dr Robertson

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Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after midnight on 9th August	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Course Rationale

Surfaces are what mathematicians call two-dimensional objects, like soap bubbles and origami models. This course is based on my PhD, which found new ways to change surfaces into other surfaces that had never been discovered before.
In this course, we'll learn how mathematicians tell different surfaces apart (and why they can get their doughnut confused with their coffee cup!). Along the way, we'll work out what coins to use for your hamster's new car, the equation of a bridge, and lots more about surfaces, like how to stick them together or identify their different features.
The course will give you a taste for what university-style learning is like, and prepare you better should you choose to make the journey yourself.

Uni Pathways Mark Scheme 2020

	Subject Knowledge	Critical Thinking	Written Communication
1st	The work shows a depth of knowledge and understanding of key concepts and mathematical methods, drawing on sources of evidence where appropriate. Knowledge is used to build and support highly effective mathematical arg uments and explanations.	Analyses the available data/evidence and identifies all the relevant mathematical information. Identifies and critically evaluates available mathematical approaches and evidence, deciding on their credibility, strength and relative significance. Makes connections between different pieces of information and puts them together to form accurate solutions and verify answers and assumptions.	Mathematical arguments are complete, and the reasoning behind each step has been clearly communicated to the reader. The work has a coherent flow and is well structured. The writing style is appropriate; mathematical language and key mathematical terms are used accurately and effectively to support the arguments and explanations made. There are no, or very few, errors in spelling or grammar. Consistent referencing, appropriate paragraphing and use of correctly labelled tables and figures; matching the style taught in the course.
2:1	The work shows an understanding of key concepts and mathematic al methods, drawing on sources of evidence where appropriate. Knowledge is used to build and support effective mathematical arg uments and explanations.	Analyses the available data/evidence and identifies all the relevant mathematical information. Engages with multiple approaches and critically evaluates their utility, forming judgements and using reasoning and evidence to inform problem solving strategy. Shows some understanding of the relative value of evidence and data.	Mathematical arguments are clearly made, with few gaps in logical reasoning. The work is well-structured. The writing style is appropriate; mathematical language and key terms are used correctly. There are few errors in spelling or grammar. Mostly consistent referencing and use of tables and figures; matching the style taught in the course.
2:2	The work shows an understanding of key concepts and mathematical methods, with no major misconceptions. Beginning to apply this knowledge to build and support effective mathematical arg uments and explanations.	Identifies and uses basic mathematical evidence and reasoning. Evaluates the quality of mathematical reasoning and evidence, forming judgements. Not yet showing understanding of the relative value of evidence and reasoning.	Mathematical arguments have been constructed, but there is little explanation from step to step. The work has some structure. The writing style can sometimes be informal; occasionally key terms are not used when it would be appropriate to do so. There are some errors in grammar and spelling, but these do not get in the way of communicating the content. Referencing has some consistency; matching the style taught in the course. Limited use of tables and figures.
3rd	Shows a developing understanding of key concepts and mathematical methods, with some misconceptions. Not yet applying this knowledge to build and support mathematical argu ments and explanations.	Beginning to analysis mathematical evidence and reasoning. Describes or attempts to solve the problem without any/or little evaluation of whether the approach was reasonable or accurate.	Mathematical arguments to solve problems are poorly sequenced, with little or no explanation. The grammar, spelling, style, and structure of the work need improving in order to communicate ideas to the reader. Key terms are not always used correctly. Limited, or no use of referencing, tables and figures.

Baseline Assignment: Pupil Feedback Report

Name of Pupil	
Name of School	
Name of RIS teacher	
Title of Assignment	

How your assignment is graded:

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Marks

OVERALL MARK / 100		FINAL MARK / 100 (including any deductions)	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Mark Breakdown and Feedback

Subject knowledge	
	mark
Critical thinking	
	mark
Written communication	
	mark

Problem Set	
Comment	*mark*

Final Assignment: Pupil Feedback Report

Name of Pupil	
Name of School	
Name of RIS teacher	
Title of Assignment	

How your assignment is graded:

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Marks

OVERALL MARK / 100		FINAL MARK / 100 (including any deductions)	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Mark Breakdown and Feedback

Subject knowledge	
	mark
Critical thinking	
	mark
Written communication	
	mark
Problem Set	
Comment	*mark*

Subject Vocabulary

Word	Definition	In a sentence
Surface	A two-dimensional shape	The sphere and the torus are surfaces
Orientable	Able to be given a consistent outside and inside	A surface is orientable if it does not contain a Möbius band
Genus	The number of doughnut holes in a surface	The genus of a figure-8 balloon is 2
Boundary	The edge or rim of a surface	The boundary of a cylinder is two circles
Deform	To change the shape of without breaking	A coffee cup can be deformed into a doughnut In a deformation, you can stretch and bend, but not cut or glue together
Theorem	A mathematical statement which can be proven to be true	Pythagoras' theorem helps us calculate the third side length in a right-angled triangle
Classify	Put into classes depending on whether they are the same in some way	Surfaces can be classified up to deformation by their genus, orientability and boundary.
Closed surface	A surface with no boundary	The torus is a closed surface, but the disc is not
Compact surface	A surface that fits in a finite space	

Launch Event

The launch event is your opportunity to learn more about this course and Uni Pathways. You will watch some videos, listen to your teacher and have some discussion. Below is some room to make notes for some of the videos you will watch during the event.

Virtual Campus Tours

Notes…

The Scholars Programme Alumni

Notes…

Student Ambassadors

Notes…

Tutorial 1 – What is a surface?

What is the purpose of Tutorial 1?

- I can recall the definition of a surface
- I can reproduce examples of surfaces
- I can identify instances of surfaces being the same under deformation

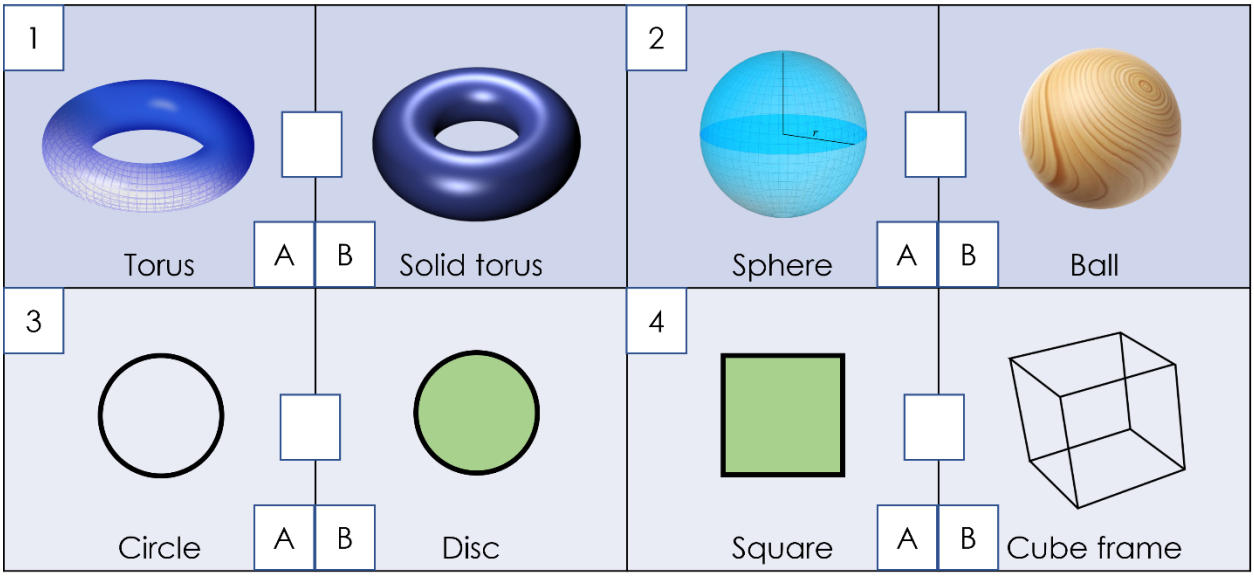
Task 1 – Mini whiteboards

Respond to the questions about shapes in 3D with your mini whiteboard. Use the space below for your notes.

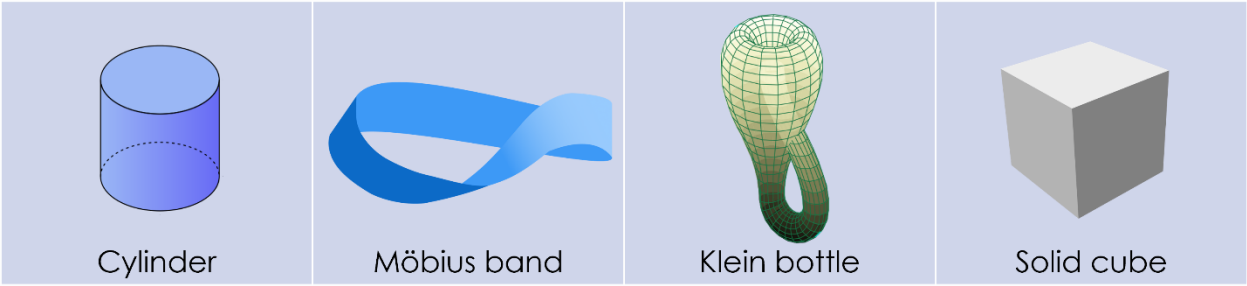
Task 2 – Examples of surfaces

A **surface** is a shape which is **two-dimensional** at every point.

In each pair, only one of the shapes is a surface – which one? Write A or B in the box.



Q5. Which of the following are surfaces? Tick (✓) the ones that are surfaces.



Task 3 – Fill in the blanks

Fill in the blanks during the mini lecture about deforming surfaces.

To deform a surface means to change the _____ of it.

The cube can be _____ into the sphere.

In a deformation, we are allowed to _____ and bend, but not cut or _____.

Task 4 – Think, pair, share

Use the space below to record your thoughts and your discussion with your partner.

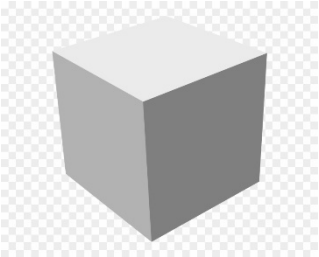
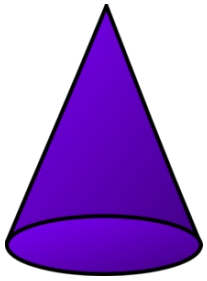
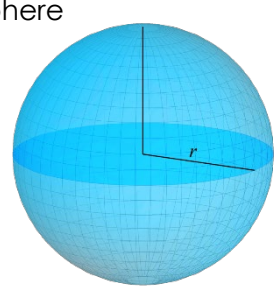
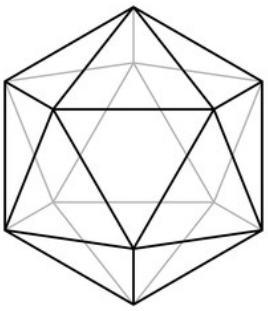
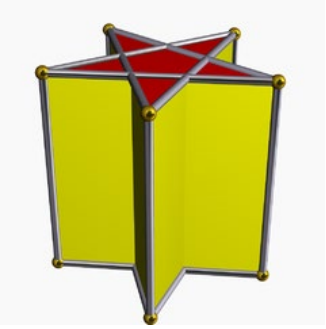
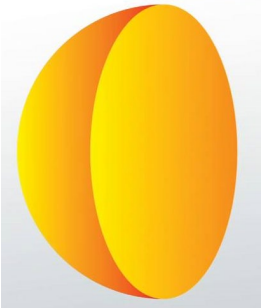
Homework 1

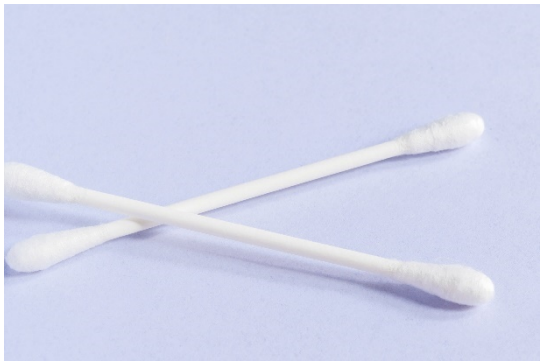
Baseline assignment question: **What is a 3D shape?**
This assignment comes in two parts, writing a paragraph and then four problems. Try to write something for every question, even if you're not sure of the final answer.

Success criteria – how to structure your assignment and what to include:
Write a paragraph (around 250 words) in response to the above question. You should include content on at least four examples, a description of edges and faces, and whether you would describe the shapes as round or straight.

Problem set

1. How many edges and faces do each of the following 3D shapes have?

a) Cube 	b) Cone 	c) Sphere 
d) Icosahedron 	e) Star prism 	f) Hemisphere 



2. I'm planning to make the base of a car for my pet hamster by gluing a hexagonal nut to each end of two cotton buds. Mathematically, is this a good idea? Justify your answer.

3. The outline of a rugby ball is given by the equation
$$r = h\cos\left(\frac{z}{h}\right)$$
where h is a fixed number. What is z in terms of h when $r = 0.5$?

4. Are these pairs of surfaces the same up to deformation?

a) The surface of a doughnut and the surface of a mug
b) a Klein bottle and a Möbius band
c) a trefoil torus and a torus
d) an annulus and a torus

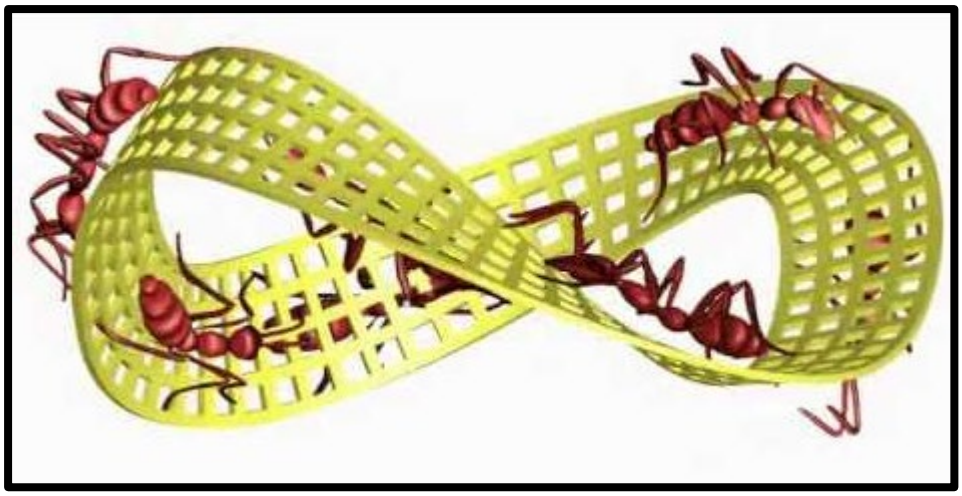
In this assignment, show me that you **understand**:

- The definition of a surface
- The definition of edges and faces of 3D shapes
- Basic properties of polygons
- The difference between length and area

Show me that you **are able to**:

- Produce examples of 3D shapes
- Manipulate trigonometric equations
- Appraise suggestions and your own solutions
- Formally state mathematical concepts and communicate ideas effectively

Tutorial 2 – How many sides?



What is the purpose of Tutorial 2?

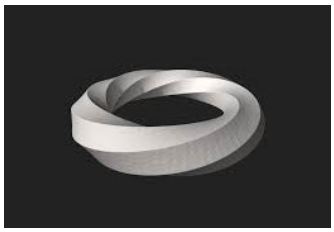
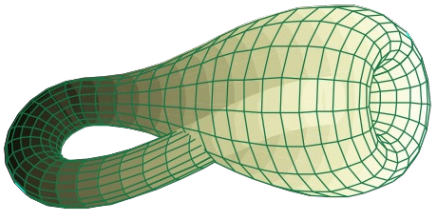
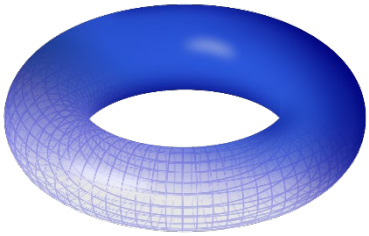
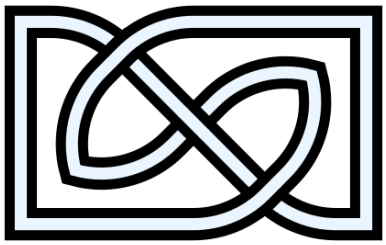
- I can differentiate between length, area and volume
- I can identify whether a surface is orientable
- I can identify the boundary of a surface

Task 1 – Mini whiteboards

Respond to the questions about length, area and volume with your mini whiteboard. Use the space below for your notes.

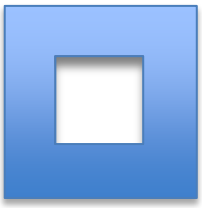
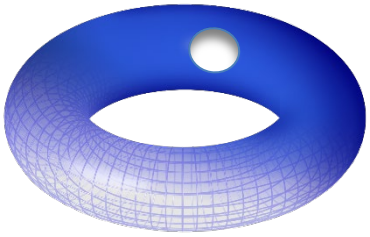
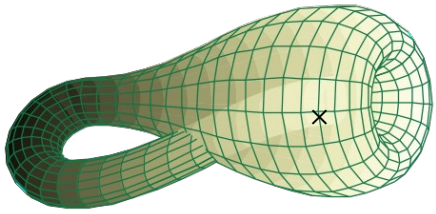
Task 2 – Orientability

Which of the following surfaces are orientable?

a) Twisted torus 	b) Möbius strip	c) Helicoid
d) Klein bottle 	e) Torus 	f) Trefoil knot 

Task 3 – Boundary

How many boundary components do the following surfaces have?

a) Disc	b) Annulus	c) Möbius strip
d) Square annulus 	e) Torus with hole 	f) Punctured Klein bottle 

Task 4 – Möbius strip exploration

Items required: Möbius strip template, glue stick, highlighter

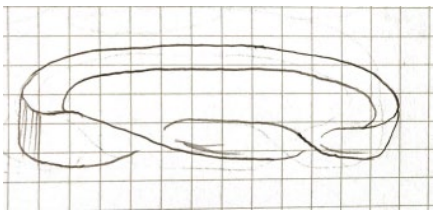
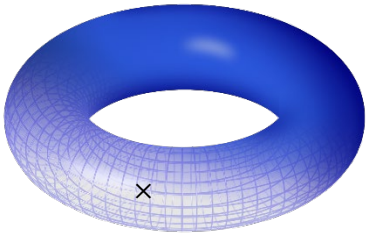
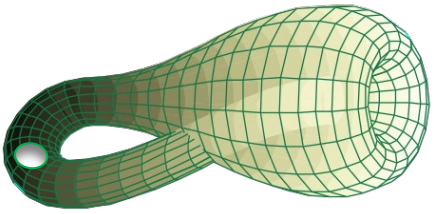
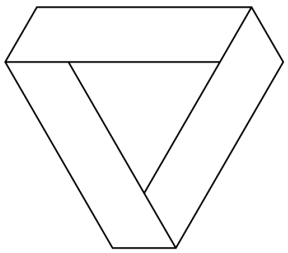
a) Take your strip of paper and match the letters together so they touch. Glue the ends together in this way to form a twisted loop. This is your Möbius strip!

b) With your pencil, start drawing a line around the middle of your strip, and continue until you get back to where you started. Look at the side underneath where you started drawing. What do you notice about your line?

c) With your highlighter, start drawing a line along the boundary of your strip, and continue until you get back to where you started. How many boundary components does your strip have? What shape is each component?

Homework 2

1. Which of the following surfaces are orientable?

a) Double twisted Möbius strip 	b) Triple twisted Möbius strip	c) Punctured torus 
d) Klein bottle with hole 	e) Folded triangle 	f) Triple folded triangle

2. How many boundary components does each of the above surfaces have?

3. Can you draw an orientable surface with no boundary components?

Tutorial 3 – How many holes?

What is the purpose of Tutorial 3?

- I can recall the main properties of Reuleaux polygons
- I can identify the genus of a surface
- I can interpret the connected sum operator algebraically and geometrically

Task 1 – Reuleaux polygons quiz

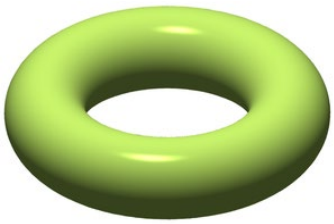
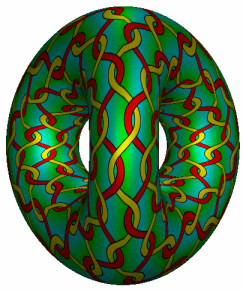
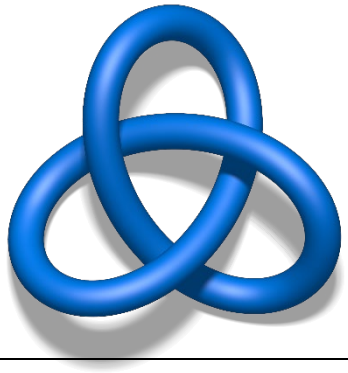
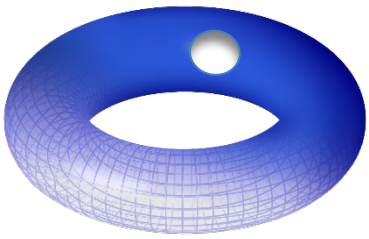
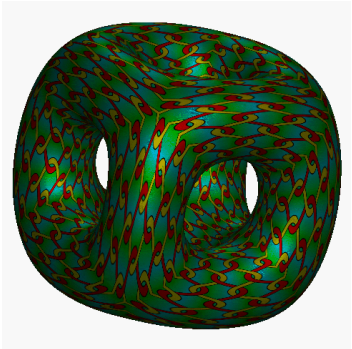
Why do we usually use circular wheels?

What property of Reuleaux polygons makes them useful as rollers?

What property of Reuleaux polygons makes them less useful as fixed wheels?

Task 2 – Genus

What is the genus of each of the following surfaces?

a) 	b)	c) 
d) 	e) 	f) 

Task 3 – Connected sum

What is the connected sum of each of the following pairs?

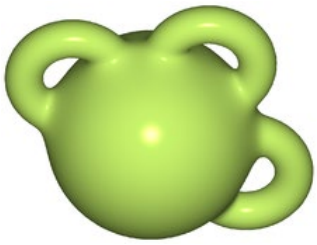
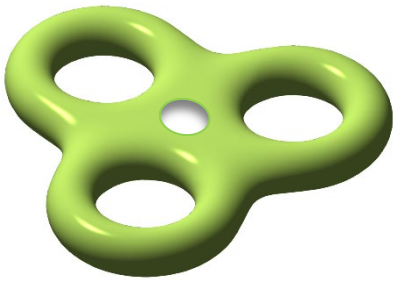
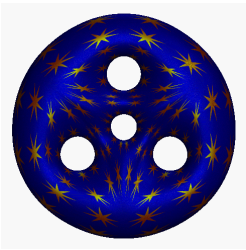
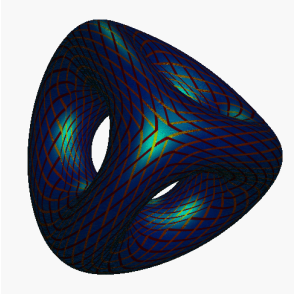
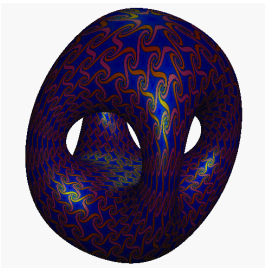
1. (a) # (c)
2. (e) # (d)
3. Torus and Möbius strip

What is the connected sum of a torus with a sphere?

Can you generalise this to the connected sum of any surface with a sphere?

Homework 3

1. What is the genus of each of the following surfaces?

a)	b) 	c) 
d) 	e) 	f) 

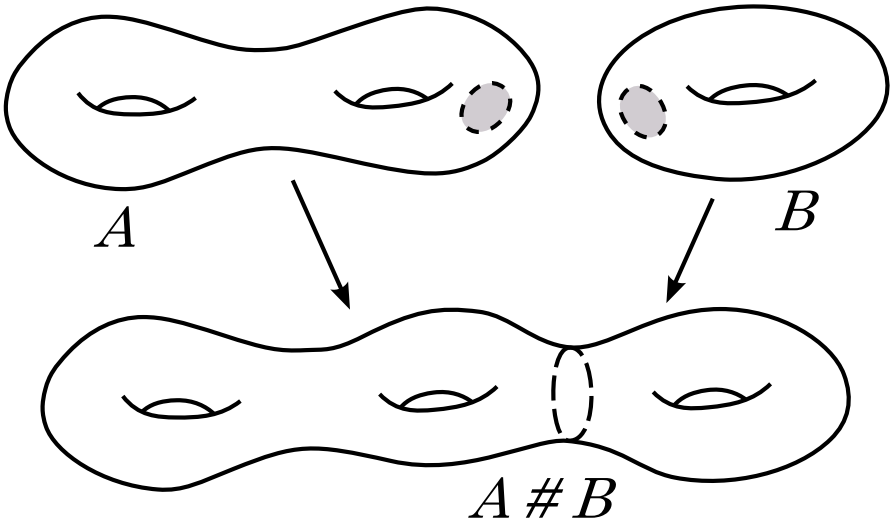
2. Draw a surface with genus two.

3. What is the connected sum of each of the following?

i) (a) # (b)
ii) A Möbius strip with another Möbius strip
iii) A Klein bottle and a cube

4. Write a few words on how you might prove the connected sum is unique.

Tutorial 4 – Classified



What is the purpose of Tutorial 4?

- I can classify surfaces by their orientability, genus and boundary
- I can state and apply the classification theorem for closed surfaces
- I can state and apply the classification theorem for compact surfaces

Task 1 – Key words

Complete the definitions of each of the following key words.

The genus of a surface is how many _____ _____ it has.

A surface is orientable if it does not contain a _____ _____.

The boundary of a surface is its _____. The pieces that make up the boundary are called _____ _____.

The connected sum of two surfaces is formed by cutting a _____ out of each of them and _____ them together along the new boundary components.

Task 2 – Mini whiteboards

Respond to the questions about reference surfaces with your mini whiteboard. Use the space below for your notes.

Task 3 – Classifying the alphabet

Use the space below to classify the 26 surfaces in the box by their boundaries.

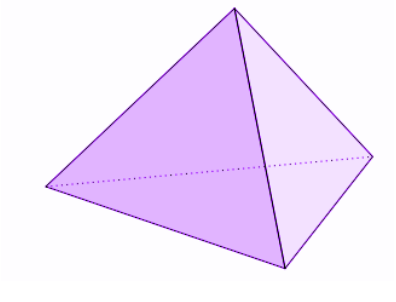
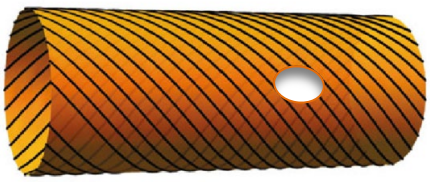
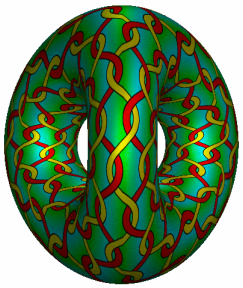
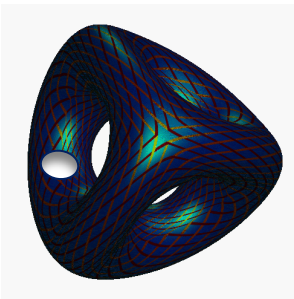
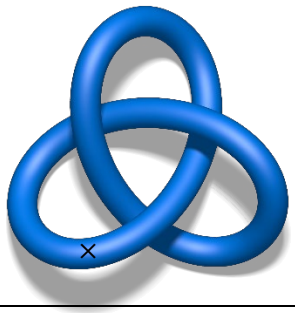
A B C D E F G H I J

K L M N O P Q R

S T U V W X Y Z

Task 4 – Applying the classification theorems

Which reference surface is each of the following classified by?

a) 	b) 	c)
d) 	e) 	f) 

Homework 4

1. A birthday party company sells ten different types of balloon, with each in the shape of a different numerical digit. Can you classify them according to their genus?

2. What is the connected sum of the following pairs? Which reference surface can it be deformed into? (Draw the pictures!)

a) A cylinder with a hole # a torus
b) A punctured Klein bottle # an annulus
c) A tetrahedron # a cube
d) A Möbius strip # a Klein bottle

3. How many orientable surfaces are there with genus 2 and one boundary component?

4. How many orientable surfaces are there with no boundary component?

Tutorial 5 – Bubbles and chain links

What is the purpose of Tutorial 5?

- I can recognize catenaries and catenoids with real-life examples
- I can manipulate the hyperbolic cosine function
- I can design a draft essay

Task 1 – Catenaries and catenoids

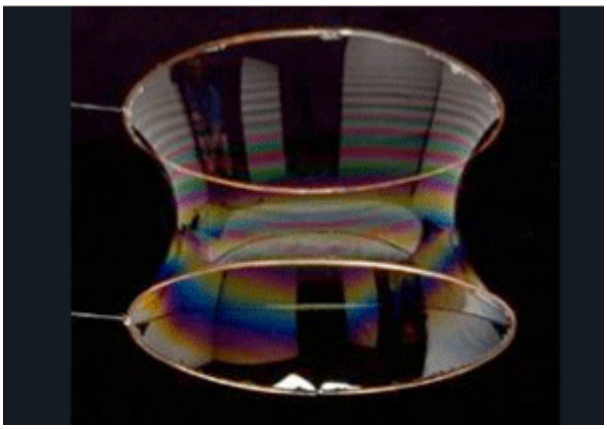
Classify each of the following as a catenary, catenoid, or neither.

$$y = x^2 - 4$$

$$y = 7\cosh x - 5$$

$$y = \frac{1}{\cosh x}$$

$$r = 4\cosh\theta$$



Task 2 – Paper chain

Record the measurements of the class paper chain here.

What is the equation of our paper chain?

Task 3 – Essay design

Use the space below to start your essay plan.

Homework 5

Final assignment question: **What is a surface?**
The final assignment is split into two parts, a short essay and a problem set. Each part will count equally towards your final grade.

Success criteria – how to structure your assignment and what to include:
Write a short essay (between 500 and 1000 words) in response to the above question. You should include content on each of the following:

- The **definition** of a surface, and what it means for surfaces to be **homeomorphic**
- The **orientability**, **genus** and **boundary** of a surface
- The **connected sum** of two surfaces
- The **classification theorem** for closed / compact surfaces
- Lots of **examples** of surfaces to demonstrate these – feel free to research your own!
-

Problem set

1. What is the genus of the following surfaces? How many boundary components does each have?

Pretzel

Straw

Half-bagel

Carrier bag

Swiss cheese

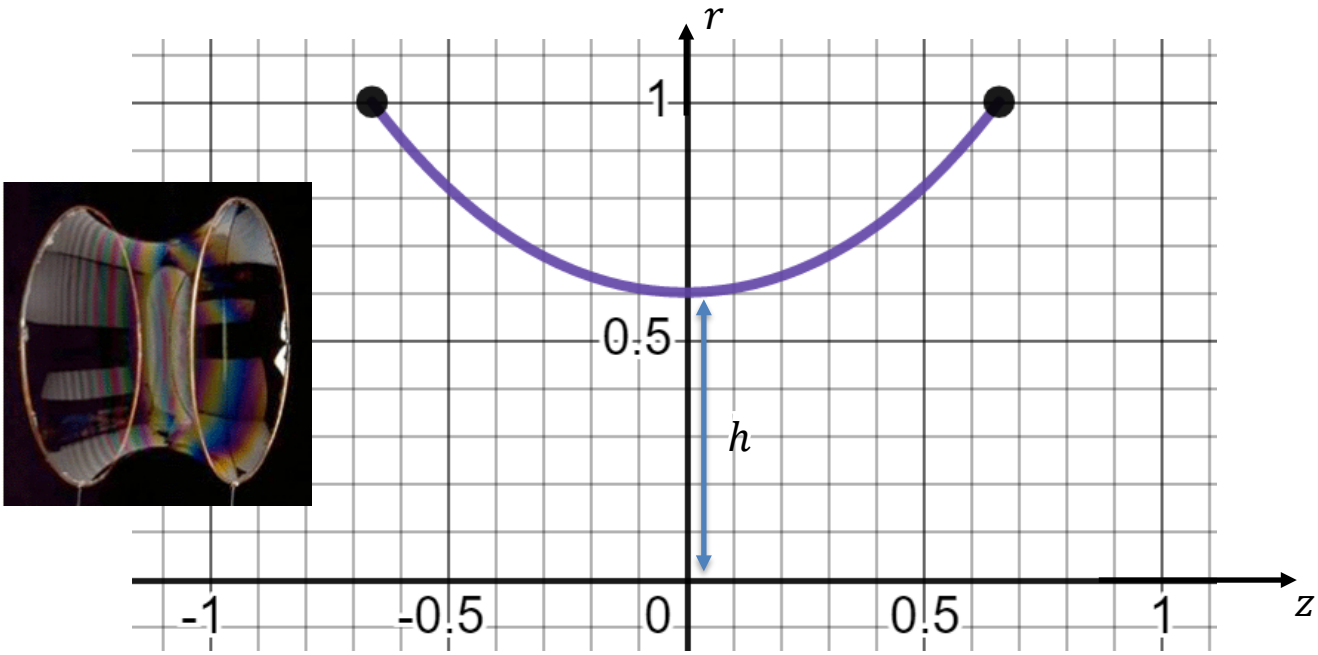
2. Write two equations between some of the surfaces in Q1 using the connected sum.

3. I'm planning to make the base of a car for my pet hamster by gluing a 50p coin to each end of two cotton buds. Mathematically, is this a good idea? Justify your answer.

4. A soap film is suspended between two rings of radius 1, which is given by the equation

$$r = h\cosh\left(\frac{z}{h}\right)$$

where h is the distance from the centre to the middle of the bubble. How far apart are the two rings spaced, in terms of h?



5. What is the connected sum of a Möbius band and a torus; a Klein bottle and a disc? Describe any differences between the two resulting surfaces.

6. How many straight 1mm lines can you fit inside a circle of radius 2mm without overlapping? Justify your answer.

In this assignment, show me that you **understand**:

- The definitions of surface, homeomorphic, orientability, genus, boundary, connected sum
- Basic properties of Reuleaux polygons, including width and centre of rotation
- The difference between length and area
- The statement and consequences of the classification theorem for closed / compact surfaces

Show me that you **are able to**:

- Identify the genus / boundary of a compact orientable surface
- Use the connected sum operator
- Manipulate equations similar to those in class
- Produce examples to demonstrate understanding of core concepts
- Appraise suggestions and your own solutions
- Formally state mathematical concepts and communicate ideas effectively

Tutorial 6 – Feedback tutorial

What is the Purpose of Tutorial 6?

- To receive feedback on your final assignment
- To respond to the feedback from your Uni Pathways teacher
- To write targets for improvement on your final assignment

Final assignment feedback from your Uni Pathways Teacher

(Remember to look at the mark scheme to help you understand what you have done well so far, and how you can do even better in your final assignment)

Here are three things that my Uni Pathways Teacher thought I did well in my draft assignment
•   •   •

Here are three things that my Uni Pathways Teacher thinks that I could do to get a higher mark in my final assignment
•   •   •

Tasks from my Uni Pathways Teacher to do during the feedback tutorial to help me improve
•   •   •
My response:

Actions I will take to improve my final assignment after this tutorial…
•   •   •

Hand in date for my final assignment:	

Tutorial 7 – Final tutorial

What is the Purpose of Tutorial 7?

- To receive feedback and a grade on your final assignment.
- To reflect on the programme including what you enjoyed and what was challenging.
- To ask any questions you may have about university.

Final assignment feedback from my Uni Pathways Teacher

Final mark:	University style grade:
Feedback: Here are three things that my Uni Pathways teacher thought I did well in my final assignment • • •	

Here are three things that my Uni Pathways teacher thinks I should remember for when I am doing this kind of study in the future
• • •

University

What questions do you still have about University after taking part in Uni Pathways?
•   •   •

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
•   •   •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Appendix 1 – Referencing correctly

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

Why should I reference?

Referencing is important in your work for the following reasons:

- It gives credit to the authors of any sources you have referred to or been influenced by.
- It supports the arguments you make in your assignments.
- It demonstrates the variety of sources you have used.
- It helps to prevent you losing marks, or failing, due to plagiarism.

When should you use a reference?

You should use a reference when you:

- Quote directly from another source.
- Summarise or rephrase another piece of work.
- Include a specific statistic or fact from a source.

How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. **A marker in your assignment**: After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].
2. **Bibliography**: This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

    a. **Websites** – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

    E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

    b. **Books** – Author, date published, title of book (in italics), pages where the information came from.

    E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

    c. **Articles** – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

    E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

Notes

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