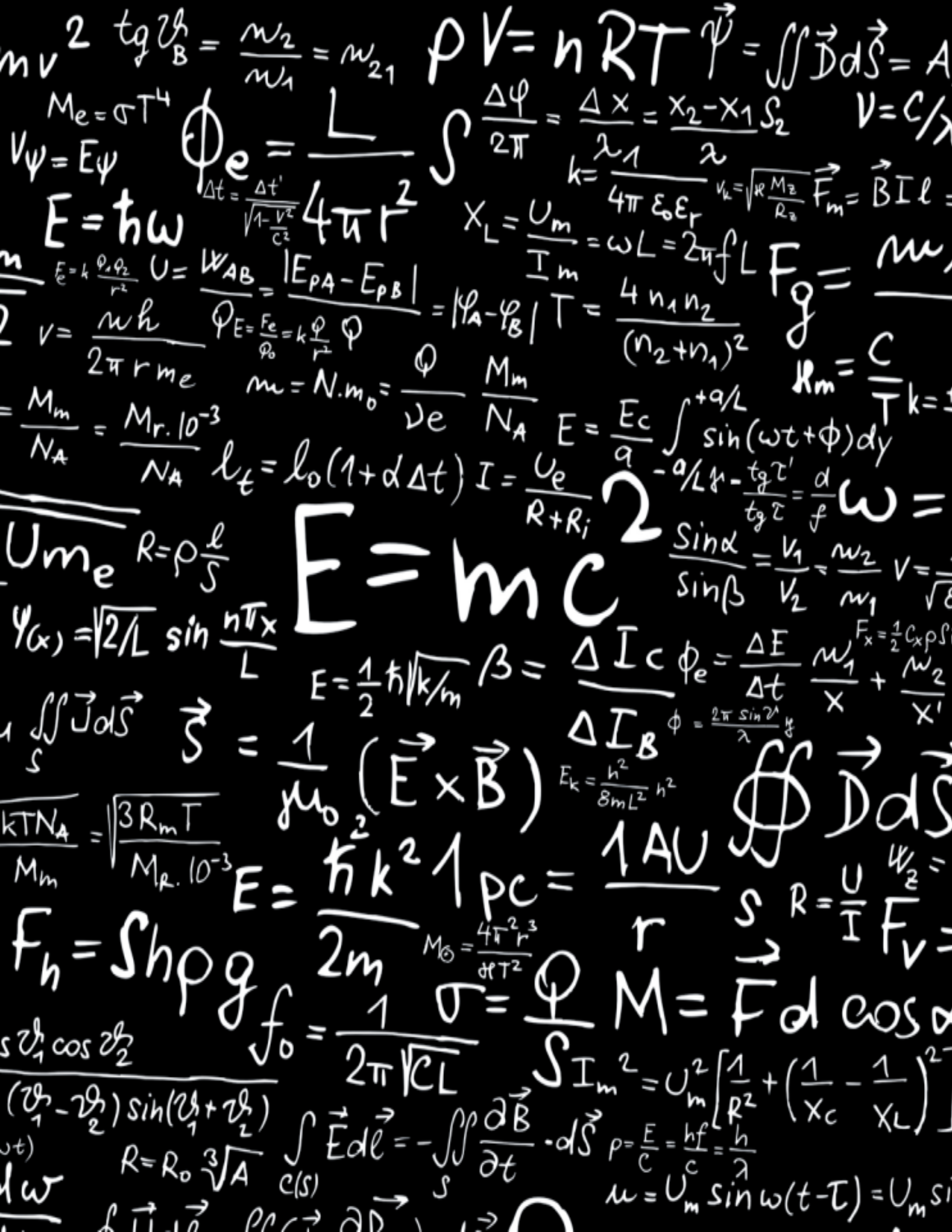
E = mc 2
pV = nRT
E = ħω
F = Shρg
E = ħk²/2m
1 pc = 1 AU / r
Φe = L / 4πr²
E = 1/2 ħ√(k/m)
S = 1/μ0 (E × B)
∮∮ D dS
F = Fd cos α
Fg = mu
ω =

University Pathways

Congratulations! You have been specially selected to take the *University Pathways Course* with here at Outwood Academy Adwick. Your teachers believe that you have the potential to change the world. The academy has enlisted the help of Dr. Reid - a theoretical physicist at the University of Aberdeen and King's College London - to help you achieve your full potential. A bright future awaits you!

Timetable and Assignment Submission

Fill this in with Dr Reid.

Table 1: Timetable and Assignment Submission

Tutorial	**Date**	**Time**	**Location**
1			
2			
3			
4			
5			
6			

Table 2: Timetable - Homework Assignments

Homework Assignment	**Description**	**Due Date**
Tutorial 1	Revision questions	
Tutorial 2	Mass-energy equivalence I	
Tutorial 3	Mass-energy equivalence II	
Tutorial 4	Assignment	
Tutorial 5	Mass-energy equivalence with `Mathematica`	

Table 3: Assignment Submission - Lateness and Plagiarism

Lateness	
Submission after 4:00 pm on	10 marks deducted

Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extensive plagiarism	Automatic fail

KS4 Programme 2016-17 – Pupil Feedback Report

Grade	**Marks**	**What this means**
1st	70+	Performing to an excellent standard at A-level
2 : 1	60-69	Performing to a good standard at A-level
2 : 2	50-59	Performing to an excellent standard at current key stage
3rd	40-49	Performing at a good standard at current key stage
Towards pass	0-39	Performing below a good standard at current key stage
No submission	NS	No assignment received by The Brilliant Club

Name of Teacher	Dr Reid
Title of Assignment	$E = mc^2$
Name of Student	

Original Mark /100	**Final Mark** /100
Deducted Marks	**Final Grade**

If marks have been deducted (*e.g.* late submission, plagiarism) Dr Reid will explain below:

Learning Feedback Comment 1 - *Algebraic Skills*

What you did in relation to this Key Learning Priority:

How you could improve in future:

Learning Feedback Comment 2 - *Physical Problem Solving*

What you did in relation to this Key Learning Priority:

How you could improve in future:

Learning Feedback Comment 3 - *Scientific Computation*

What you did in relation to this Key Learning Priority:

How you could improve in future:

Resilience Comment

How you showed learning resilience during the course:

How you could build resilience in the future:

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Tutorial 4 - Assignment 12
Tutorial 5 - Mass-Energy Equivalence with `Mathematica` 13

Course Rationale

The mass-energy equivalence relationship, $E = mc^2$, is one of the cornerstones of modern physics. Its discovery will forever shape the course of history. The technology developed with an understanding of $E = mc^2$ has the power to annihilate the human race or provide us with boundless clean energy. In this course, pupils will discover the secrets of $E = mc^2$, and develop their numerical, algebraic, computational and physical problem solving skills along the way.

The course begins with an historical overview of this important equation. Why is it so important? And why did it forever change history? Thereafter, students will learn to perform direct calculations with $E = mc^2$. Students will then be introduced to the concept of *relativistic mass* and to so-called *Fermi problems* for example:

> *Could the energy contained within a mere litre of hydrogen gas power Outwood Adwick Academy for an entire year?*

$E = mc^2$ will tell us the answer. Finally, pupils will undertake a numerical exploration of $E = mc^2$ using the cutting edge scientific software package `Mathematica` a package not otherwise encountered until postgraduate level at most UK universities.

Mark Scheme

Table 4: Mark Scheme

Key	$1^{\mathbf{st}}$	$2:2$
Algebraic manipulation	Students will be able to algebraically manipulate nonlinear functions with complicated arguments. First class honours will be awarded for complete or near-complete accuracy.	Students will be able to algebraically manipulate nonlinear functions with simplified arguments with accuracy. Partial accuracy is sufficient to attain lower-second class honours.
Mass-energy equivalence	Students will have a comprehensive understanding of the concept of mass-energy equivalence and can. First class honours will be awarded when students can adeptly solve conceptually challenging problems in physical systems with a relativistic mass.	Students have a basic awareness of the concept of mass-energy equivalence, and can articulate the most salient points with a modicum of skill. Lower-second class honours will be attained if the students can solve rudimentary problems in conceptually simplified physical systems.
Scientific Computation	Students will be able to demonstrate distinct competence in the the use of the WebMathematica software package to solve mass-energy equivalence problems. First class honours will be awarded for complete or near-complete syntactic knowledge and accuracy in the homework assignment.	Students will be able to demonstrate moderate competence in the the use of the WebMathematica software package to solve mass-energy equivalence problems. Lower-second class honours will be awarded for partial syntactic knowledge and reasonable accuracy in the homework assignment.

Tutorial 1 - Revision Questions

These questions are designed to establish a baseline from which we will work. The University Pathways course is designed to radically accelerate your learning in only six weeks, and the assessment is also handy to see how much progress we've made! Try your best to answer all questions, but don't worry about getting them all right - just do your best and we'll improve on this over the coming weeks. As in university, you'll have to use the internet to help you figure out some of the answers.

Question 1: Change the subject of the equation to x for each of the expressions below. Give your answer in its simplest form.

$$y = mx + c, \qquad a = \frac{x - b}{c + d}, \qquad f = \frac{\sin x}{\cos x} \tag{1}$$

Question 2: This main topic in the course is the equation

$$E = mc^2 \tag{2}$$

where energy E is measured in Joules, mass m in kilograms and the speed of light, c, is a constant approximately equal to 3×10^6 m/s. By substituting values into equation (2), determine how much energy is contained in two grams (2×10^{-3} kg) of matter.

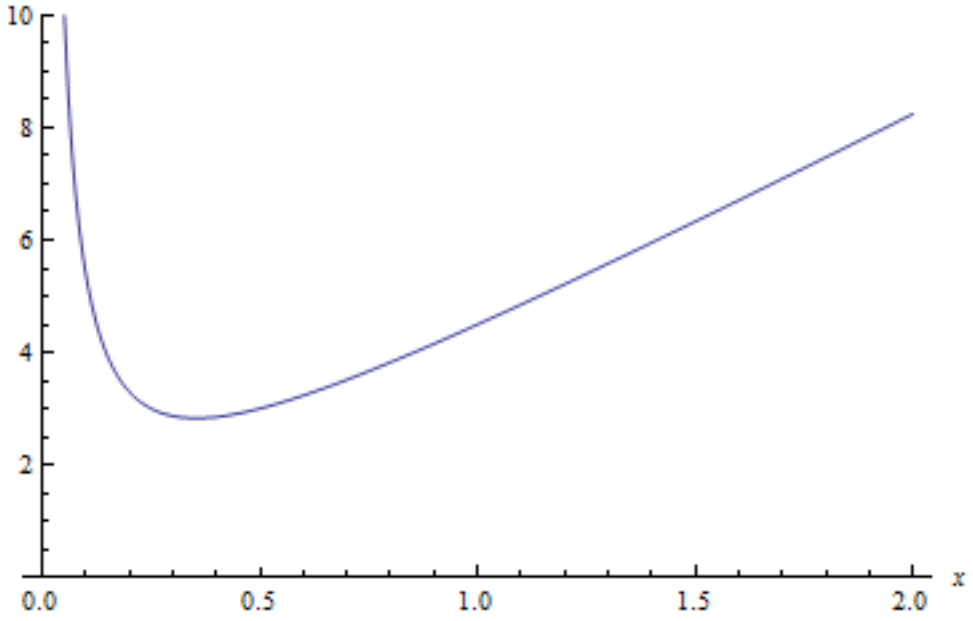
Question 3: Order the following energies from largest to smallest

$$10 \text{ fJ}, \quad 1.5 \text{ kJ}, \quad 18 \text{ eV}, \quad 0.95 \text{ TJ}, \quad 2\pi \text{ pJ}, \quad 100 \text{ GeV} \tag{3}$$

where the unit eV is called the electron volt. **Hint:** use Wikpedia to help you convert electron volts to Joules.

Question 4: Examine the graph of the function on the next page. How would you describe the behaviour of the function when

1. $x < 0$
2. $0 < x \leq 0.4$
3. $x > 0.4$


10
8
6
4
2
0.0
0.5
1.0
1.5
2.0
x

Tutorial 2 - Mass-Energy Equivalence I

In this question sheet we will write a short essay and perform some calculations using the concept of *mass-energy equivalence* introduced in tutorial 2. As previous, it is expected that you are resourceful in order to find all the answers. This is great training for university, and starts you off ahead of the pack.

Question 1: The allied nuclear bombing of Nagasaki occurred on the 9th of August, 1945. Imagine that someone asks you:

How could a bomb only a metre or so wide cause such tremendous destruction?

Your answer should be written in full sentences and of course contain reference to $E = mc^2$. It ought to include the basic ideas from nuclear physics you can read about on Wikipedia and summarise in your own words. Five hundred words should be sufficient to explain the physics, but don't worry if you're significantly under or over. Understanding is always more important than a word count.

Question 2: What is the mass equivalent of...

1. 1 J of energy
2. 100 GJ of energy

Question 3: *Cosmology* is the field of theoretical physics that deals with the large scale structure of the universe. There is a cosmological theory called *modified gravity* that believes that the speed of light was different in the distant past. Suppose that a physicist working on modified gravity determines that the speed of light shortly after the big bang was not $c = 3 \times 10^8$ m/s, but one of the three values below:

1. $c = 2 \times 10^{21}$ m/s
2. $c = 6 \times 10^4$ m/s
3. $c = 3 \times 10^{13}$ m/s

She wishes to know which of values of c would result in the largest value of...

1. The energy content of a physical system of mass 0.5 grams,
2. The mass of a particle with 1 eV of energy

Which value(s) would you tell the physicist to further her research? Explain how you determined your answer(s).

Tutorial 3 - Mass-Energy Equivalence II

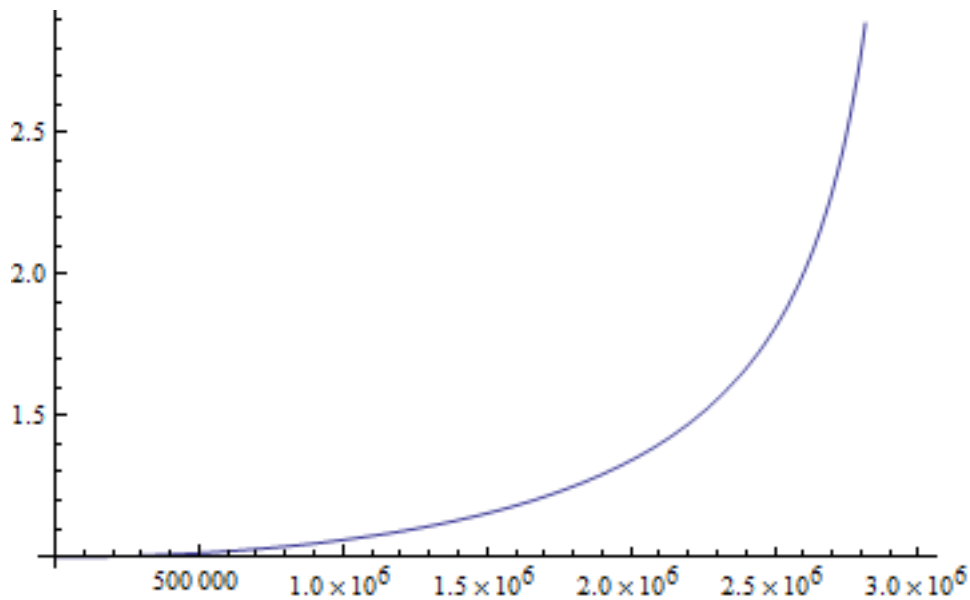
Question 1: Recall that the relativistic mass m of a particle is related to its rest mass $m_{0)}$ according to the equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

where v is the velocity of the particle and c the speed of light. This equation is sometimes written as $m = \gamma(v) \times m_0$ where $\gamma(v)$ is called the *Lorentz factor*

$$\gamma(v) := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

The graph of the Lorentz factor $\gamma(v)$ against v looks like this:



where the units on the v-axis are metres per second. With this is mind, what is the relativistic mass of an apple of mass 0.25 kg falling at a speed of 1m/s from a tree? Does this example explain why relativistic effects are not relevant for objects with velocities much smaller than the speed of light? Explain your answer.

Question 2: You pick up a stone and throw it over a cliff. Does it have a different mass when it leaves your hand compared to when you found it on the ground? Explain your answer.

Question 3: I make a chicken pie and bake it in the oven. Assuming that none of its

physical constituents are lost to the atmosphere during the cooking process, does its relativistic mass change? Explain your answer.

Question 4: One joule of energy is removed from a massive system. How many grams of relativistic mass does the system lose?

Question 5: Consider two moles of hydrogen reacting with a mole of oxygen to give a mole of water. Is rest mass conserved in this reaction? If so why, if not why not?

Question 6: What does the radiation from a twenty kiloton nuclear bomb weigh? **Hint:** in other words, calculate the relativistic mass of the energy released by the bomb, using Wikipedia for the definition of *kiloton*.

Question 7: Discuss the validity of the statement

> *Whenever energy is added to a system, the system gains mass.*

Tutorial 4 - Assignment

Question 1: How much energy is contained within a 1 kg mass?

Question 2: What is the mass of a particle with relativistic energy content 20MeV?

Question 3: Does a photon have...

1. Rest mass?
2. Relativistic mass? Explain your answer

Question 4: All masses gravitationally attract. Would I therefore be gravitationally attracted to a stream of photons? Explain your answer.

Question 5: In 2012, the UK reported that electricity usage for the UK was 274801 GWh. What is the mass equivalent of this energy?

Question 6: In 2011, Yorkshire used 24123 GWh of electricity over the course of the whole year. What is the mass equivalent of this energy?

Question 7 - Extra Credit: A light bulb, suspended from the ceiling, is lowered into an ideal container (*i.e.* one from which no radiation can escape.) Is the relativistic mass of the container more, less or the same when the bulb is switched on? **Hint:** You may wish to recall the full form of the mass-energy equivalence relation. Explain your answer.

Question 8 - Extra Credit: Using the mass-energy equivalence relation,

$$E^2 = \left(m_0 c^2\right)^2 + p^2 c^2 \tag{6}$$

derive an expression for the relativistic mass of a photon. Thence use the fact that $E = hf$, where h is a number called Planck's constant, to find a relationship between the frequency of a photon and its relativistic mass.

Tutorial 5 - Mass-Energy Equivalence with `Mathematica`

Congratulations - you've made it through the assignment to the final tutorial of the course! This last tutorial is not assessed, as it designed to give you an even deeper appreciation for the magnitude of the numbers occurring when studying $E = mc^2$.

For each of the questions, take a screengrab using the `PrtSc` key and paste the grab into Microsoft Paint. Save the picture with convention `studentname-question`$[n]$`.jpg`

Question 1: Use `WebMathematica` to plot the Lorentz factor

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{7}$$

from tutorial 3.

Question 2: Consider a meteor travelling at 10^3 m/s towards the Earth. Calculate the Newtonian kinetic energy $E = \frac{1}{2}mv^2$ and the relativistic kinetic energy. What about if the meteor was travelling at only 10 m/s?