Should We Really Trust The Maths Teachers?

Key Stage 4 Programme

Pupil Name

Coursebook Designed by Dr Nicholas Winstone

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Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after midnight on 9th August	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Course Rationale

"1+1=2 and that's a fact!"

But why are we so sure about this? How do we really know that anything we are taught is absolute fact? Why do maths teachers and mathematicians seem so willing to trust that what they are teaching is indisputably true?

All reasonable questions, and ones that we will aim to investigate over the course of 7 tutorials. Looking at how we can develop mathematical ideas which aren't yet shown to be certain, we will begin to understand the concept of proof.

Writing proofs is a mathematician's bread and butter, and you can assume that almost every mathematical statement ever written has a proof specifically for it. With the near countless numbers of proofs that have existed at some point throughout history, there is no way that any one person could read them all, let alone check them all for mistakes.

We will build an understanding of mathematical proofs, and how they are used to build on previous knowledge to develop new ideas. We distinguish fact from guess, theorem from conjecture, and consider how seemingly obvious mathematical statements can be used to mislead us. We will apply this to poor representations of statistics and graphs in the media. Ultimately, we want to gain an understanding as to how mathematicians can be so confident that the maths that they know is correct.

This course will be an opportunity for students to reflect on their mathematical education and allow them to question whether they should have always been so trusting of what was taught. This will lead them to write a short essay providing their own thoughts on the question:

"To what extent should we trust that the mathematics we are taught in school is correct"

Should we really trust the maths teachers?

Uni Pathways Mark Scheme 2020 – maths-based assignments

	Subject Knowledge	Critical Thinking	Written Communication
1st	The work shows a depth of knowledge and understanding of key concepts and mathematical methods, drawing on sources of evidence where appropriate. Knowledge is used to build and support highly effective mathematical arguments and explanations.	Analyses the available data/evidence and identifies all the relevant mathematical information. Identifies and critically evaluates available mathematical approaches and evidence, deciding on their credibility, strength and relative significance. Makes connections between different pieces of information and puts them together to form accurate solutions and verify answers and assumptions.	Mathematical arguments are complete, and the reasoning behind each step has been clearly communicated to the reader. The work has a coherent flow and is well structured. The writing style is appropriate; mathematical language and key mathematical terms are used accurately and effectively to support the arguments and explanations made. There are no, or very few, errors in spelling or grammar. Consistent referencing, appropriate paragraphing and use of correctly labelled tables and figures; matching the style taught in the course.
2:1	The work shows an understanding of key concepts and mathematical methods, drawing on sources of evidence where appropriate. Knowledge is used to build and support effective mathematical arguments and explanations.	Analyses the available data/evidence and identifies all the relevant mathematical information. Engages with multiple approaches and critically evaluates their utility, forming judgements and using reasoning and evidence to inform problem solving strategy. Shows some understanding of the relative value of evidence and data.	Mathematical arguments are clearly made, with few gaps in logical reasoning. The work is well-structured. The writing style is appropriate; mathematical language and key terms are used correctly. There are few errors in spelling or grammar. Mostly consistent referencing and use of tables and figures; matching the style taught in the course.
2:2	The work shows an understanding of key concepts and mathematical methods, with no major misconceptions. Beginning to apply this knowledge to build and support effective mathematical arguments and explanations.	Identifies and uses basic mathematical evidence and reasoning. Evaluates the quality of mathematical reasoning and evidence, forming judgements. Not yet showing understanding of the relative value of evidence and reasoning.	Mathematical arguments have been constructed, but there is little explanation from step to step. The work has some structure. The writing style can sometimes be informal; occasionally key terms are not used when it would be appropriate to do so.

			There are some errors in grammar and spelling, but these do not get in the way of communicating the content. Referencing has some consistency; matching the style taught in the course. Limited use of tables and figures.
3rd	Shows a developing understanding of key concepts and mathematical methods, with some misconceptions. Not yet applying this knowledge to build and support mathematical arguments and explanations.	Beginning to analysis mathematical evidence and reasoning. Describes or attempts to solve the problem without any/or little evaluation of whether the approach was reasonable or accurate.	Mathematical arguments to solve problems are poorly sequenced, with little or no explanation. The grammar, spelling, style, and structure of the work need improving in order to communicate ideas to the reader. Key terms are not always used correctly. Limited, or no use of referencing, tables and figures.

Baseline Assignment (problem-set): Pupil Feedback Report

Name of Pupil	
Name of School	
Name of RIS teacher	
Title of Assignment	

How your assignment is graded:

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Marks

OVERALL MARK / 100		FINAL MARK / 100 (including any deductions)	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Mark Breakdown and Feedback

Subject knowledge	
	mark
Critical thinking	
	mark
Written communication	
	mark

Problem Set	
Comment	*mark*

Final Assignment (problem-set): Pupil Feedback Report

Name of Pupil	
Name of School	
Name of RIS teacher	
Title of Assignment	

How your assignment is graded:

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Marks

OVERALL MARK / 100		FINAL MARK / 100 (including any deductions)	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Mark Breakdown and Feedback

Subject knowledge	
	mark
Critical thinking	
	mark
Written communication	
	mark
Problem Set	
Comment	*mark*

Subject Vocabulary

Word	Definition	In a sentence
Integers	Whole numbers, either positive, negative or 0.	
Rational Numbers		
Irrational Numbers		
Real Numbers		
Recurring	Repeating; A recurring decimal is a decimal which has an endlessly repeating part.	
Contradiction		
Bias		

Launch Event

The launch event is your opportunity to learn more about this course and Uni Pathways. You will watch some videos, listen to your teacher and have some discussion. Below is some room to make notes for some of the videos you will watch during the event.

Virtual Campus Tours

Notes…

The Scholars Programme Alumni

Notes…

Student Ambassadors

Notes…

Tutorial 1 – A time to think *rationally*

What is the Purpose of Tutorial 1?

- Students can define both rational numbers and irrational numbers.
- Students can prove that some infinite decimal numbers are rational.
- Students can name examples of irrational numbers.

What types of number are there?

_____

_____

_____

_____

Convert the following decimals to fractions in their simplest form

0.1= 0.75= 0.125= 0.0001= 0.123456789=

Any decimal number that can be converted to a fraction is called a

_____ number.

When can we convert a decimal number to a fraction?

_____

_____

_____

_____

What do we mean by recurring?

$0.\dot{3} = 0.3333333333333\cdots$ $0.\dot{1}2\dot{3} = 0.123123123123123\cdots$

Fill in the proof

Let $x = 0.\dot{3} = 0.333333333\cdots$

Therefore $10x =$ _____

Therefore $10x - x =$ _____

Therefore $9x =$ _____

Therefore $x = \dfrac{\square}{\square} = 0.\dot{3} = 0.333333333\cdots$

Try some for yourself!

Let $x = 0.\dot{1} = 0.111111111\cdots$

Therefore $10x =$ _____

Therefore $10x - x =$ _____

Therefore $9x =$ _____

Therefore $x = \dfrac{\square}{\square} = 0.\dot{1} = 0.11111111\cdots$

Let $x = 0.\dot{0}\dot{9} = 0.0909090909\cdots$

Therefore $100x =$ _____

Therefore $100x - x =$ _____

Therefore $99x =$ _____

Therefore $x = \dfrac{\square}{\square} = 0.\dot{0}\dot{9} = 0.0909090909\cdots$

Challenges

Let $x = 0.\dot{1}2\dot{3} = 0.123123123123\cdots$

Therefore $1000x =$ _____

Therefore $1000x - x =$ _____

Therefore $999x =$ _____

Therefore $x = \frac{\square}{\square} = \frac{\square}{\square} = 0.\dot{1}2\dot{3} = 0.123123123123\cdots$

Let $x = 0.\dot{1}23\dot{4} = 0.1234123412341234\cdots$

Therefore $10000x =$ _____

Therefore $10000x - x =$ _____

Therefore $9999x =$ _____

Therefore $x = \frac{\square}{\square} = \frac{\square}{\square} = 0.\dot{1}23\dot{4} = 0.12341234\cdots$

Can all decimal numbers be converted to fractions?

_____

_____

_____

_____

_____

Can you think of any decimal numbers that cannot be converted into a fraction?

_____

_____

_____

_____

_____

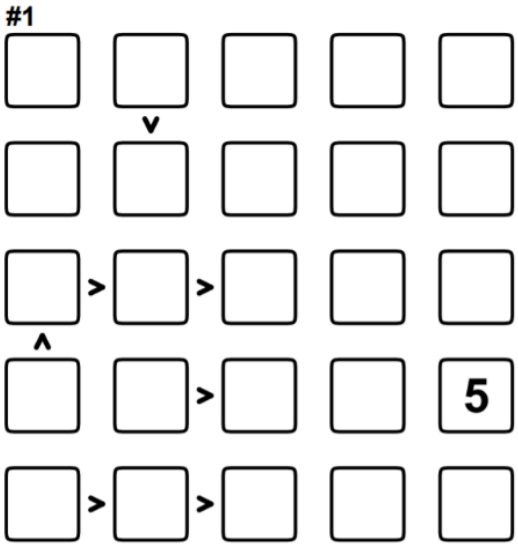
Baseline assignment question: **When the first examples of irrational numbers were discovered, there were some mathematicians (followers of Pythagoras) who refused to believe that they existed. Is this response justifiable?**	
Details of the assignment: Whilst in school, mathematics is taught as fact. However, in 400BC much of mathematics was still being defined. The introduction of the concept of irrational numbers caused uproar in the mathematical community, and had some disastrous consequences for those that claimed irrational numbers existed. In this essay, you are asked to question whether this is a natural response or one that is unacceptable, and provide your own opinion as to how justifiable this response is.	
Success criteria – how to structure your assignment and what to include: • An introduction to rational and irrational numbers (around 50 words) • An argument for and against why the reactions of the Pythagoreans was justified • You are encouraged to research the historical context for yourself • A conclusion including **your own opinion** as to whether maths is discovered or invented. Your work should be around 250 - 500 words.	
In this assignment, show me that you **understand**: • That you know the difference between rational and irrational numbers.	Show me that you **are able to**: • Use correct mathematical language • Demonstrate the ability to consider both points of view.

Problem Set

Recall that $0.\dot{1}2\dot{3} = 0.123123123123123123123123123123\cdots$

1. Write down the definition of a rational number and find an example of
   a. A rational number
   b. An irrational number
2. State whether the following decimal numbers are rational or irrational
   a. 0.25
   b. $0.\dot{2}$
   c. π (Pi)
   d. $0.\dot{1}2345678\dot{9}$
3. Prove that $\frac{2}{3} = 0.\dot{6}$ by taking $x = 0.\dot{6}$, and using the structure shown in tutorial.
4. Prove that $\frac{2}{11} = 0.\dot{1}\dot{8}$ by taking $x = 0.\dot{1}\dot{8}$, and using the structure shown in tutorial.
5. Find a decimal expression for the fraction $\frac{1}{7}$

Tutorial 2 – Proof by Contradiction



Source: krazydad.com

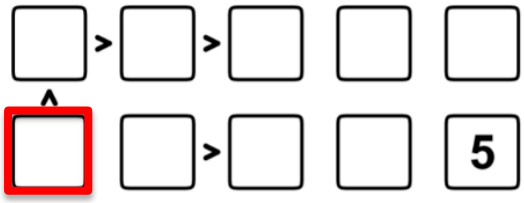
What is the Purpose of Tutorial 2?

- Understand that there are different methods of proof
- Students can prove the following using "*Proof by Contradiction*"
    - There are an infinite number of prime numbers
    - The square root of 2, $\sqrt{2}$, is irrational

Consider the Futoshiki above.

The numbers 1 to 5 must each appear once in each row and each column.
Wherever an arrow, <, sits between two squares, the arrow points to the smaller number.

- Complete the puzzle!
- Consider the third and fourth rows:



Discuss:

- What numbers ***can*** go in the red square?
- What happens if we put in each number?
- Which numbers give a **Contradiction?**
    - If any numbers do give a contradiction, what rule do they contradict?

What is a contradiction?

_____

_____

_____

_____

Come up with a contradiction to the following statements:

1. If a whole number n is one more than a multiple of 4, then n is prime.
2. No written number contains the letter 'a'.
3. Every country name (written in English) contains more than 1 vowel.

Proof by Contradiction

"Suppose for a contradiction"
Make a guess.

→

Deduce an obviously false statement

→

Contradiction!
The guess must have been incorrect!

Arrange the cards to find a contradiction

There are 2 sets of cards. Use them to prove that

- $\sqrt{2}$ is irrational
  - Start with: "Suppose for contradiction that $\sqrt{2} = \frac{a}{b}$, a rational number that is fully simplified"
- There are infinitely many prime numbers
  - Start with: "Suppose for contradiction that there are a finite number of prime numbers"

Once you have checked the order is correct, write out the proofs below.

$\sqrt{2}$ is irrational	There are infinitely many prime numbers
Suppose for contradiction that $\sqrt{2} = \frac{a}{b}$, a rational number that is fully simplified.	Suppose for contradiction that there are a finite number of prime numbers.

Tutorial 2 Homework

When completing these Futoshiki, try to take note of the times you use *contradictions* to solve the puzzle.

#1

4				
			5	2
>				
		<		
				^
>	3	>		

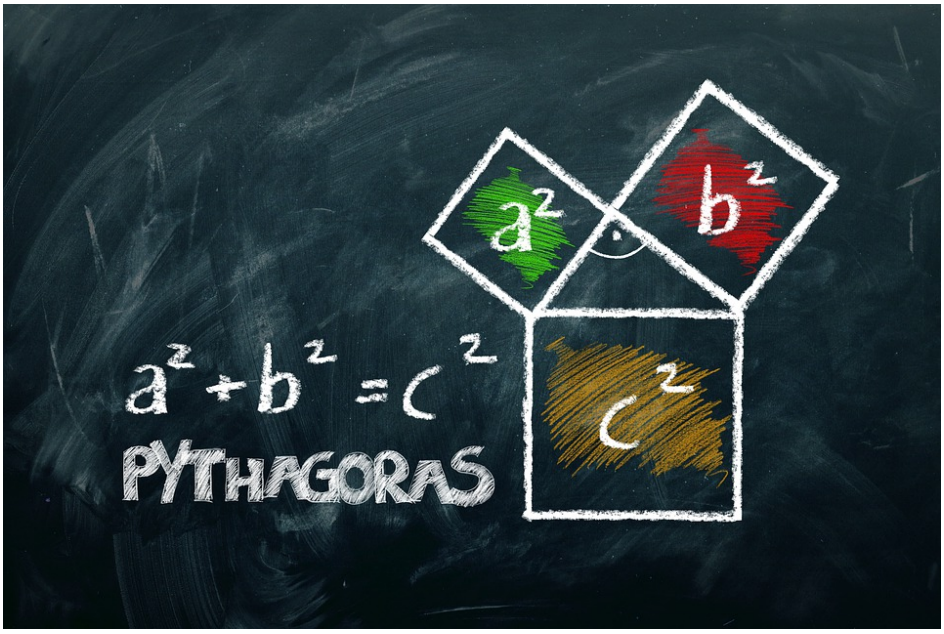
#2

		5		
v			v	
	^			
				1
<			>	
	<	<		

Source: krazydad.com

Prove by contradiction that $\sqrt{3}$ is irrational

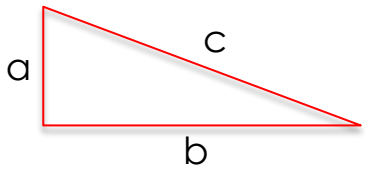
Tutorial 3 – Theorem vs Conjecture



What is the Purpose of Tutorial 3?

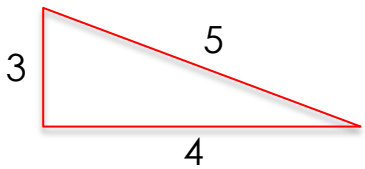
- To see some deductive proofs of Pythagoras' Theorem
- To learn the difference between Theorem and Conjecture
- To learn the statement behind Fermat's Last Theorem

What is Pythagoras' Theorem



"The square on the hypotenuse is equal to the sums of the squares on the other two sides"

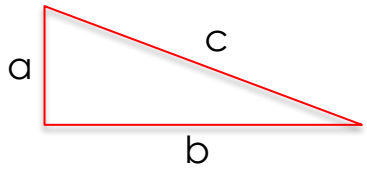
What does this mean?



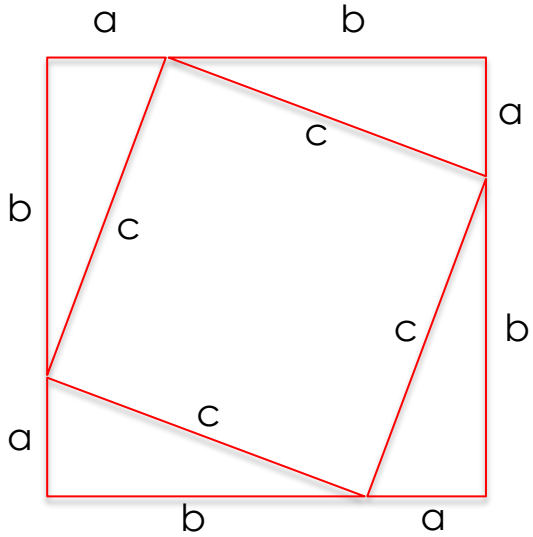
$^{2} + \quad ^{2} = \quad ^{2}$

A deductive proof of Pythagoras' Theorem

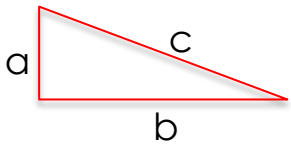
Consider this right-angled triangle



Put 4 copies of this triangle to form a square like so.

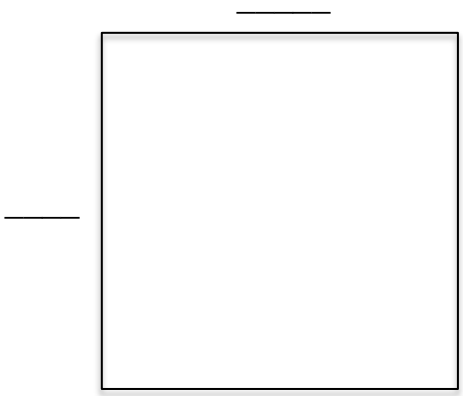


What is the area of each triangle?



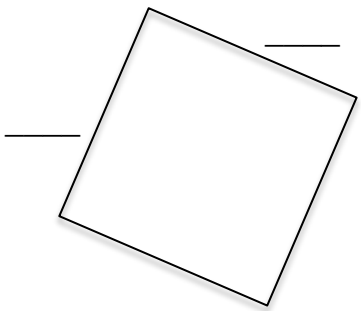
_____

What is the area of the larger square?

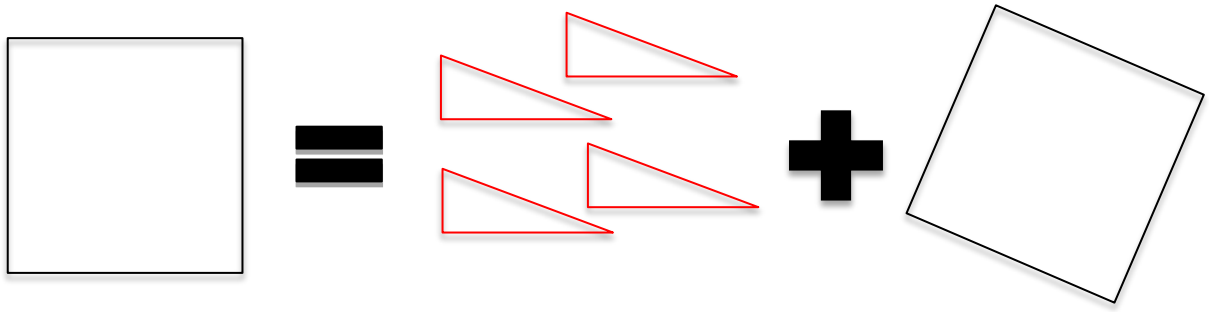


_____

What is the area of the smaller square?



_____



_____ = 4(_____) + _____

This equation simplifies to

$$a^2 + b^2 = c^2$$

We have now proved Pythagoras' Theorem.

What is a theorem?

Theorem:

"A formula, proposition, or statement in mathematics or logic deduced or to be deduced from other formulas or propositions"

- https://www.merriam-webster.com/dictionary/theorem

In your own words:

_____

_____

What is a Conjecture?

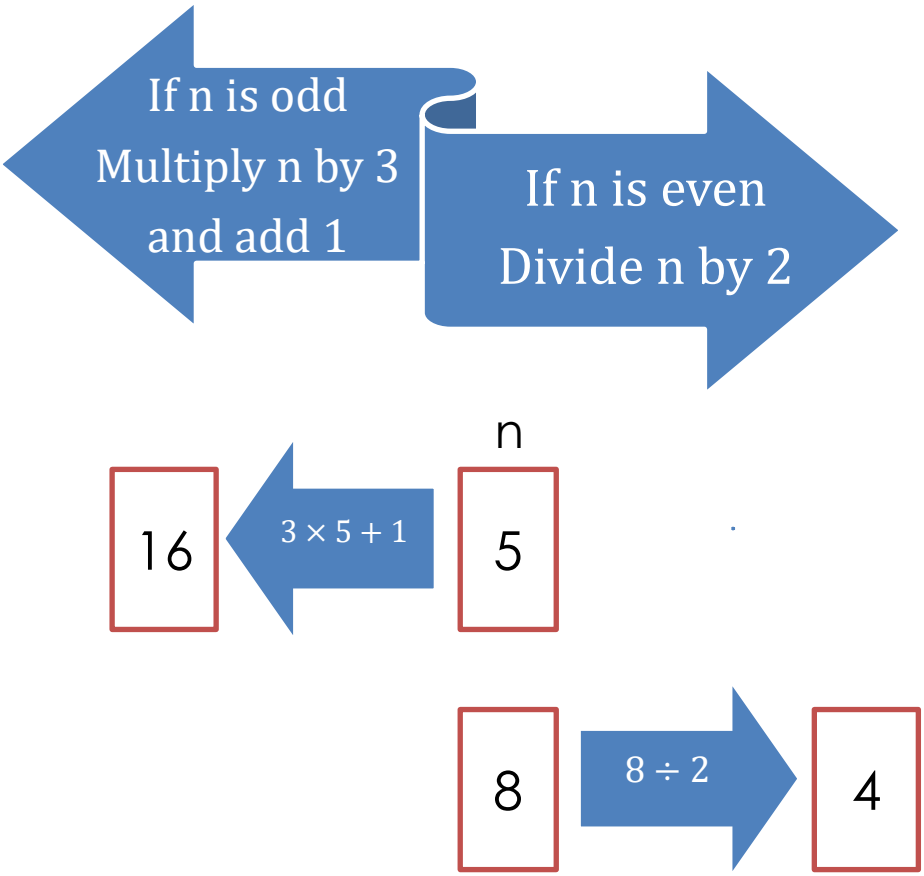
Conjecture:

_____

_____

The Collatz conjecture

Pick a whole number n which is bigger than 0. Then…



Once you have a new number, you can repeat the process.

The Collatz conjecture:
For any positive whole number n, this process will eventually reach the number 1.

Test this conjecture out on:

- 8 → 4 →
- 5 → 16 →
- 3 →
- 13 →

Tutorial 3 Homework:
Which of the following numbers takes the most steps in the Collatz conjecture to get to 1?

a) 9 b) 11 c) 17 d) 21 e) 24

Tutorial 4 – Fake news mathematics; bad proofs and intentional misuse

What is the Purpose of Tutorial 4?

- To see an example of an incorrect 'proof'
- To look at examples of incorrect mathematics in the media
- To be able to define the term 'Jargon'

Proving that 1=2

Let $a = 1$ and $b = 1$.

Step	Statement	What has happened?	True?
Step 1	$a = b$		✓
Step 2	$a^2 = ab$		
Step 3	$a^2 - b^2 = ab - b^2$		
Step 4	$(a + b)(a - b) = b(a - b)$		
Step 5	$b + b = b$		
Step 6	$2b = b$		
Step 7	$2 = 1$		✗

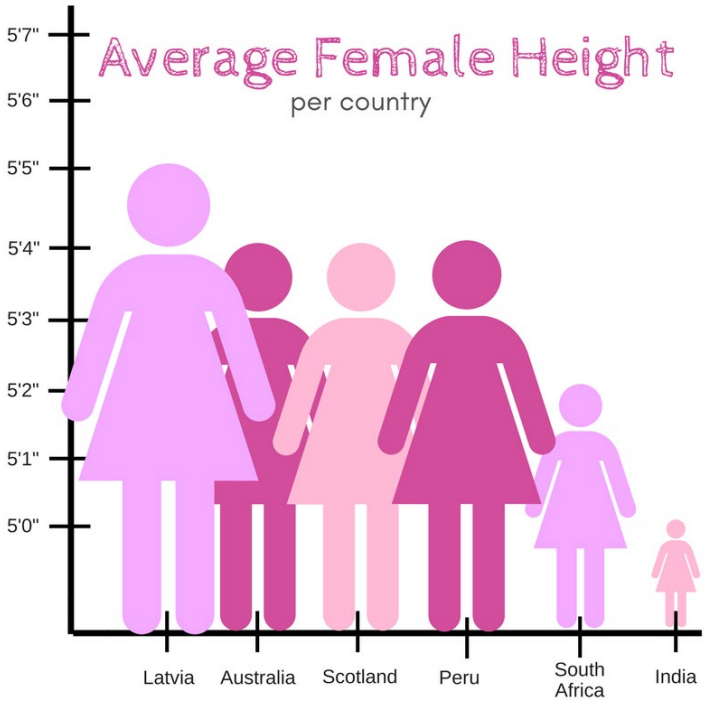
At which stage is the statement no longer true.

Misleading Maths in Media

What information is this graph telling us?

What works well about it?

What could be misleading?



How can we avoid being misled?

What do mathematicians do to avoid false proofs?

Homework: Find a piece of statistics that you have seen in the news/media and bring it to next lesson ready to analyse.

Tutorial 5 – School of Mathematical Lies?

What is the Purpose of Tutorial 5?

- To acknowledge some mistruths in mathematics we might once have believed.
- To explore how each subject is taught to include some simplifications
- To develop the skills of planning and writing essays

Discussion time

Has a maths teacher ever taught you something which was incorrect?

Has a teacher from another subject ever taught you something which was incorrect?

Why do you think anybody would teach something that isn't correct?

Final Assignment

Problem Set:

Recall that $0.\dot{1}2\dot{3} = 0.123123123123123123123123123123\cdots$

1. x is an even number if there is a whole number m such that $x = 2m + 1$. Prove that the sum of any two even numbers is an even number.

2. What does it mean for a number to be rational?

3. Prove that $\frac{7}{9} = 0.\dot{7}$ by taking $x = 0.\dot{7}$, and considering $10x$
4. Prove by contradiction that $\sqrt{7}$ is irrational.

5. Recall the Collatz conjecture

    a. What does the Collatz conjecture state will always happen eventually?
    b. Write the first 5 steps in the Collatz sequence starting with 18

6. Prove that the sum of three consecutive whole numbers is a multiple of 3

Final assignment question:

To what extent can we trust that the maths we are taught is correct?

Details of the assignment:

The subject of mathematics is perhaps the only subject taught as guaranteed fact, and not based on theories or simplified models. Using historical evidence and knowledge of the process in which we learn and develop mathematics, argue for and against the trustworthiness of the mathematics which you are taught at school.

Success criteria – how to structure your assignment and what to include:

- An introduction to how mathematics is developed using proofs.
- A balanced argument for and against the certainty that the maths we learn is correct.
- You should consider historical examples in which the mathematicians were incorrect, or did not know enough to get to an answer.
- A conclusion including **your own opinion** as to how much we can trust maths to be correct.
- Your work should be around 500 - 1000 words.

In this assignment, show me that you **understand**:	Show me that you **are able to**:
• That you know what makes a good proof or a bad proof. • At least one historical method of a proof • How mathematicians reference other mathematicians' work.	• Use correct mathematical language • Use examples of good and bad proofs • Demonstrate your own ability to consider both points of view

Tutorial 6 – Feedback tutorial

What is the Purpose of Tutorial 6?

- To receive feedback on your final assignment
- To respond to the feedback from your Uni Pathways teacher
- To write targets for improvement on your final assignment

Final assignment feedback from your Uni Pathways Teacher

(Remember to look at the mark scheme to help you understand what you have done well so far, and how you can do even better in your final assignment)

Here are three things that my Uni Pathways Teacher thought I did well in my draft assignment
•   •   •

Here are three things that my Uni Pathways Teacher thinks that I could do to get a higher mark in my final assignment
•   •   •

Tasks from my Uni Pathways Teacher to do during the feedback tutorial to help me improve
•   •   •
My response:

Actions I will take to improve my final assignment after this tutorial…
•   •   •

Hand in date for my final assignment:	

Tutorial 7 – Final tutorial

What is the Purpose of Tutorial 7?

- To receive feedback and a grade on your final assignment.
- To reflect on the programme including what you enjoyed and what was challenging.
- To ask any questions you may have about university.

Final assignment feedback from my Uni Pathways Teacher

Final mark:	University style grade:
Feedback: Here are three things that my Uni Pathways teacher thought I did well in my final assignment • • •	

Here are three things that my Uni Pathways teacher thinks I should remember for when I am doing this kind of study in the future
• • •

University

What questions do you still have about University after taking part in Uni Pathways?
•   •   •

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
•   •   •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Appendix 1 – Referencing correctly

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

Why should I reference?

Referencing is important in your work for the following reasons:

- It gives credit to the authors of any sources you have referred to or been influenced by.
- It supports the arguments you make in your assignments.
- It demonstrates the variety of sources you have used.
- It helps to prevent you losing marks, or failing, due to plagiarism.

When should you use a reference?

You should use a reference when you:

- Quote directly from another source.
- Summarise or rephrase another piece of work.
- Include a specific statistic or fact from a source.

How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. **A marker in your assignment**: After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].
2. **Bibliography**: This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

    a. **Websites** – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

    E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

    b. **Books** – Author, date published, title of book (in italics), pages where the information came from.

    E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

    c. **Articles** – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

    E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

Notes

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