Quantum teleportation

How is this a thing?

Key Stage 4 Programme

Pupil Name

Handbook Designed by Dr Pietro Liuzzo Scorpo

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1	Reading: *The man without a body.* Three-slide presentation about the Telepomp	
Tutorial 2	Three exercises about quantum superposition. One mini-essay about quantum superposition	
Tutorial 3	One exercise about bipartite states. **Optional** reading about quantum entanglement.	
Tutorial 4	Revision of all four tutorials. Schematic summary of each tutorial.	
Tutorial 5	Slide presentation about quantum teleportation.	

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after midnight on due date	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Uni Pathways – Pupil Feedback Report

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Name of RIS teacher			
Title of Assignment			
Name of Pupil			
Name of School			
ORIGINAL MARK / 100		FINAL MARK / 100	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the tutor should give an explanation in this section:

Knowledge and Understanding

Critical Evaluation/Problem Solving

Structure and Presentation

Resilience Comment

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Course Rationale

In this course we will talk about quantum physics, a branch of science which investigates nature at a very small scale: the *quantum world*. The extraordinary thing about this quantum world is that it escapes our common sense: quantum objects behaves in ways that we cannot imagine because we cannot experience them in our everyday life. For example, quantum objects can have one characteristic and its opposite at the same time: an electron can spin clockwise and counter-clockwise at the same time, or it can pass through two separated slits at the same time (it can be here and there simultaneously). Another difference with the world we experience every day, and that still puzzles physicists, is that when we measure a quantum object we change it. We can say that measurements are invasive actions that can change the very properties they are supposed to estimate. Last but not least, quantum systems composed by more quantum objects can be quantum correlated, which means that a measurement on one of the components affects the other components, it changes them; some quantum systems are so intertwined with each other (we say that they are *entangled*) that we cannot describe them separately, any description of a single component would be so incomplete to be worthless.

Thanks to these extraordinary properties, the quantum world has become the obvious playground for scientists to develop cutting-edge, sci-fi-like technologies, such as teleportation! Because of its pop-culture representations, the idea of teleportation is well rooted in our minds: it's the transfer of an object from one place to another without physically moving it; most of the times the object is de-materialized here to be re-materialized there. Well, the idea behind quantum teleportation is very similar, however, physicists don't teleport objects -they cannot teleport a chocolate bar nor a human(oid) being- but rather some of their physical properties, the "information" the physical object carries. The interesting thing, where the usefulness of teleportation really lies, is that the properties one teleports can be unknown to the sender, and to the receiver!

This may seem all very confusing, but don't worry: it *is* very confusing! If it weren't confusing we wouldn't need all these tutorials, right? We will tackle quantum mechanics and try the get away with the "juice", the main ideas behind it, and a better understanding of this technology which is finally dawning: quantum teleportation!

Mark Scheme

Skills	1st (70-100)	2:1 (60-69)	2:2 (50-59)
Knowledge and Understanding	• All content included is relevant to the general topic and to the specific question/title • Good understanding of all the relevant topics. • Scientific terms are defined and used accurately throughout • Clear justification on how the content included is related to the specific issues that are the focus of the assignment	• Most of the content included is relevant to the general topic and to the specific question/title • Good understanding of most the relevant topics • Scientific terms are used accurately but not always clearly defined. • Adequate justification on how the content included is related to the specific issues that are the focus of the assignment	• Some of the content included is relevant to the general topic and to the specific question/title • Good understanding on some of the relevant topics but occasional confusion on others. • Scientific terms are used mostly accurately with occasional confusion and often not defined. • Some justification on how the content included is related to the specific issues that are the focus of the assignment
Critical Evaluation / Problem solving	• Moved beyond description to an assessment of the value or significance of what is described • Evaluative points are consistently explicit, systematic, reasoned, justified • Effective critiques on the reliability of sources provided • Consistently demonstrate clear, analytical and logical steps to solving problems	• Mostly description but some assessment of the value or significance of what is described • Evaluative points are mostly explicit, systematic, reasoned, justified. • Some evidence of critiques on the reliability of sources provided • Some examples of solving problems but not consistently clear, analytical and logical	• Only description with minimal assessment of the value or significance of what is described • Evaluative points are at times explicit, systematic, reasoned, justified • Limited evidence of critiques on the reliability of sources provided • Demonstrate clear steps to solving problems but not consistently analytical or logical
Structure and Presentation	• Ideas are presented and arranged in a logical structure that is appropriate for the assignment • The introduction clearly outlines how the essay/presentation will deal with the issues	• Ideas are presented in paragraphs and arranged in a structure that is mostly appropriate for the assignment • The introduction adequately describes how the	• Ideas are presented in paragraphs and arranged in a structure • The introduction mentions how the essay/presentation will deal with the issues

	• The conclusion summarises all the main points clearly and concisely • All calculations, formulas and methods are clearly structured, clear to follow and correct • Tables and graphs are effectively constructed including appropriate headings, units and scales. • All sources are referenced correctly in an agreed format	essay/presentation will deal with the issues. • The conclusion summarises most of the main points clearly • Calculations, formulas and methods are mostly structured, clear to follow and correct • Most tables and graphs are well constructed • Most sources are referenced correctly in an agreed format	• The conclusion summarises some of the main points clearly • Calculations, formulas and methods are not always structured, clear to follow and correct. • Some tables and graphs are well constructed but contains some errors • Some sources are referenced correctly in the agreed format with occasional errors

Subject Vocabulary

Word	Definition	In a sentence
System	A physical object, or a set of physical objects regarded as a whole, whose physical characteristics (e.g. position, volume, mass, speed, spin, etc.) can be measured	The Solar **System** is composed by the Sun and by the objects orbiting it.
State	The description of a physical system with respect to some of its measurable properties. It is often used in the following expressions, which mean the same thing: "the state of the system is…" or "the system is in the state…"	The **state** of the bike is: 1km North-East from the School, moving at 10km/h towards West. We could summarize this by writing $B=(1,NE,10,W)$ where B represents "the state of the bike"
Discrete	Set of values, each separated from the others. We can numerate the values in the set using whole numbers.	- Can I get 8.22222 in my GCSE Maths? - Nope, you can only get a **discrete** value between 1 and 9.
Continuous	Set of values which cannot be separated from each other, therefore, these values cannot be numerated using whole numbers.	After the sun disappeared below the horizon, the temperature fell **continuously** (without jumps) by 10 degrees in less than one hour.
Simultaneous	Happening at the same time.	The swimmers won the gold medal for synchronised swimming by ending their routine jumping **simultaneously** out of the water.
Quantum	Adjective used to describe systems and/or physical phenomena which obey quantum rules. Usually, measurable quantities (and their exchange between physical systems) involved in these phenomena are discrete.	An electron is a **quantum** object, therefore it can exchange only discrete amounts of energy with the surroundings.
Superposition	When a physical quantum system is simultaneously in more than one state of some measurable quantity.	The electron's spin is in a **superposition** of up and down: it is behaving like it was spinning both clockwise and counter-clockwise at the same time.
State collapse	It occurs when a quantum superposition is measured: after the measurement the system is not in a superposition anymore, but its state is the state which corresponds to the measurement outcome.	We measured the z-component of the spin of an electron in a quantum superposition of up and down; since we obtained up, the state **collapsed** and it's now in the up state.
Bipartite	Composed by two parts (partitions).	We can consider the Earth-Moon system as a **bipartite** system: one partition is the Earth, the other partition is the Moon.
Correlation	A mutual relationship or connection between two or more things.	The **correlation** between his mood and the weather is evident: he is happy with a clear sky and grumpy when it rains.
Entangled state	Quantum superposition of bipartite states.	Given that the two spins are in an **entangled state**, if we measure one of them, we would cause the collapse of the other spin's state.
Teleportation	Disembodied (without using a "body", i.e. a physical object) transfer of information from a sender to a remote receiver.	Alice **teleported** the secret quantum state to Bob from Weston to Glasgow.
Protocol	A set of rules and procedure for carrying out a scientific experiment.	- I'm not sure of what I need to do to measure this. What if I do something wrong and ruin everything? - You have nothing to worry about: just follow the **protocol** to the letter and everything will be fine!

Knowledge Organiser

This is how we indicate a quantum state

$$|\psi\rangle = \sqrt{\frac{1}{4}}\,|\uparrow\rangle_z + \sqrt{\frac{3}{4}}\,|\downarrow\rangle_z$$

It can be written as a sum of states: it's a quantum superposition, in this case this is a superposition of up (counter-clockwise spinning) and down (clockwise spinning).

A rotation (when an object rotates around itself, this rotation is called spin) can be represented by a vector (a quantity with a direction and an orientation), depicted as an arrow. To represent the rotation with an arrow we must follow the right-hand-rule.

These are the probabilities of obtaining up or down when measuring the z-component of the spin

When we measure a superposition, the state collapses into one (and only one) of the possible states associated with the measurement outcomes. In the example above, the outcomes of the measurement (z-component of the spin) can be either up or down. If we obtain

UP (with probability 1/4=25%), the state after the measurement will simply be up

DOWN (with probability 3/4=75%), the state after the measurement will simply be down.

$$|\psi\rangle = |\uparrow\rangle_z$$

$$|\psi\rangle = |\downarrow\rangle_z$$

Using a Z-pulse we can flip the orientation of an electron's spin: if it's UP, after the Z-pulse becomes DOWN, and viceversa.

If we measure the z-component of this state we would get UP with certainty and the state would not change.

If we measure the z-component of this state we would get DOWN with certainty and the state would not change.

Using an X-pulse we can change the state of an electron's spin by making it point in a perpendicular direction: if it's UP in the z-direction, after the X-pulse becomes UP in the x-direction and viceversa

$$|\psi\rangle = |\rightarrow\rangle_x$$

$$|\psi\rangle = |\leftarrow\rangle_x$$

A state associated with a definite measurement outcome, i.e. not in a superposition with respect to that measurement, can be in a superposition with respect to another measurement!

$$|\rightarrow\rangle_x = \sqrt{\frac{1}{2}}|\uparrow\rangle_z + \sqrt{\frac{1}{2}}|\downarrow\rangle_z$$

$$|\uparrow\rangle_z = \sqrt{\frac{1}{2}}|\rightarrow\rangle_x + \sqrt{\frac{1}{2}}|\leftarrow\rangle_x$$

An **entangled state** is a bipartite quantum state for which the outcome of a measurement on one part is inextricably linked to the outcome of a measurement on the other part: if we measure the z-component of spin A of the system described by the bipartite state

$$|\psi_{AB}\rangle = \sqrt{\frac{1}{2}}|\uparrow_A\rangle_z\,|\downarrow_B\rangle_z + \sqrt{\frac{1}{2}}|\downarrow_A\rangle_z\,|\uparrow_B\rangle_z$$

we could obtain UP with probability 1/2=50%, or DOWN with probability 1/2=50%. If we obtain UP for spin A, any subsequent measurement of the z-component of spin B will yield DOWN with certainty.

This is because and entangled state is a quantum superposition of bipartite states. Therefore, measuring one subsystem will cause the collapse of the state on the corresponding bipartite state. For example, in the case of obtaining UP by measuring the z-component of spin A for the state above, the state after the measurement would be

$$|\psi_{AB}\rangle = |\uparrow_A\rangle_z\,|\downarrow_B\rangle_z$$

Bipartite states that are not in a superposition are not entangled, and they are called **separable states**. The states of the single partitions of a separable state can be in a superposition:

$$|\psi_{AB}\rangle = \left(\sqrt{\frac{1}{2}}|\uparrow_A\rangle_z + \sqrt{\frac{1}{2}}|\downarrow_A\rangle_z\right)\left(\sqrt{\frac{1}{2}}|\uparrow_B\rangle_z + \sqrt{\frac{1}{2}}|\downarrow_B\rangle_z\right) = |\rightarrow_A\rangle_x|\rightarrow_B\rangle_x$$

Tutorial 1 – Discrete, continuous and spinning

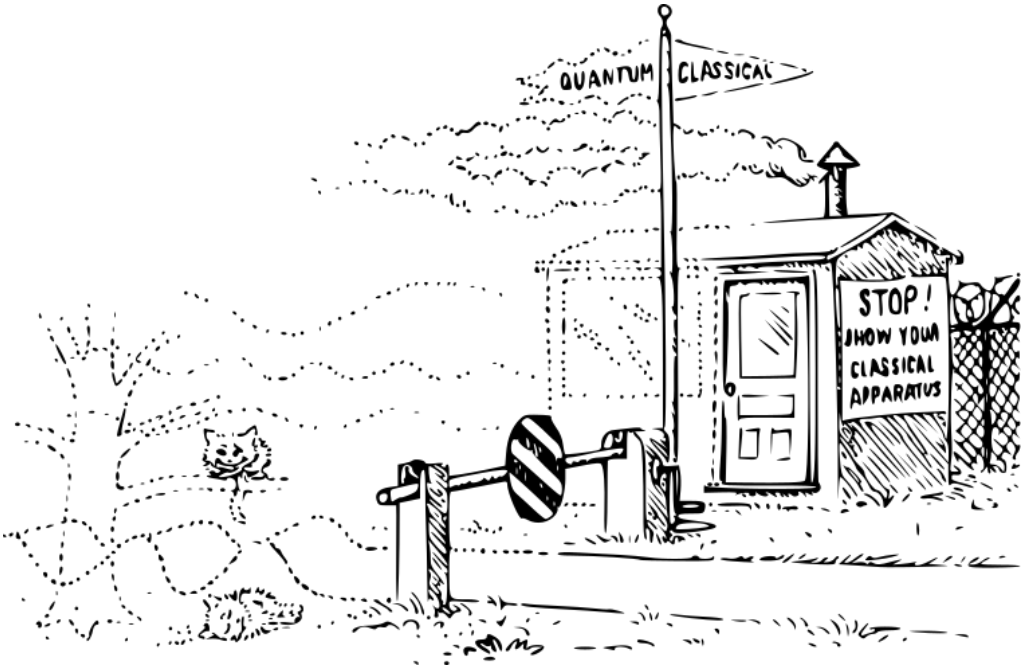


Figure 1, from the article *Decoherence and the transition from Quantum to Classical*, Wojciech H. Zurek, Physics Today, 44:36–44 (1991)

What is the Purpose of Tutorial 1?

- Understanding the difference between discrete and continuous quantities
- Representing rotations with vectors
- Understanding what "quantum" means

1.1 Measuring

Measuring is the act of obtaining a number as an answer to a question about an object without "working it out".
Example: Q: How long is the desk you are sitting at?

A: .....

To obtain the answer, you have to **measure** the *length* of the desk. The answer is the **measured quantity**.

1.2 Discrete VS Continuous quantities

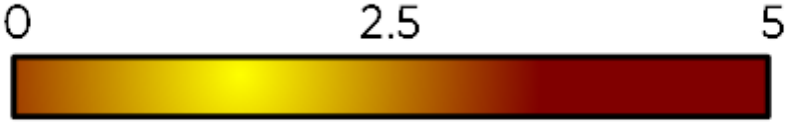
A measured quantity can be **discrete** or **continuous**. It is discrete if it consists in unconnected, distinct parts; in many circumstances a discrete quantity can be represented by a **whole number** (1, 10, 17 etc.). It is continuous if it can extend without a break and it can be represented by a number with decimal figures (1.234, 10.0, 17.3333333 etc.).

What is your level of understanding of what a **discrete quantity** is? Circle your answer.

I have no idea of what you are talking about, Sir! 0 1 2 3 4 5 Of course I understand what it is! What kind of question is this, Sir?

What is your level of understanding of what a **continuous quantity** is? Indicate your answer with a vertical line on the bar below

I have no idea of what you are talking about, Sir!



Of course I understand what it is! What kind of question is this, Sir?

What could you measure?

Table

Continuous quantities

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Discrete quantities

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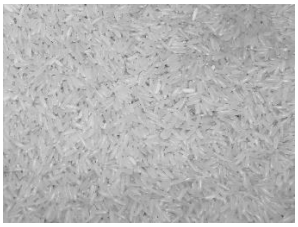
Handful of rice



Continuous quantities

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Discrete quantities

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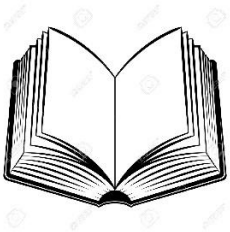
Book



Continuous quantities

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Discrete quantities

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Lego tower

Continuous quantities

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Discrete quantities

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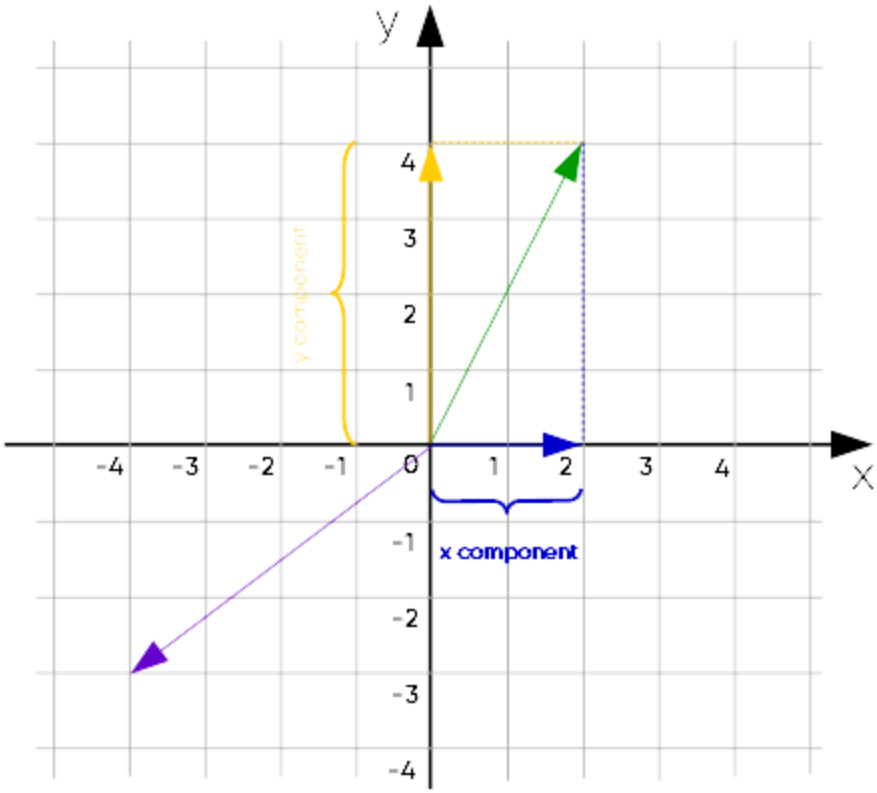
1.3 Vectors and rotations

A **vector** is a quantity with a direction. It is usually represented by a straight arrow. Given a reference frame (x and y axis for example), we can identify the **components** of the vector in this reference frame:

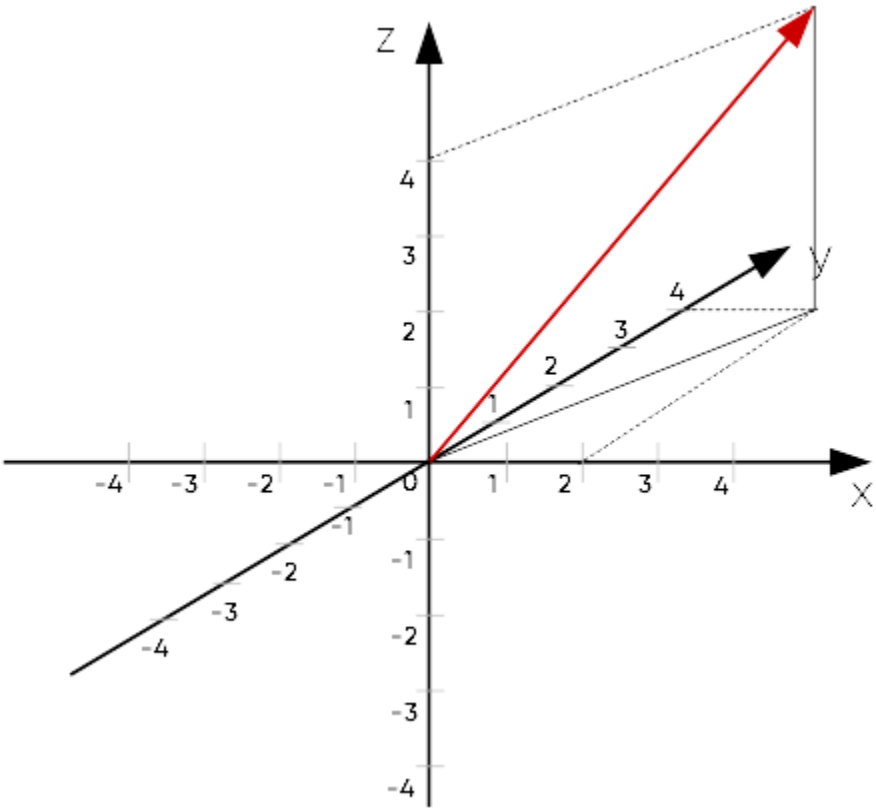


In the figure above the x component of the green vector is 2, while its y component is .....
What are the x and y components of the purple vector?

x component = ... y component = ...

We can reconstruct a vector if we know its components!

Exercise: Complete the plot below with the missing green and blue vector

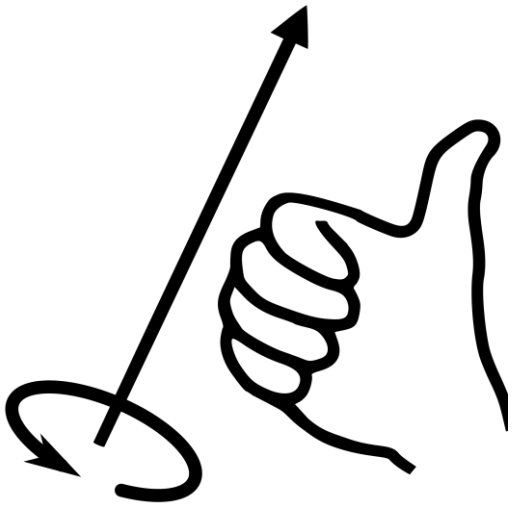


Red vector:
x component: 2
y component: 4
z component: 4

Green vector:
x component: 1
y component: -2
z component: -3

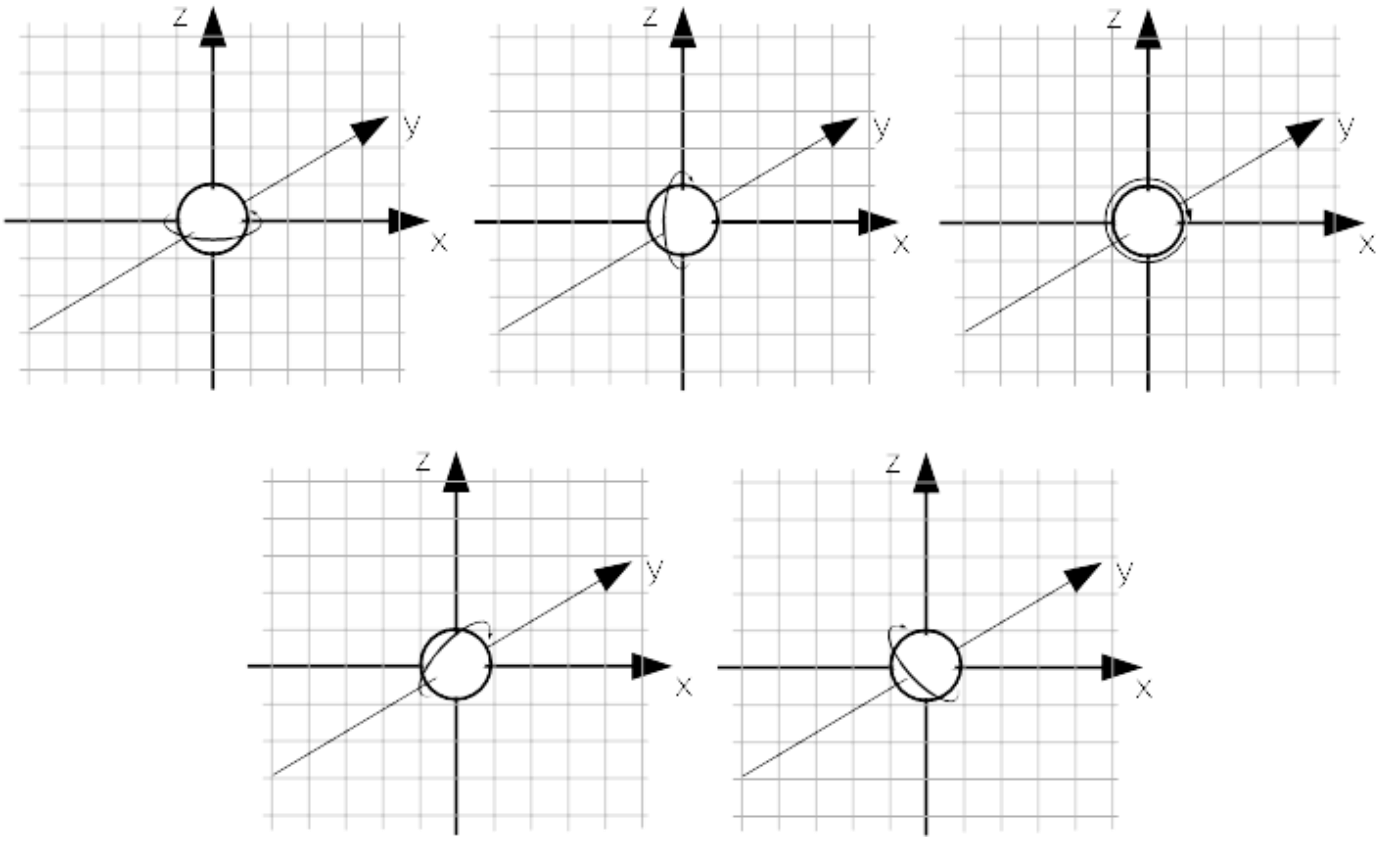
Blue vector:
x component: -2.5
y component: 1.5
z component: 0

It is possible to represent the rotation of an object with a vector by using the *right hand rule*.



The line along which the object rotates is called **rotation axis**.
This vector is called **angular momentum**.
The longer the arrow, the faster the object is rotating.

Exercise: Draw the angular momentum for each of the following rotations. Use the right hand rule!



Question: Do you think the angular momentum components are discrete or continuous quantities?

1.4 What's "quantum"?

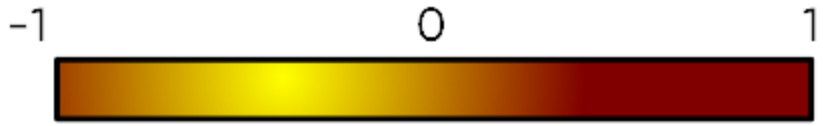
Quantum physics is a branch of science which studies very small objects: atoms, sub-atomic particles and light. The word "quantum" comes from latin and it means "how much". This is because, at these small scales, all the measured quantities are discrete! In other words, we can say that the result of a measurement will always be a multiple of some unit: physical quantities come in 'indivisible 'packets", the **quanta**.

Very small objects obey different rules than big objects, we can say that they behave in a quantum manner. These **quantum rules** are very different to the rules of our big, macroscopic world! The rules of quantum physics are surprising and, most of the time, counter-intuitive!

Follow this reasoning and complete the last bullet point:

- Sub-atomic particles, like **electrons**, have an angular momentum called **spin**.
- Sub-atomic particles obey **quantum laws (rules)** and are therefore considered **quantum objects**.
- When we measure a quantum objects we obtain a **discrete** quantity.
- The best way to take note of the spin of a sub-atomic particle is (pick one from the two images below: A or B)

A



B

Baseline assignment

Read the following extract from the short story *The man without a body*, written in **1877** by Edward Page Mitchell, an American editorial and short story writer. This story is considered to be perhaps the **first** sci-fi story to depict the teleportation of objects and living beings by using a device constructed specifically for this purpose.

"I promised to tell you how I happen to be a Man without a Body. You see that some three or four years ago I discovered the principle of the transmission of sound by electricity. My telephone, as I called it, would have been an invention of great practical utility if I had been spared to introduce it to the public. But, alas-"

"Excuse the interruption," I said, "but I must inform you that somebody else has recently accomplished the same thing. The telephone is a realized fact."

"Have they gone any further?" he eagerly asked. "Have they discovered the great secret of the transmission of atoms? In other words, have they accomplished the Telepomp?"

"I have heard nothing of the kind," I hastened to assure him, "but what do you mean?"

"Listen," he said. "In the course of my experiments with the telephone I became convinced that the same principle was capable of indefinite expansion. Matter is made up of molecules, and molecules, in their turn, are made up of atoms. The atom, you know, is the

unit of being. The molecules differ according to the number and the arrangement of their constituent atoms. Chemical changes are effected[1] by the dissolution[2] of the atoms in the molecules and their rearrangements into molecules of another kind. This dissolution may be accomplished by chemical affinity or by a sufficiently strong electric current. Do you follow me?"

"Perfectly."

"Well, then, following out this line of thought, I conceived a great idea. There was no reason why matter could not be telegraphed, or, to be etymologically[3] accurate, 'telepomped.' It was only necessary to effect at one end of the line the disintegration of the molecules into atoms and to convey the vibrations of the chemical dissolution by electricity to the other pole, where a corresponding reconstruction could be effected from other atoms. As all atoms are alike, their arrangement into molecules of the same order, and the arrangement of those molecules into an organization similar to the original organization, would be practically a reproduction of the original. It would be a materialization–not in the sense of the spiritualists' cant[4], but in all the truth and logic of stern science. Do you still follow me?"

"It is a little misty," I said, "but I think I get the point. You would telegraph the Idea of the matter, to use the word Idea in Plato's sense[5]."

"Precisely. A candle flame is the same candle flame although the burning gas is continually changing. A wave on the surface of water is the same wave, although the water composing it is shifting as it moves. A man is the same man although there is not an atom in his body which was there five years before. It is the form, the shape, the Idea, that is essential. The vibrations that give individuality to matter may be transmitted to a distance by wire just as readily as the vibrations that give individuality to sound. So I constructed an instrument by which I could pull down matter, so to speak, at the anode and build it up again on the same plan at the cathode. This was my Telepomp."

"But in practice–how did the Telepomp work?"

*"To perfection**!** In my rooms on joy Street, in Boston, I had about five miles of wire. I had no difficulty in sending simple compounds, such as quartz, starch, and water, from one room to another over this five- mile coil. I shall never forget the joy with which I disintegrated a three-cent postage stamp in one room and found it immediately reproduced at the receiving instrument in another. This success with inorganic matter emboldened me to attempt the same thing with a living organism. I caught a cat–a black and yellow cat–and I submitted him to a terrible current from my two-hundred-cup battery. The cat disappeared in a twinkling. I hastened to the next room and, to my immense satisfaction, found Thomas there, alive and purring, although somewhat astonished. It worked like a charm."*

Prepare a three-slide presentation about the Telepomp. A slide presentation is composed by two main parts: the **slides** and the **explanation**. The slides must be just a graphical prompt for the explanation: they should help the audience follow the reasoning of the speaker and get a better grasp of the presented concepts. For this assignment you will need to write the explanation rather than presenting it (you can think of it as a script).

Slides - In each of the slides, draw a schematic representation of the ideas you want to present. You can write labels in your slides, but, as a general rule, you should not to write too much on them: you want the audience to listen to you, not to get lost reading what's written on the slides and not paying attention to the explanation. In some cases, it is necessary to write a sentence on a slide. Even in these cases though, keep it simple, with only the strictly necessary information. The only slide where you can write in full sentences is the last one, where you are asked to express your original thoughts. However, also in this case, keep your sentences simple and organize them in a bullet point list.

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[1] Effected: executed, produced, or brought about
[2] Dissolution: the undoing or breaking of a bond
[3] Etymology: the derivation of a word. In this case, *telepomp* etymologically mean "sending far away": from Greek *tele* "far off, afar, at or to a distance" + *pomp* from Greek *pompe* "sending away,"
[4] Cant: insincere, false, or hypocritical statements:
[5] An "idea" for Plato is an eternal, changeless and abstract entity. The objects and phenomena we observe are just a "reflection", an "expression" of their idea.

Explanation – Script what you want to say in the space below the slides. You will find some questions that will help you structure your explanation. The explanation should refer to what is depicted in the slides and guide the audience's understanding.

You will be evaluated on your

- **understanding**: how the content in the explanation is relevant to the slides titles and to the questions underneath; how appropriately you define and use specific terms used in the text;
- **critical evaluation**: how consistent, clear and logical your evaluative points, in particular those expressed in the last slide, are;
- **structure and presentation**: how well the explanation refers to the slide; how clear and coherent the slides are;
- **spelling, punctuation and grammar.**

For a more detailed mark scheme, see page 6.

The Telepomp device

How many parts is the Telepomp device made of?
What are these parts?
How are they connected with each other?
Information about the device is scattered all through the text, but most of it can be found in the last paragraph: lines 49-57.

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Operating principles of the Telepomp

What is the purpose of each part of the Telepomp device?
What happens, at the *anode*, to the object that one wants to "telepomp"?
What happens at the *cathode*, i.e. the receiving end?
Information about the operating principles of the Telepomp is scattered all through the text, but most of it can be found between lines 14 and 33.

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Is the Telepomp feasible?

Feasible: possible and practical to do easily or conveniently.
Read again from line 38 to line 45: can you say that the object at the cathode **is** the same object that was at the anode before this was Telepomp? What is transmitted through the cable?
Do you think such a technology could be built now or in the foreseeable future? Where do you think the biggest obstacle to the construction of the Telepomp device lies?

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Recommended revision for the next tutorial:

- Probability.
- Mutually exclusive events.
- How to transform fractions into percentages.

Tutorial 2 – Quantum measurements and superposition

Anyone who is not shocked by quantum theory has not understood it.

Niels Bohr, one of the forefathers of Quantum Physics

What is the Purpose of Tutorial 2?

- Understanding how quantum measurements differ from classical ones
- Understanding quantum superposition
- Learning the Dirac notation

2.1 Measuring quantum objects

Let's suppose that we have a closed box with a rotating (non-quantum) sphere inside (whose radius and mass are known). What questions would you ask in order to be able to reconstruct from the answers to these questions the angular momentum of the rotating sphere spinning in the box? Or, in other words, what do you need to measure?
Hint: You only need three questions!

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What answers do you expect if, after you received the answers to the above questions, you ask the same questions again?

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Let's suppose now that in our closed box there is a quantum particle, for example an electron. In this case, we cannot use the same strategy as in the case above (from now on, we will refer to non-quantum objects and laws as **classical**). Why? Let's see an example:
We know that in the box there is an electron whose spin components can only be +1 or -1 (the result of a measurement of a quantum object is discrete, remember?).
We decide to start by measuring the *z* component, and we obtain

+1

We then measure the *x* component:

+1

Finally, we measure the *y* component and we get

−1

Since this is our first time measuring the spin of an electron, and we are not sure we did everything correctly, we decide to repeat the measurements, just to be sure. So we measure the *z* component again and this time we obtain:

−1

But that is different from our first result! We measure it again:

−1

Which confirms our last measurement. We measure it again, just to be sure, and we obtain

−1

Quite confident on our *z* measurement, we measure the *x* component again:

+1

Which confirms the first measurement. We then check the *y* component:

+1

Also in this case, as for the *z* measurement, the result of this second measurement contradicts the first. We therefore decide to repeat it again

+1

and again

+1

Out of curiosity, we measure again the *z* component and

+1

What's going on here??? We continue measuring and measuring. Look at the table below for a summary of all our measurements (the first row tells us what component we decided to measure, the second row is the obtained measurement result)

z	x	y	z	z	z	x	y	y	y	z	z	x	x	x	y	y	x	x	z
+1	+1	−1	−1	−1	−1	+1	+1	+1	+1	+1	+1	−1	−1	−1	−1	−1	+1	+1	+1

Question: What do you think it's happening? Can you notice any pattern in this series of measurements?

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What would you predict for the following series of measurements?

x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
+1																			

z	z	z	z	z	z	z	z	z	z	z	z	z	z	z	z	z	z	z	z
+1																			

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2.2 Quantum superposition

To fully understand what is going on, we have to talk about another very curious property of quantum systems: **quantum superposition**. What do we mean by this?
We just discovered, in the previous section, that once we measure one spin component, all subsequent measurement of the same component will give the same result. In the last example in the previous page, we measured the z-component and we obtained +1. It doesn't matter how many times we measure it, **if we don't measure any other components**, we will always obtain +1. This means that the z-component of the electron's spin **is** +1, which means that the electron is spinning counter-clockwise. In this case we say that

The quantum state of the electron's spin is up in the z direction.

We can write this mathematically as (Dirac notation)

$$|\psi_{\text{electron}}\rangle = |\uparrow\rangle_z$$

This is a greek letter which reads **psi**

At this point we may ask: What is the quantum state of the electron's spin in the x direction? Well, this is where things get interesting (and a bit complicated, but don't worry, remember Bohr's sentence!).

$$|\psi_{\text{electron}}\rangle = |\uparrow\rangle_z = \sqrt{\frac{1}{2}}|\rightarrow\rangle_x + \sqrt{\frac{1}{2}}|\leftarrow\rangle_x$$

This reads as

> *The quantum state of the electron's spin "up in the z direction"* $|\uparrow\rangle_z$ *corresponds to a* ***superposition*** *of up* $|\rightarrow\rangle_x$ *and down* $|\leftarrow\rangle_x$ *in the x direction.*

But what does this mean? And what are those numbers?!?!

2.3 Measuring a quantum superposition

When the electron's spin state is in a superposition of up and down in one of the three directions, then we can say that in that direction **the electron is spinning clockwise and counter-clockwise simultaneously.** However, we have seen that if we measure the spin component in that particular direction we obtain only one of the two: either +1 or -1.
Which one?
It's a matter of chance. We could find that it's spinning clockwise (=down=-1) with a certain **probability** and counter-clockwise (=up=+1) with another probability! The probability with which we obtain either up or down is given by the number into the square root in front of that state.
In the example above,

$$|\psi_{\text{electron}}\rangle = |\uparrow\rangle_{\text{z}} = \sqrt{\frac{1}{2}}|\rightarrow\rangle_{\text{x}} + \sqrt{\frac{1}{2}}|\leftarrow\rangle_{\text{x}}$$

when measuring the z-component of the electron's spin we have that the probability of obtaining counter-clockwise (=+1) is *P(+1)=1*, i.e. it is a certain event. On the other hand, when we measure the x-component of the electron's spin, we have an equal probability of finding clockwise or counter-clockwise: the numbers into the square roots are equal to 1/2=0.50=50%.

A state associated with a definite measurement outcome, i.e. not in a superposition with respect to that measurement (the z-component of the spin in the example above), **can be in a superposition with respect to another measurement** (the x-component of the spin in the example above)!

Look at the state below. Suppose we want to measure the spin in the x direction. Is it more likely that we'll get +1 (=up=counter-clockwise) or -1 (=down=clockwise)? Can you say what are the probabilities of getting one or the other?

$$|\psi_{electron}\rangle = \sqrt{\frac{3}{4}}|\rightarrow\rangle_x + \sqrt{\frac{1}{4}}|\leftarrow\rangle_x$$

P(+1)=..... P(-1)=.....

Now, we remember from Sections 2.1 and 2.2 that, after we measured a certain component, the outcome of **subsequent measurements of the same component** would yield the same outcome. Why is this? Let's suppose that we measure the x-component of the spin of the electron described by the state above and that we obtain +1. Since, as we just said, any subsequent x-component measurement will yield the same outcome, +1, with certainty (i.e. *P(+1)=1*), we can write the state of the system after the first measurement as

$$\psi_{\text{electron}}^{\text{after meas }+1}\rangle = \sqrt{1}\ |\rightarrow\rangle_x = |\rightarrow\rangle_x$$

Has the state of the system changed after the measurement?!? Yes! **We can think that the state of the system has changed exactly because of the measurement!** We say that, because of the measurement, the state of the electron **has collapsed** into one of the two directions!

$$|\psi_{\text{electron}}\rangle = \sqrt{\frac{3}{4}}|\rightarrow\rangle_x + \sqrt{\frac{1}{4}}|\leftarrow\rangle_x \quad \Rightarrow \quad \psi_{\text{electron}}^{\text{after meas }+1}\rangle = |\rightarrow\rangle_x$$

What actually happens when we measure a quantum superposition is still a mystery! Physicists are still very puzzled by this "collapse". There are many possible ways to explain it, all equivalent in the predictions, but very different for the philosophical implications. Each of these explanations gave rise to a certain **interpretation of quantum mechanics**. The most famous ones are the so-called ***Copenhagen Interpretation*** and the ***Many-World Interpretation***. According to the former, the collapse of the quantum state is caused by the very action of measuring[6]; in particular, it is caused because of the interaction of a macroscopic object, the measuring apparatus, with a quantum one, the measured system. On the other hand, for the Many World Interpretation, the collapse doesn't take place, however, whenever a quantum measurement is carried out, the "Universe splits" into parallel realities: one for each measurement outcome!

[6] In this course we are following the Copenhagen approach: we asserted several times that "measurements change the state of the measured system". Even though I think that this is the easiest way to introduce quantum mechanics to new learners, I prefer to maintain an ***agnostic*** position with regards to interpretations of quantum mechanics: I don't know which interpretation is the "correct" one and I think I will probably never know. I think that quantum mechanics is a successful scientific theory because it can be used to predict experimental results, and this is what matters.

Homework

Exercise 1: Knowing that if the electron's spin state is up in the x-direction then it is in the following superposition of up and down states in the z-direction,

$$|\rightarrow\rangle_x = \sqrt{\frac{1}{2}}|\uparrow\rangle_z + \sqrt{\frac{1}{2}}|\downarrow\rangle_z$$

and recalling that the z-up states is a superposition of up and down states in the x-direction,

$$|\uparrow\rangle_z = \sqrt{\frac{1}{2}}|\rightarrow\rangle_x + \sqrt{\frac{1}{2}}|\leftarrow\rangle_x$$

explain this series of measurements (suppose that the initial state of the electron's spin is up in the z direction):

z	x	z	x
+1	+1	+1	-1

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Exercise 2: Write the probability of obtaining +1 and the probability of obtaining -1 when measuring the z component of the spin of an electron when the electron is in the states

$$|\psi_{\text{electron}}\rangle = \sqrt{\frac{4}{5}}|\uparrow\rangle_z + \sqrt{\frac{1}{5}}|\downarrow\rangle_z$$

P(+1)=..... P(-1)=.....

$$|\psi_{\text{electron}}\rangle = \sqrt{\frac{1}{3}}|\uparrow\rangle_z + \sqrt{\frac{2}{3}}|\downarrow\rangle_z$$

P(+1)=..... P(-1)=.....

$$|\psi_{\text{electron}}\rangle = \sqrt{\frac{3}{10}}|\uparrow\rangle_{\text{z}} + \sqrt{\frac{7}{10}}|\downarrow\rangle_{\text{z}}$$

P(+1)=..... P(-1)=.....

Exercise 3:
a) Write down a state of an electron in a superposition of up and down (in the **z-direction**) such that the probability of measuring up is P(+1)=3/5, and the probability of measuring down is P(-1)=2/5:

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b) Write down a state of an electron in a superposition of up and down (in the **x-direction**) such that the probability of measuring up is P(+1)=1/42, and the probability of measuring down is P(−1)=41/42:

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Exercise 4: Can you explain the comic strip below?

Help: "exists in a simultaneous state" = "is in a quantum superposition"
"observe" = "measure"

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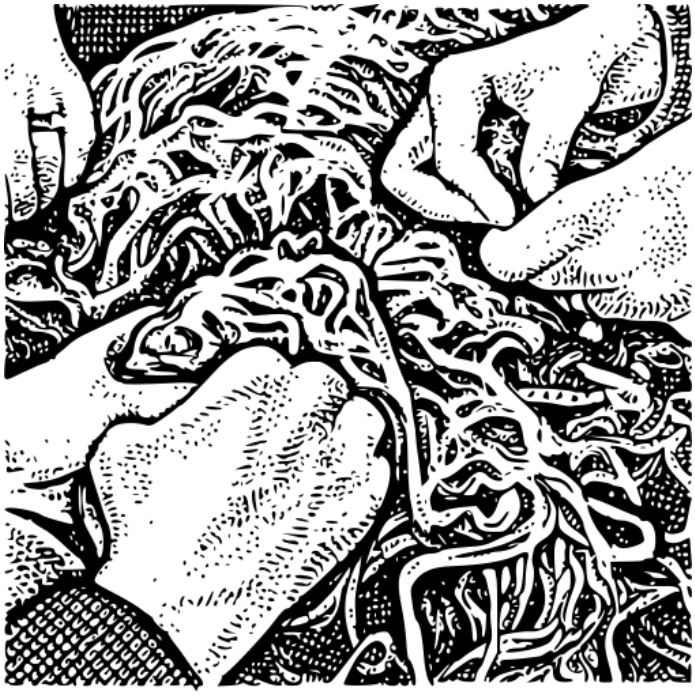
Tutorial 3 – The "spooky action at distance"

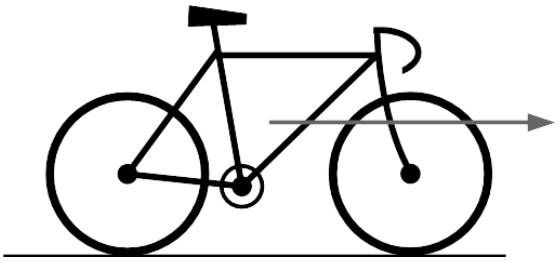


What is the Purpose of Tutorial 3?

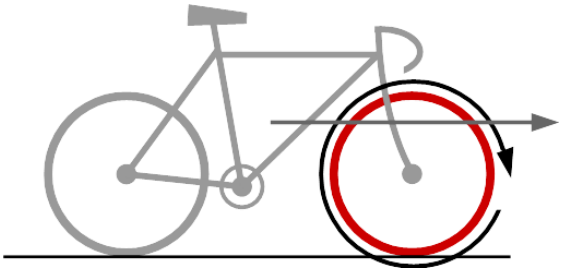
- Understanding what correlations are
- Understanding what quantum entanglement is

3.1 Bipartite systems

Picture a **moving** bike in your mind.



This is made of many different pieces. We can think of the bike as a whole, or we can think of the individual pieces. For example, we can focus our attention on the front wheel only.



What aspects of this wheel can we measure?

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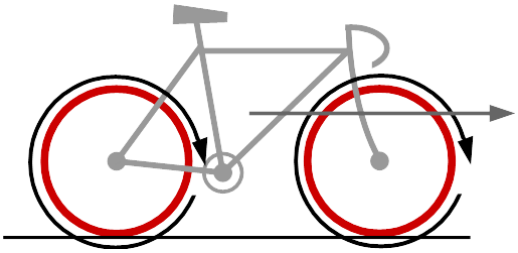
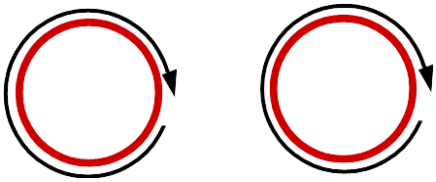
Now, instead of focusing on the whole bike, or on a single wheel, we focus on both the front and the back wheels at the same time.



Now, with some imagination we make the bike's frame, the street and the rest of the universe, become invisible such that we can see only the two wheels spinning.



This is a **bipartite system**, i.e. an object composed by two parts. Like the bike, we can think of it as a whole, but it is composed of two pieces which we can observe independently.

3.2 Correlations

At this point we invite our friends, Alice and Bob, to participate in an experiment. We tell Alice to observe and measure the rotation of one of the two wheels, let's say the front one, and we tell Bob to do the same with the other wheel. We don't tell them that the two wheels are part of the same (invisible) bike moving on the (invisible) road, rather we ask them if they can find anything interesting about these two rotating wheels.
They accept the challenge and immediately start to measure the rotation of their respective wheels. After a bit, they are very puzzled "What can possibly be interesting in a spinning wheel? Sometimes it spins faster, sometimes slower. What is this all about?" Bob asks. Alice nods at first, agreeing with her friend, but then her eyes open wide and she asks Bob "I think I figured it out! What are the results of your measurements?'.
They start comparing their measurements and after a couple of minutes Alice exclaims "You got an invisible bike! Cool!". Bob is very surprised "How do you know?"
Can you explain to Bob how Alice figured it out?

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If they kept their results for themselves they wouldn't have noticed anything interesting: it would have appeared to them just a sequence of random numbers. By comparing their measurements results, Alice found something much more interesting than just a bunch of numbers.
Alice's and Bob's measurement results are indeed connected. We say that they are **correlated**.

After this ride we finally move back into the quantum realm. Brace yourself!

3.3 Superposition of bipartite quantum states

We can have quantum bipartite systems too! Two electrons, for example. Like the two wheels, we can think of these two electrons as a whole but we can focus our attention only on one of the two if we want.
We have seen in the previous tutorials that if we measure the spin of an electron in, for example, the z direction we can obtain either +1 (up) or -1 (down), and this tells us that the electron's state can be up, down, or a superposition of up and down.
When we measure the z component of the spin of two electrons, let's call these electrons A ("A" of Alice!) and B ("B" of Bob!) we have four possible results.

		SPIN B	
		+1	−1
SPIN A	+1	+1 +1	+1 −1
	−1	−1 +1	−1 −1

Like for the single electron case, this tells us that the electrons' state (i.e. the state of the whole, bipartite system!) can be:

both electrons up

$$|\psi_{\mathrm{AB}}\rangle = |\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

electron A up and electron B down

$$|\psi_{\mathrm{AB}}\rangle = |\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

electron A down and electron B up

$$|\psi_{AB}\rangle = \ldots\ldots\ldots\ldots\ldots\ldots$$

Both electrons down

$$|\psi_{AB}\rangle = \ldots\ldots\ldots\ldots\ldots\ldots$$

or a superposition of these four

$$|\psi_{\mathrm{AB}}\rangle = \sqrt{\frac{1}{2}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{8}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{8}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{4}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

3.2 Measuring a superposition of bipartite quantum states

We measure the z component of the two spins when their state is described by the superposition above and we find that they are both up. What was the probability of obtaining this result? (Have a look at Exercise 2 in your last homework if you need to remember how to find the probability of obtaining a certain outcome when measuring a quantum superposition)

P(A=+1, B=+1) = ...

What do you think the state of the bipartite system is after this measurement?

$$|\psi_{AB}\rangle = \ldots\ldots\ldots\ldots\ldots\ldots$$

Let's consider now the following bipartite quantum superposition

$$|\psi_{\mathrm{AB}}\rangle = \sqrt{\frac{1}{2}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{2}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

Could you tell what the probability of finding +1 is when measuring the z-component of electron A's spin only?

P(A=+1) = ...

What do you think the state of the bipartite system is after this measurement?

$$|\psi_{AB}\rangle = \ldots\ldots\ldots\ldots\ldots\ldots$$

What do we obtain if, after this measurement, we measure the z-component of electron B's spin? What is the outcome? What is the state of the bipartite system?

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3.3 Entanglement

Isn't this weird? We haven't even touched electron B, but its state has changed: if instead of measuring A the first time, we measured B, we could have obtain either +1 or -1; but after we measured A we obtain -1 with certainty! This means that by measuring A we changed electron B's state!
This is what Einstein called *"the spooky action at distance"* (the two electrons could be very far apart, for example, one in Bristol and one in London). Einstein and two colleagues (Podolsky and Rosen), in 1935, wrote a whole article about this fact to state their perplexities. They couldn't get their head around it, they couldn't believe such a thing could happen. Their article sparked a big debate which was resolved only many years later (1964) thanks to Bell, who theorized that this spooky action must be true, otherwise, some experimental predictions made using quantum mechanics theory would be rubbish. He did this using a very simple argument (read the text in Appendix 2 if you wish to know more about it). His argument was proven true only in 1981, thanks to an experiment carried out by Aspect (a variation of Bell's argument was already proven experimentally in 1972 by Freedman and Clauser).

This spooky action at distance is due to a special type of correlation that only quantum objects can have: **entanglement**. We can say that **entanglement is a type correlation between two quantum systems that causes one system to change** (meaning that the outcomes of future measurements can be affected) **by measuring the other one.**

Don't worry if you are confused. I am still confused about it too. Einstein (yes! the most famous physicist of all times) was confused by all this. Nature can be very, very weird at the quantum scale!

Homework

Exercise 1: Consider a bipartite system composed by two electrons in the following state

$$|\psi_{AB}\rangle = \left(\sqrt{\frac{47}{100}}|\uparrow_{\mathrm{A}}\rangle_z + \sqrt{\frac{53}{100}}|\downarrow_{\mathrm{A}}\rangle_z\right)|\uparrow_B\rangle_z$$

What outcomes can you obtain if you measure electron A's spin's z-component? With what probabilities?

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What is the state of the bipartite system after this measurement if you obtained -1?

$$|\psi_{AB}\rangle = \ldots\ldots\ldots\ldots\ldots$$

What result would you get if now (i.e. after you already measured A and got -1) you measure electron B's spin's z-component?

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If instead of electron A, you measured electron B first, would you have got a different result for electron B's spin z-component? What do you think this means? (Are they entangled?)

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Optional (but recommended) reading

In *Appendix 2: Entanglement, an* Alice and Bob, *story*, you can find an extract from the *Lecture Notes on Quantum Information and Computation*, written by Professor John Preskill for his Quantum Information and Computation course at the California Institute of Technology.
This part of his notes is written as a dialogue/story between two fictional characters, Alice and Bob, and the theoretical physicist John Stewart Bell. Don't be fooled, even though this is written as a story, it is a very challenging reading, meant for university students. However, I think that it would be good for you to read it and try to grasp the main ideas presented therein!
Do you want to read it? Great!!! That's the spirit! Before you do, it's better to explain/revise a few things:

Estimating the probability of random events (frequency)
Let's suppose there is a machine with one button and three lights: one red, one yellow and one green (and no! It's not a traffic light). When we press the button, one (and only one!) of the three lights flashes. It seems there is no logic to which of the three lights flashes: sometimes is the red one, sometimes the yellow and sometimes the green. Every time we press the button we record what light flashed, and we fill in the table below. After 10 trials we get

	# of flashes	*Frequency = (# of flashes)/TOTAL*
Red	4	4/10 = 2/5 = 0.40 = 40%
Yellow	3	3/10 = 0.30 = 30%
Green	3	3/10 = 0.30 = 30%
TOTAL	**10**	**4/10+3/10+3/10 = 1 = 100%**

Does this mean that the probability of having the red light flashing is 2/5=40%, while the probability of getting a yellow or green flash is 30%? No! As it is not true that if flipping a coin 4 times we get 3 tails and 1 head, then the probability of getting head is 1/4=25% and the probability of getting tail is 3/4=75%. The probability of getting head or tail is always 50%. What does this mean? It means that if we repeat the experiment many, many times, we'll notice that the number of times we get head (or tail) is very close to be half of the total number of times we flipped the coin. **The more trials we do, the more accurate our estimate will be.**
Going back to our three lights experiment... After 100 trials we get

	# of flashes	*Frequency = (# of flashes)/TOTAL*
Red	33	33/100 = 0.33 = 33%
Yellow	38	38/100 = 0.38 = 38%
Green	29	29/100 = 0.29 = 29%
TOTAL	**100**	**33/100+38/100+29/100 = 1 = 100%**

Can we stop here? Maybe some more trials... After 1,000 trials

	# of flashes	*Frequency = (# of flashes)/TOTAL*
Red	327	327/1,000 = 0.327 = 32.7%
Yellow	385	385/1,000 = 0.385 = 38.5%
Green	288	288/1,000 = 0.288 = 28.8%
TOTAL	**1000**	**327/1,000 +385/1,000 +288/1,000 = 1 = 100%**

Our estimate is getting better. Just a few more... After 1,000,000 trials

	# of flashes	*Frequency = (# of flashes)/TOTAL*
Red	328,732	328,732/1,000,000 = 0.328732 = 32.8732%
Yellow	384,782	384,782/1,000,000 = 0.384782 = 38.4782%
Green	286,486	286,486/1,000,000 = 0.286486 = 28.6486%
TOTAL	**1000**	**328,732/1,000,000 +384,782/1,000,000 +286,486/1,000,000 = 1 = 100%**

Can we stop here? It depends how precise we need to be (and how tired of pressing a button we are!). However, we can be reasonably confident that our estimate is not going to change much. If we want our probability to be expressed as a percentage with one decimal figure, we can stop here: it's likely that the first decimal figure is not gonna change. So we can conclude that, with good enough accuracy, our probabilities are

$$P(R)\approx 32.9\% \quad P(Y)\approx 38.5\% \quad P(G)\approx 28.6\%$$

It is worth noticing that when we sum up the frequencies for the three events we always get 1=100%. This is because the three events are **mutually exclusive**, i.e. they cannot happen at the same time.
REMEMBER: The probabilities of **all** possible mutually exclusive events sum up to **1**!

What is a probability distribution?
At one point in the dialogue, Bell mentions a "probability distribution P(x,y,z), (with x,y,z ∈ {H,T})". A probability distribution **(for a set of mutually exclusive events)** is just a list of numbers which tells us what are the probabilities of each possible event.
Let's consider a fair coin: if we flip it, we can obtain either head (H) or tail (T), both with probability ½ = 0.5 = 50%. In this case the probability distribution P(x) (the letter "x" represents the result of flipping a coin, by writing x ∈ {H,T} we are simply saying "x (the result) can be either head (H) or tail (T)) is

$$P(x=H) = \frac{1}{2} = 0.5 = 50\%$$
$$P(x=T) = \frac{1}{2} = 0.5 = 50\%$$

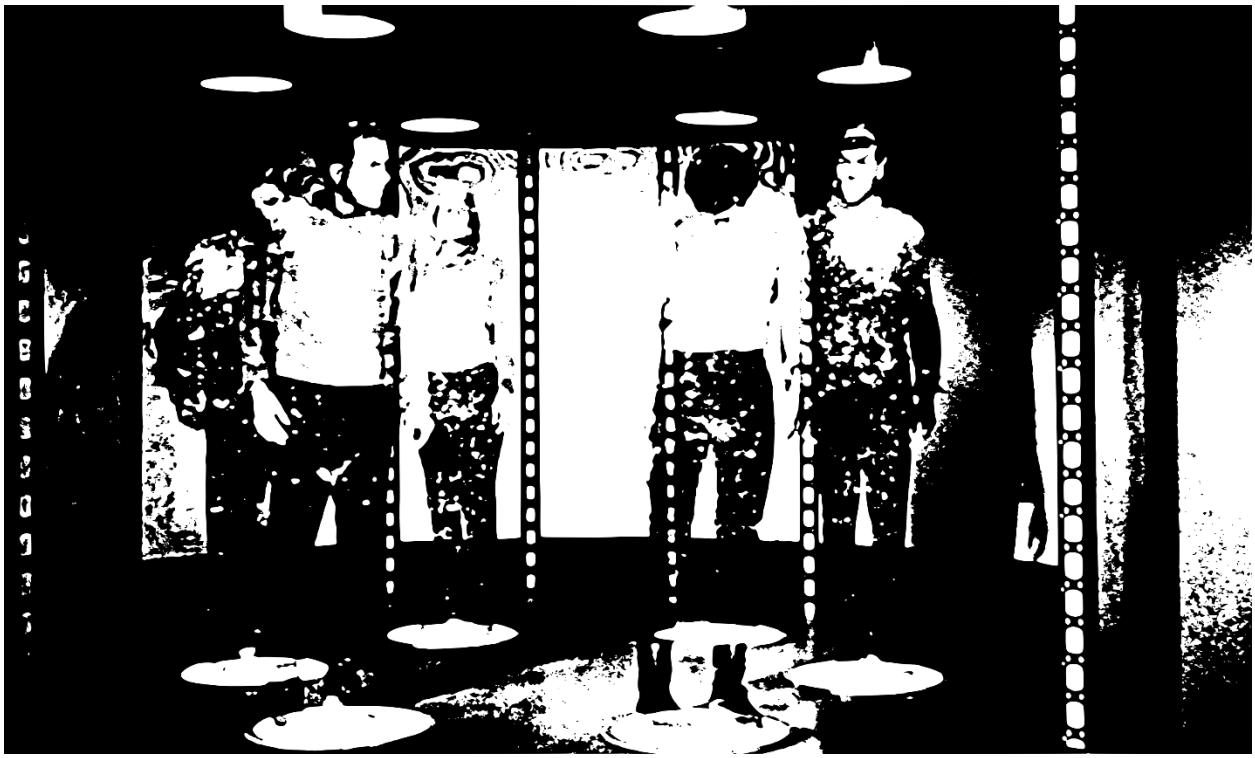
If we flip two fair coins we can get HH, HT, TH, TT. In this case the probability distribution P(x,y) (since we have two coins we use two letters: "x" represents the result of flipping one of the two coins, "y" represents the result of flipping the other coin) is

$$P(x=H,y=H) = P(HH) = \frac{1}{4} = 0.25 = 25\%$$
$$P(x=H,y=T) = P(HT) = \frac{1}{4} = 0.25 = 25\%$$
$$P(x=T,y=H) = P(TH) = \frac{1}{4} = 0.25 = 25\%$$
$$P(x=T,y=T) = P(TT) = \frac{1}{4} = 0.25 = 25\%$$

(Remember that to *calculate* the probability of a certain event to happen, we divide the number of outcomes favourable to that particular event, by the total number of possible outcomes.)
As we said before, the probabilities of all possible **mutually exclusive** events sum up to 1, so, when we sum all the numbers in the probability distribution, we obtain 1.

When Bell says "P(x,y,z), (with $x,y,z \in \{H,T\}$)" is because he doesn't want to assume that the coins are fair (we say that "he is considering the most general case"), so he doesn't specify the list (by giving numbers) as we did above, but he is assuming that such a list exists, and therefore the sum of all its elements equals 1.

Tutorial 4 – Quantum Teleportation



What is the Purpose of Tutorial 4?

- Understanding how quantum systems can be used for secure communication
- Understanding why teleportation is a convenient way of communicating securely
- Understanding the teleportation protocol

4.1 Using quantum systems for secret messages (quantum cryptography)

In Tutorial 2 we have seen that when we measure a quantum state in a superposition, we collapse it into one of the states corresponding to the possible measurement outcomes. However, we have also seen that a quantum state could be in a superposition with respect to one measurement, but not in a superposition with respect to another measurement: in Tutorial 2 we have seen, for example, that

$$|\psi\rangle = |\uparrow\rangle_z = \sqrt{\frac{1}{2}}|\rightarrow\rangle_x + \sqrt{\frac{1}{2}}|\leftarrow\rangle_x$$

Which means that, if we measure the spin's component in the x-direction, we can get either +1 or -1 with a 50%-50% chance. On the other hand, if we decide to measure the z-component, we would get +1 with certainty. Moreover, the state, after the measurement of the z-component, is not changed.

This is a property of any quantum state: we can always find some quantity to measure which will give us a certain outcome. In the case of an electron's spin, this means that there exists a special direction such that when we measure the spin's component in that direction we will obtain a result with probability 1 and the state of the electron's spin after the measurement would be the same as it was before the measurement, we could say that the electron's spin state collapsed into itself.

Let's suppose that Alice wants to give Bob a secret message, for example a series of numbers (maybe the combination of a safe?). What they can do is to agree on a direction in which to measure. Alice can therefore prepare a bunch of electrons' spins which are up and down in that direction (this operation is called **encoding**) and give these electrons to Bob. Bob can then read the message (**decoding**) by measuring the spin of those electrons in the agreed direction: if he gets -1 he writes down a 0, if he gets a +1 he writes down 1. We can write any number (or letter) using only 0s and 1s (bits) by using the so-called Binary Code.

What is the advantage of doing this? Well, secrecy! Indeed, if a curious Eve wanted to read the message, without knowing in what direction to measure, she would fail. Can you tell why?

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4.2 Sending secret messages: Teleportation!

But what if Alice and Bob are not in the same place? Maybe Alice is in Weston-super-Mare and Bob is in Glasgow. How can she send him the message and being sure that no one else intercepts it and reads it?

She could, for example, encode the message in a bunch of electron spins and have them delivered to Bob. This solution, however, is not practical: it's an eight-hour drive (without stops) from Weston to Glasgow!

The best solution would be to send the electron spins' **states** to Bob without moving the spins. This is something they can achieve with **quantum teleportation**. Let's define quantum teleportation:

> ***Quantum teleportation consists in the disembodied*** (without using a "body", i.e. a physical object) ***transfer of information from a sender*** (Alice in our case) ***to a remote*** (far away) ***receiver*** (Bob).

To implement this teleportation, they need to:

1) share as many entangled states (see Tutorial 3) as the number of bits the message is composed of
2) have a line of communication (for example their respective phone numbers)
3) be able to measure quantum systems

If they don't have one of these three things they cannot teleport the message.

4.3 Quantum teleportation protocol

Let's suppose that the first number of the secret message is encoded in the following electron spin's state (we call this **input**)

$$|\psi_{\mathrm{in}}\rangle = \sqrt{p_\uparrow}|\uparrow_{in}\rangle_z + \sqrt{p_\downarrow}|\downarrow_{in}\rangle_z$$

The symbols $p_\uparrow$ and $p_\downarrow$ represent the probability of getting +1 and -1, respectively, when measuring the z-component of the electron spin. It does not matter what these numbers are, it is sufficient to know that, since they are probabilities, they are positive and sum up to 1 (getting +1 or -1 are mutually exclusive events):

$$p_\uparrow + p_\downarrow = 1$$

We also know, from the first section of this Tutorial, that if we measure this electron's spin in some direction (for example $\nearrow$), we would get +1 with certainty. We can indicate this fact by writing

$$|\psi_{\mathrm{in}}\rangle = |\nearrow_{\mathrm{in}}\rangle$$

As we said in the previous section, Alice and Bob must share an entangled state:

$$|\psi_{\mathrm{AB}}\rangle = \sqrt{\tfrac{1}{2}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\tfrac{1}{2}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

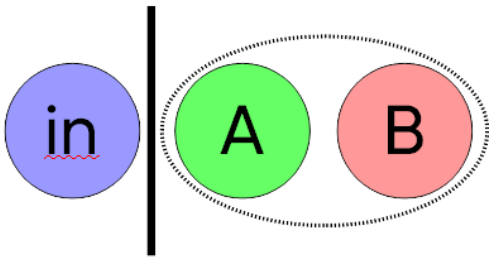
There are three electrons in total: the input, Alice's electron of the entangled state and Bob's electron of the entangled state. Overall, the total state of the composite system is

$$|\psi_{\mathrm{inAB}}\rangle = |\psi_{\mathrm{in}}\rangle|\psi_{AB}\rangle = \left(\sqrt{p_\uparrow}|\uparrow_{in}\rangle_z + \sqrt{p_\downarrow}|\downarrow_{in}\rangle_z\right)\left(\sqrt{\frac{1}{2}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{2}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}}\right)$$

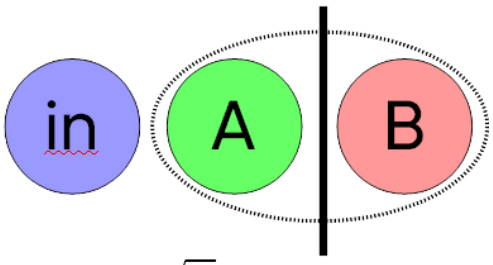
With some algebraic manipulation we can rewrite this state as:

$$|\psi_{\mathrm{inAB}}\rangle = \sqrt{\frac{1}{4}}|\Phi^{+}{}_{\mathrm{inA}}\rangle|\nearrow_{\mathrm{B}}\rangle + \sqrt{\frac{1}{4}}|\Phi^{-}{}_{\mathrm{inA}}\rangle|\swarrow_{\mathrm{B}}\rangle + \sqrt{\frac{1}{4}}|\Psi^{+}{}_{\mathrm{inA}}\rangle|\searrow_{\mathrm{B}}\rangle + \sqrt{\frac{1}{4}}|\Psi^{-}{}_{\mathrm{inA}}\rangle|\nwarrow_{\mathrm{B}}\rangle$$

The state is the same as before, only written differently. What "changed" is the bipartition we consider: we have three systems, before the separation was between the input and the shared entangled state, while in the last equation the separation is between the input with Alice's part of the entangled state, and Bob's part of the entangled state.



$$|\psi_{\mathrm{in}AB}\rangle = |\psi_{\mathrm{in}}\rangle\,|\psi_{AB}\rangle$$



$$\begin{aligned}|\psi_{\mathrm{in}AB}\rangle = \quad & \sqrt{\frac{1}{4}}|\Phi^{+}_{\mathrm{in}A}\rangle|\nearrow_{B}\rangle + \\ & + \sqrt{\frac{1}{4}}|\Phi^{-}_{\mathrm{in}A}\rangle|\swarrow_{B}\rangle + \\ & + \sqrt{\frac{1}{4}}|\Psi^{+}_{\mathrm{in}A}\rangle|\searrow_{B}\rangle + \\ & + \sqrt{\frac{1}{4}}|\Psi^{-}_{\mathrm{in}A}\rangle|\nwarrow_{B}\rangle\end{aligned}$$

Notice that since *A* and *B* are entangled, the (composite) system *inA* is entangled with system *B*: this means that if we measure the system *inA* we would change the state of system *B*.

Before moving on with the protocol, we should have a look at every state in this superposition. We start by focussing on the *inA* part. These four states are called Bell states (yes, that Bell) and they have this very useful characteristic: let's consider two spins, we first measure whether they are spinning in the same direction along z, then we measure whether they are spinning in the same direction along x; this particular measurement (which we will call *zx-alignment*) has four possible outcomes, summarized in the table below

		Are they spinning in the same direction along x?	
		Yes	No
Are they spinning in the same direction along z?	Yes	Y Y	Y N
	No	N Y	N N

The Bell states are not changed by the *zx-alignment measurement* (as $|\uparrow\rangle_z$ is not changed by the z-component measurement): if we measure the zx-alignment of the state $|\Phi^{+}{}_{\mathrm{inA}}\rangle$ we would get **Y Y** as a result

with probability 1. We summarise the measurement outcomes of the zx-alignment on the Bell states in the table below

State	zx-alignment
$\lvert\Phi^{+}{}_{\mathrm{inA}}\rangle$	Y Y
$\lvert\Phi^{-}{}_{\mathrm{inA}}\rangle$	Y N
$\lvert\Psi^{+}{}_{\mathrm{inA}}\rangle$	N Y
$\lvert\Psi^{-}{}_{\mathrm{inA}}\rangle$	N N

Therefore, if Alice measures the zx-alignment of the two electron spins in her possession (input and A) and finds that they are aligned in both directions (Y Y), the state of the total system (input, A and B) after the measurement would be

$$|\psi_{inAB}\rangle = \ldots\ldots\ldots\ldots\ldots\ldots$$

What is the probability of this happening?

$$\mathrm{P(Y\ Y)} = \ldots\ldots\ldots\ldots\ldots\ldots$$

At this point, Alice calls Bob on the phone and tells him the outcome of her measurement. If now Bob measures the electron's spin in the agreed direction he would get +1 with certainty, exactly the result he would have got by measuring the input! He would finally be in possession of the first bit of the secret message.

But this happens only in 25% of the cases: when Alice measures Y Y. What happens in the remaining 75% of the cases? Well, if the outcome of Alice's measurement is Y N, the total state would be

$$|\psi_{inAB}\rangle = |\Phi^{-}{}_{\mathrm{inA}}\rangle|\swarrow_{\mathrm{B}}\rangle$$

Which means that if Bob measures the spin component of his electron in the agreed direction, he would get -1. So, after Alice communicates him that she got Y N, Bob knows that to get his electron in the same state as the input, he has to "flip it". We haven't discussed this, however, it is sufficient to know that one can easily "flip" a spin (invert its rotation direction: clockwise ↔ counter-clockwise), no matter in which direction it's spinning, by hitting the electron with a laser-pulse called **Z-pulse**: the spin is rotating clockwise in the x-direction? One hits it with this Z-pulse and it starts rotating counter-clockwise. Is it rotating counter-clockwise in the y-direction? One hits it with the same Z-pulse and now it rotates clockwise. Is it rotating counter-clockwise in an unknown direction? Well, one hits it with the Z-pulse and it ends up rotating clockwise in that same unknown direction!

State before	State after Z-pulse
$\lvert\uparrow\rangle_z$	$\lvert\downarrow\rangle_z$
$\lvert\leftarrow\rangle_x$	$\lvert\rightarrow\rangle_x$
$\lvert\nearrow\rangle$	$\lvert\swarrow\rangle$
$\lvert\searrow\rangle$	$\lvert\nwarrow\rangle$

If Alice gets N Y or N N, Bob's state is rotating clockwise and counter-clockwise, respectively, in a direction which is perpendicular to the agreed direction $\nearrow$. Bob can, in these cases, hit his electron with another laser-pulse called **X-pulse** which changes the rotation direction to the perpendicular one. Is the spin rotating clockwise in an unknown direction? Well, one hits it with the X-pulse and it ends up rotating clockwise in a direction which is perpendicular to the one it was rotating before.

State before	State after X-pulse
$\lvert\uparrow\rangle_z$	$\lvert\rightarrow\rangle_x$
$\lvert\leftarrow\rangle_x$	$\lvert\downarrow\rangle_z$
$\lvert\searrow\rangle$	$\lvert\nearrow\rangle$

So, after Alice's measurement, Bob can always find a way to transform his state into a state which is the same state as the input. To do this he needs to know the outcome of Alice's measurement, that is why they need the communication line.

Let's summarise the protocol of teleporting an input state from Alice to Bob:

1) Alice and Bob must share the entangled state

$$|\psi_{AB}\rangle = \sqrt{\frac{1}{2}}|\uparrow_A\rangle_z|\uparrow_B\rangle_z + \sqrt{\frac{1}{2}}|\downarrow_A\rangle_z|\downarrow_B\rangle_z$$

2) Alice measures the zx-alignment of the input and her part of the shared entangled state
3) She communicates the outcome of the measurement (Y Y, Y N, N Y or N N) to Bob
4) Depending on the result communicated to him by Alice, Bob acts on his part of the shared entangled state according to the following table

Outcome of Alice's measurement	Bob's state after Alice's measurement	Bob's series of operation on his part of the entangled state		Final state
Y Y	$\lvert\nearrow_B\rangle$	-	-	$\lvert\nearrow_B\rangle$
Y N	$\lvert\swarrow_B\rangle$	Z-pulse	-	$\lvert\nearrow_B\rangle$
N Y	$\lvert\searrow_B\rangle$	X-pulse	-	$\lvert\nearrow_B\rangle$
N N	$\lvert\nwarrow_B\rangle$	Z-pulse	X-pulse	$\lvert\nearrow_B\rangle$

The teleportation is complete. Bob ended up with the input state without the input electron being moved. Alice and Bob can therefore repeat the protocol for every bit in the secret message.

4.3 A few important remarks

Secrecy

In our narrative, Alice knew the input state, and Bob was the designated receiver. However, it is important to consider the case in which they don't have all of the three "must-have for teleportation" and they had to resort to a third party to implement the protocol. So, let's suppose that, for example, they don't share an entangled state. They know, however, that Charlie (in Weston where Alice is) and Dianne (in Glasgow where Bob is) have all the necessary to implement the teleportation. Alice gives the input electrons to Charlie. Charlie teleports them to Dianne, who then gives them to Bob, who can therefore read the secret message.

Why can Alice and Bob be sure that Charlie and Dianne won't find out the content of the secret message? Well, to answer this question we can ask ourselves:
Does Charlie need to know anything about the input electron to make the necessary measurement?

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Does Charlie get any information about the input state because of the measurement he carries out?

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Is he left, at the end of the protocol, with a copy of the input? What is the state he is left with in the case he got N N by measuring the zx-alignment? And in the case he got N Y?

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Does Dianne need to know anything about the input state to complete the protocol and recreate it in her lab?

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Entanglement lies at the heart of the protocol

The teleportation protocol works because of the entanglement shared between sender and receiver. There are, in total, three systems involved: input, A and B. The entangled state is the bipartite system AB, and we know that since, because of the entanglement, by measuring one of the two subsystems, let's say A, we modify the state of the other one, B.

We have seen that the sender doesn't measure system A only, but they measure the system input+A. However, since A and B are entangled, also input+A and B are entangled, therefore, by measuring this composite system (input+A), they change the state of B. The way in which the state of B is changed reflect the fact that the information contained in the input state has somehow been transferred to B.

Homework

Read again the coursebook from the beginning. Make sure you write down any question, any doubt, any curiosity you may have and you want to ask me.

Preparation for the final assignment

Create a schematic summary (e.g. a concept map) for each Tutorial where you present (either pictorially or in short sentences organized in a bullet point list) the main concepts and ideas of the 4 tutorials as you understood them.

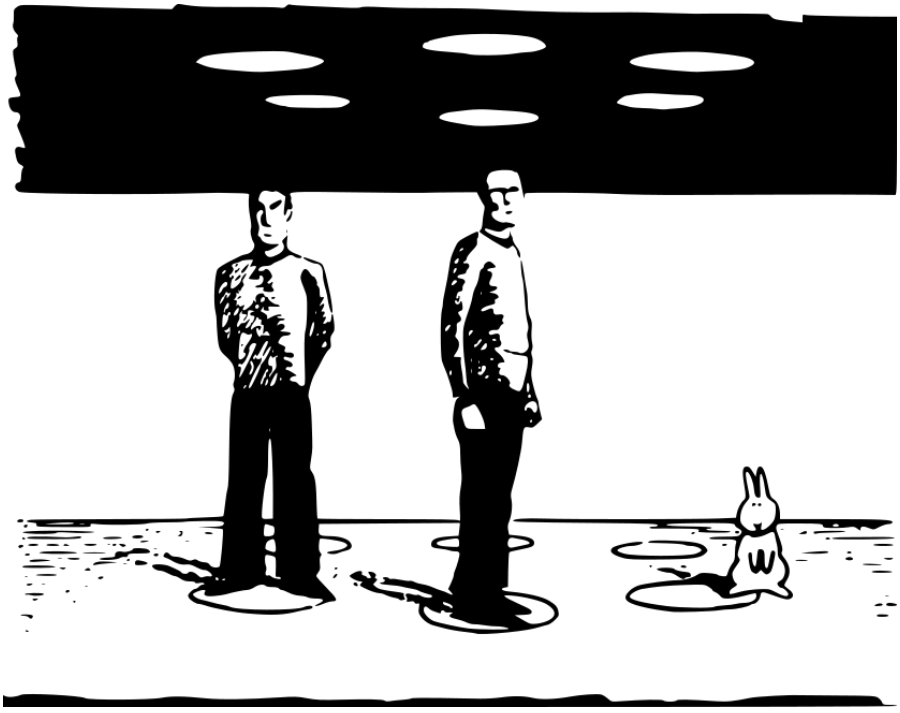
TUTORIAL 1: Discrete, continuous and spinning

TUTORIAL 2: Quantum measurement and superposition

TUTORIAL 3: The “spooky action at distance”

TUTORIAL 4: Quantum teleportation

Tutorial 5 – Imperfect quantum teleportation



What is the Purpose of Tutorial 5?

- Understanding that bipartite states can be more entangled than others
- Understanding that quantum states can be "similar"
- Understanding how limited entanglement can affect the teleportation protocol

5.1 Limited entanglement

We have seen in the last tutorial that entanglement is indispensable for implementing the teleportation protocol. What we haven't mentioned is that quantum states can be more entangled than others: the state we have seen Alice and Bob using in the teleportation protocol

$$|\psi_{\mathrm{AB}}\rangle = \sqrt{\frac{1}{2}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{2}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

is one of the so-called **maximally entangled states**. Between states with no entanglement and maximally entangled states there is a whole world of more or less, entangled states. For example, consider the state

$$|\psi_{\mathrm{AB}}\rangle = |\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

This is not entangled since a measurement on spin A (or spin B) won't change the state of the other spin. If, on the other hand, we consider the following state

$$|\psi_{\mathrm{AB}}\rangle = \sqrt{\frac{999}{1000}}|\uparrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\uparrow_{\mathrm{B}}\rangle_{\mathrm{z}} + \sqrt{\frac{1}{1000}}|\downarrow_{\mathrm{A}}\rangle_{\mathrm{z}}|\downarrow_{\mathrm{B}}\rangle_{\mathrm{z}}$$

we see that by measuring one spin we change the state of the other, and this means that they are entangled. However, we also notice that when we measure this state, 999 times out of 1000 we would get a state that is very similar to the non-entangled one. In other words, it would take us a lot of time to figure out, by measuring it over and over, that the last state is not the non-entangled state above: they are difficult to distinguish from each other.

In general, it is possible to quantify entanglement, i.e. to associate a number to an entangled state which tells us "how much entangled" a bipartite quantum state is: the maximally entangled state has entanglement 1=100% (which is therefore the maximum value entanglement can reach), the last state we considered has entanglement 0.003=0.3%.

So, what happens when we try to teleport a quantum state using a state which is not 100% (maximally) entangled?

4.2 Distinguishability of quantum states

The idea at the heart of the teleportation protocol is that the state Bob gets at the end of the protocol is exactly the same state Alice had before the protocol. What do we mean when we say that the two states are "exactly the same"? Well, we can say that they are indistinguishable: provided with infinite copies of the two states and carrying infinite measurements on both, we wouldn't notice any difference.

One can quantify (i.e. associate a number to) the distinguishability of states. The quantity which tells us how different or how similar two quantum states are is called "fidelity". Fidelity is equal to 1 only when the two state we are considering are the same, and it is 0 when with only one measurement we can tell with certainty which state is which:

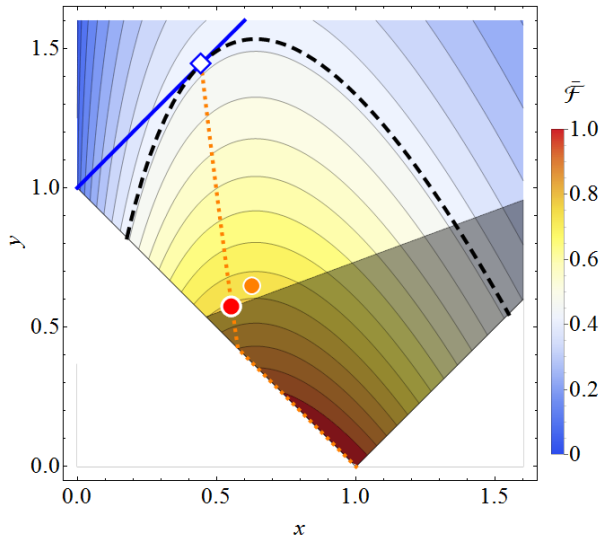
$$F(|\uparrow\rangle_z, |\uparrow\rangle_z) = 1 = 100\%$$

$$F(|\uparrow\rangle_z, |\downarrow\rangle_z) = 0 = 0\%$$

$$F(|\uparrow\rangle_z, |\rightarrow\rangle_x) = 0.5 = 50\%$$

When we teleport a quantum state using a bipartite state which is not maximally entangled, the fidelity between the output and the input is going to be less than 100%.

4.3 Recent research results



During my PhD I studied the teleportation protocol of a particular type of states called Gaussian states. The question which I answered was the following: what is the best achievable input-output fidelity if the entanglement shared between sender and receiver is limited? What is the best bipartite state one can use to achieve the optimal result?

In the figure above, you can see part of my results: each point in the coloured portion of plane, corresponds to an entangled state, the warmer the colour, the bigger the input-output the fidelity is, the gray shaded area represents all the states sender and receiver can share with higher entanglement. The red dot represents the state I found to be the best for implementing the teleportation protocol of these type of special states (Gaussian) I was considering.

Final assignment

Prepare a 10-minute slide-presentation (between 8 and 12 slides) about Quantum Teleportation. You will deliver the presentation in the next tutorial.

Remember that a slide presentation is composed by two main parts: the **slides** and the **explanation**. The slides must be just a graphical prompt for the explanation: they should help the audience follow the reasoning of the speaker and get a better grasp of the presented concepts. As for the baseline assignment, also for this final assignment you will need to write the explanation. You will deliver the explanation, **without reading it**, in the next tutorial.

Slides – In each of the slides, draw a schematic representation of the ideas you want to present. You can write labels in your slides, but, as a general rule, you should not to write too much on them: you want the audience to listen to you, not to get lost reading what's written on the slides and not paying attention to the explanation. In some cases, it is necessary to write a sentence in a slide. Even in these cases though, write it simple, with only the strictly necessary information. The only slides where you can write in full are the ones where you express your original thoughts. However, also in this case, keep your sentences simple and organize them in a bullet point list.

Explanation – Script what you want to say. Below, you will find some questions that will help you structure your explanation. The explanation should refer to what is depicted in the slides and guide the audience's understanding.

You will be evaluated in particular for your

- **understanding**: how the content in the explanation is relevant to the content covered in the tutorial and to the prompt questions below; how appropriately you define and use specific scientific terms used in the course; how rigorous your explanation is;
- **critical evaluation**: how consistent, clear and logical your evaluative points are;
- **structure and presentation**: how well the explanation refers to the slide; how clear and coherent the slides are;
- **spelling, punctuation and grammar.**

For a more detailed mark scheme, see page 6.

Prompt questions:

- What do we talk about when we use the word **"quantum"**?
- How do quantum **measurements** differ from classical ones?
- What is quantum **superposition**?
- What is **entanglement**?
- What are possible applications of the quantum teleportation technology?
- How does the quantum teleportation **protocol** work?
- What are the similarities between quantum teleportation and the Telepomp? Read again from line 38 to line 45 of the story *The man without a body* and remember how you answered to the following questions: can you say that the object at the cathode **is** the same object that was at the anode before this was Telepomp? What is transmitted through the cable? Now, focus on quantum teleportation: in what sense is the quantum object at the receiver end **the same** as the input quantum object? What is transmitted from the sender to the receiver thanks to the shared entangled state?

Tutorial 6 – Feedback tutorial

What is the Purpose of Tutorial 6?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Tutorial 7 – Feedback tutorial

What is the Purpose of Tutorial 7?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
•   •   •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Appendix 1 – Referencing correctly

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

Why should I reference?

Referencing is important in your work for the following reasons:

- It gives credit to the authors of any sources you have referred to or been influenced by.
- It supports the arguments you make in your assignments.
- It demonstrates the variety of sources you have used.
- It helps to prevent you losing marks, or failing, due to plagiarism.

When should I use a reference?

You should use a reference when you:

- Quote directly from another source.
- Summarise or rephrase another piece of work.
- Include a specific statistic or fact from a source.

How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. A marker in your assignment: After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].
2. Bibliography: This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

    a. Websites – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

    E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

    b. Books – Author, date published, title of book (in italics), pages where the information came from.

    E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

    c. Articles – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

    E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

Appendix 2 – Entanglement, an *Alice and Bob* story

Lecture Notes for Ph219/CS219 : Quantum Information and Computation
Chapter 4
by John Preskill (California Institute of Technology)
2001

The system that Alice and Bob are studying might be described this way: Alice, in Pasadena, has in her possession three coins laid out on a table, labelled 1,2,3. Each coin has either its heads (H) or tails (T) side facing up, but it is hidden under an opaque cover, so that Alice is not able to tell whether it is an H or a T. Alice can uncover any one of the three coins, and so learn its value (H or T). However, as soon as that one coin is uncovered, the other two covered coins instantly disappear in a puff of smoke, and Alice never gets an opportunity to uncover the other coins. She has many copies of the three-coin set, and eventually she learns that, no matter which coin she exposes, she is just as likely to find an H as a T. Bob, in Chicago, has a similar set of coins, also labelled 1,2,3. He too finds that each one of his coins, when revealed, is as likely to be an H as a T.
In fact, Alice and Bob have many identical copies of their shared set of coins, so they conduct an extensive series of experiments to investigate how their coin sets are correlated with one another. They quickly make a remarkable discovery: Whenever Alice and Bob uncover coins with the same label (whether 1, 2, or 3), they always find coins with the same value– either both are H or both are T. They conduct a million trials, just to be sure, and it works every single time! Their coin sets are perfectly correlated.
Alice and Bob suspect that they have discovered something important,and they frequently talk on the phone to brainstorm about the implications of their results. One day, Alice is in an especially reflective mood:

Alice: You know, Bob, sometimes it's hard for me to decide which of the three coins to uncover. I know that if I uncover coin 1, say, then coins 2 and 3 will disappear, and I'll never have a chance to find out the values of those coins. Once, just once, I'd love to be able to uncover two of the three coins, and find out whether each is an H or a T. But I've tried and it just isn't possible – there's no way to look at one coin and prevent the other from going poof!

Bob: [Long pause] Hey...wait a minute Alice, I've got an idea...Look, I think there is a way for you to find the value of two of your coins, after all! Let's say you would like to uncover coin 1 and coin2. Well, I'll uncover my coin 2 here in Chicago, and I'll call you to tell you what I found, let's say its an H. We know, then, that you are certain to find an H if you uncover your coin 2 also. There's absolutely no doubt about that, because we've checked it a million times. Right?

Alice: Right...

Bob: But now there's no reason for you to uncover your coin 2; you know what you'll find anyway. You can uncover coin 1 instead. And then you'll know the value of both coins.

Alice: Hmmm...yeah, maybe. But we won't be sure, will we? I mean, yes, it always worked when we uncovered the same coin before, but this time you uncovered your coin 2, and your coins 1 and 3 disappeared, and I uncovered my coin 1, and my coins 2 and 3 disappeared. There's no way we'll ever be able to check anymore what would have happened if we had both uncovered coin 2.

Bob: We don't have to check that anymore, Alice; we've already checked it a million times. Look, your coins are in Pasadena and mine are in Chicago. Clearly, there's just no way that my decision to uncover my coin 2 can have any influence on what you'll find when you uncover your coin 2. That's not what's happening. It's just that when I uncover my coin 2 we're collecting the information we need to predict with certainty what will happen when you uncover your coin 2. Since we're already certain about it, why bother to do it!

Alice: Okay, Bob, I see what you mean. Why don't we do an experiment to see what really happens when you and I uncover different coins?

Bob: I don't know, Alice. We're not likely to get any funding to do such a dopey experiment. I mean, does anybody really care what happens when I uncover coin 2 and you uncover coin 1?

Alice: I'm not sure, Bob. But I've heard about a theorist named Bell. They say that he has some interesting ideas about the coins. He might have a theory that makes a prediction about what we'll find. Maybe we should talk to him.

Bob: Good idea! And it doesn't really matter whether his theory makes any sense or not. We can still propose an experiment to test his prediction, and they'll probably fund us.

So Alice and Bob travel to CERN to have a chat with Bell. They tell Bell about the experiment they propose to do. Bell listens closely, but for a long time he remains silent, with a faraway look in his eyes. Alice and Bob are not bothered by his silence, as they rarely understand anything that theorists say anyway. But finally Bell speaks.

Bell: I think I have an idea.... When Bob uncovers his coin in Chicago, that can't exert any influence on Alice's coin in Pasadena. Instead, what Bob finds out by uncovering his coin reveals some information about what will happen when Alice uncovers her coin.

Bob: Well, that's what I've been saying...

Bell: Right. Sounds reasonable. So let's assume that Bob is right about that. Now Bob can uncover any one of his coins, and know for sure what Alice will find when she uncovers the corresponding coin. He isn't disturbing her coin in any way; he's just finding out about it. We're forced to conclude that there must be some hidden variables that specify the condition of Alice's coins. And if those variables are completely known, then the value of each of Alice's coins can be unambiguously predicted.

Bob: *[Impatient with all this abstract stuff]* Yeah, but so what?

Bell: When your correlated coin sets are prepared, the values of the hidden variables are not completely specified; that's why any one coin is as likely to be an H as a T. But there must be some probability distribution P(x,y,z) (with $x,y,z \in \{H,T\}$) that characterizes the preparation and governs Alice's three coins. These probabilities must be non-negative, and they sum to one (they are mutually exclusive events!):

$$P(HHH)+P(HHT)+P(HTH)+P(THH)+P(HTT)+P(THT)+P(THH)+P(TTT) = 1$$

Alice can't uncover all three of her coins, so she can't measure P(x,y,z) directly. But with Bob's help, she can in effect uncover any two coins of her choice. Let's denote with Psame(i,j), the probability that coins i and j (i,j= 1,2,3) have the same value, either both H or both T. Then we see that

$$\begin{aligned} P_{same}(1,2) &= P(HHT)+P(TTH)+P(HHH)+P(TTT) \\ P_{same}(1,3) &= P(HTH)+P(THT)+P(HHH)+P(TTT) \\ P_{same}(2,3) &= P(THH)+P(HTT)+P(HHH)+P(TTT) \end{aligned}$$

and if we sum this together we get

$$\begin{aligned} P_{same}(1,2)+P_{same}(1,3)+P_{same}(2,3) = {} & \underline{P(HHT)}+\underline{P(TTH)}+\underline{P(HHH)}+\underline{P(TTT)}+ \\ & +\underline{P(HTH)}+\underline{P(THT)}+P(HHH)+P(TTT)+ \\ & +\underline{P(THH)}+\underline{P(HTT)}+P(HHH)+P(TTT)= \\ & =1+2P(HHH)+2P(TTT) \geq 1 \end{aligned}$$

(because of the first equation we have written, the underlined terms sum up to 1).
So, that's my prediction: P_{same} should obey the inequality

$$P_{same}(1,2)+P_{same}(1,3)+P_{same}(2,3) \geq 1$$

You can test it my doing your experiment that "uncovers" two coins at a time.

Bob: Well, I guess the math looks right. But I don't really get it. Why does it work?

Alice: I think I see.... Bell is saying that if there are three coins on a table, and each one is either an H or a T, then at least two of the three have to be the same, either both H or both T. Isn't that it, Bell?

Bell stares at Alice, a surprised look on his face. His eyes glaze, and for a long time he is speechless. Finally, he speaks:

Bell: Yes.

So Alice and Bob are amazed and delighted to find that Bell is that rarest of beasts — a theorist who makes sense. With Bell's help, their proposal is approved and they do the experiment, only to obtain a shocking result. After many careful trials, they conclude, to very good statistical accuracy that

$$P_{same}(1,2) \approx P_{same}(1,3) \approx P_{same}(2,3) \approx 1/4 = 0.25 = 25\%$$

and hence

$$P_{same}(1,2)+P_{same}(1,3)+P_{same}(2,3) = 1/4 + 1/4 + 1/4 = 3/4 < 1$$

The correlations found by Alice and Bob flagrantly violate Bell's inequality!
Alice and Bob are good experimenters, but dare not publish so disturbing a result unless they can find a plausible theoretical interpretation. Finally, they become so desperate that they visit the library to see if quantum mechanics can offer any solace...

Notes

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Quantum teleportation
How is this a thing?
Key Stage 4 Programme

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