Space Batteries... It's not rocket science

Key Stage 4 Programme

Pupil Name	
Handbook Designed by	Sam Kelly

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1	Algebra	
Tutorial 2	Logarithms	
Tutorial 3	Graph work	
Tutorial 4	Radioactivity – I	
Tutorial 5	Radioactivity - II	

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after midnight on 9th August	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

The Brilliant Club KS4 Programme – Pupil Feedback Report

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at AS-level
3rd	40-49	Performing to a good standard at AS-level
Working towards a pass	0-39	Performing below a good standard at AS-level
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Name of PhD Tutor	Sam Kelly		
Title of Assignment			
Name of Pupil			
Name of School	Cedar Mount Academy		
ORIGINAL MARK / 100		FINAL MARK / 100	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the PhD tutor should give an explanation in this section:

Learning Feedback Comment 1 - *How humans can utilise radioactivity.*	
What you did in relation to this Key Learning Priority	How you could improve in the future
Learning Feedback Comment 2 – *How to critically analyse the properties of radioactive sources*	
What you did in relation to this Key Learning Priority	How you could improve in the future
Learning Feedback Comment 3 – *How to construct a scientific argument*	
What you did in relation to this Key Learning Priority	How you could improve in the future
Resilience Comment	
How you showed learning resilience during the course	How you could build learning resilience in the future

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Course Rationale

You have been selected for a mission, not just any mission, but a mission to get to outer space. This mission is not going to be easy, and plenty of issues are going to need to be solved before we can embark. Your task will be to find an energy source that will last for the duration of the trip? Not possible you say? Well not so fast, over the next few weeks you are going to develop the skills necessary to answer such a question.

There are going to be lots of aspects to consider throughout this project. Your mathematical ability, logical reasoning and knowledge of physics will be tested. In order to solve the problem, we will also need to develop and improve our critical thinking as well as add to our knowledge base. We will begin by revising and learning some mathematics including algebra, conversion of units and how to extract information from graphs. Following this, we will look at some fundamental physics concerning the behaviour of atoms and nuclei, and the phenomenon of radioactivity.

Through a series of tutorials, you will have chance to practice all the topics discussed above and hone your skills as a mathematician, a physicist and an engineer. By completing the tasks put forward in this document, you will have the necessary tools to answer the question 'What is the source of energy for a 20-year space flight?' and present your argument to your peers.

Mark Scheme Table

	Skills	1st (70-100)	2:1 (60-69)	2:2 (50-59)
Subject knowledge	Knowledge and Understanding	○ All content included is relevant to the general topic and to the specific question/title ○ Good understanding of all the relevant topics. ○ Scientific terms are defined and used accurately throughout ○ Clear justification on how the content included is related to the specific issues that are the focus of the assignment	○ Most of the content included is relevant to the general topic and to the specific question/title ○ Good understanding of most the relevant topics ○ Scientific terms are used accurately but not always clearly defined. ○ Adequate justification on how the content included is related to the specific issues that are the focus of the assignment	○ Some of the content included is relevant to the general topic and to the specific question/title ○ Good understanding on some of the relevant topics but occasional confusion on others. ○ Scientific terms are used mostly accurately with occasional confusion and often not defined. ○ Some justification on how the content included is related to the specific issues that are the focus of the assignment
Critical thinking	Research and Evidence	○ Inclusion of rich sources of research findings, data, quotations or other sourced material as evidence for the claims/ ideas ○ Use evidence/calculations to support claims/assertions /ideas, consistently clearly and convincingly ○ Evidence of further reading beyond materials provided which were used in an appropriate context ○ Data is effectively analysed and appropriate assumptions/conclusions are reached	○ Inclusion of adequate sources of research findings, data, quotations or other sourced material as evidence for the claims/ ideas ○ Use evidence/calculations to support claims/assertions/ideas, mostly clearly and convincingly ○ Evidence of further reading beyond materials provided ○ Data is analysed and the assumptions/conclusions that are reached are mostly appropriate	○ Inclusion of some sources of research findings, data, quotations or other sourced material as evidence for the claims/ ideas ○ Use evidence/calculations to support claims/assertions/ideas, at times clearly and convincingly ○ Limited evidence of further reading beyond materials provided ○ There is an attempt to analyse data is and draw assumptions/conclusions
	Developing an Argument	○ A point of view or position in relation to the title or question is	○ A point of view or position in relation to the title or question is	○ A point of view or position in relation to the title or question is

		consistently clear ○ Argument/proof exceptionally well-developed and well-justified ○ A position is clearly established in relation to the question, and is developed effectively and consistently throughout the essay ○ Makes links effectively between subjects that have not previously been associated ○ Uses concepts from the tutorials in an unfamiliar context, and does so accurately and confidently. ○ Content is analysed effectively to support the argument	adequately clear ○ Argument/proof clear and well-developed and position justified ○ A position is established in relation to the question, and is well-developed in most of the essay ○ Some evidence of linking subjects that have not previously been associated ○ Use some concepts from the tutorials in an unfamiliar context, but not always accurate ○ Analysis of content to support the argument	somewhat clear ○ Argument/proof clear but not well-developed ○ A position is established in relation to the question, and is well-developed in parts of the essay ○ Limited evidence of linking subjects that have not previously been associated ○ Limited use of concepts from the tutorials in other contexts ○ Some analysis of content to support the argument
	Critical Evaluation	○ Moved beyond description to an assessment of the value or significance of what is described ○ Evaluative points are consistently explicit/systematic/reasoned/justified ○ Effective critiques on the reliability of sources provided	○ Mostly description but some assessment of the value or significance of what is described ○ Evaluative points are mostly explicit/systematic/reasoned/justified ○ Some evidence of critiques on the reliability of sources provided	○ Only description with minimal assessment of the value or significance of what is described ○ Evaluative points are at times explicit/systematic/reasoned/justified ○ Limited evidence of critiques on the reliability of sources provided
Written communication	Structure and Presentation	○ Ideas are presented in paragraphs and arranged in a logical structure that is appropriate for the assignment ○ The introduction clearly outlines how the essay/report will deal with the issues ○ The conclusion summarises all the main points clearly and concisely	○ Ideas are presented in paragraphs and arranged in a structure that is mostly appropriate for the assignment ○ The introduction adequately describes how the essay/report will deal with the issues ○ The conclusion summarises most of the main points clearly ○ Calculations,	○ Ideas are presented in paragraphs and arranged in a structure ○ The introduction mentions how the essay/report will deal with the issues ○ The conclusion summarises some of the main points clearly ○ Calculations, formulas and methods are not always structured, clear to follow

		○ All calculations, formulas and methods are clearly structured, clear to follow and correct ○ Tables and graphs are effectively constructed including appropriate headings, units and scales. ○ All sources are referenced correctly in an agreed format	formulas and methods are mostly structured, clear to follow and correct ○ Most tables and graphs are well constructed ○ Most sources are referenced correctly in an agreed format	and correct. ○ Some tables and graphs are well constructed but contains some errors ○ Some sources are referenced correctly in the agreed format with occasional errors
	Language and Style	○ No spelling, grammar or punctuation errors ○ Units and significant figures are presented accurately throughout ○ Writing style consistently clear, appropriate for scientific documents and easy to follow ○ Accurate and consistent use of technical language and vocabulary	○ Minimal spelling, grammar or punctuation errors ○ Units and significant figures are presented accurately throughout ○ Writing style mostly clear, appropriate for scientific documents and easy to follow ○ Some attempts of using technical language and vocab alary, but not always accurate	○ Some spelling, grammar or punctuation errors ○ Units and significant figures are presented accurately throughout with occasional errors ○ Writing style moderately clear, appropriate for scientific documents and easy to follow ○ Use of simple language and vocabulary effectively but struggles to use technical language

Glossary of Keywords

Word	Definition/Description	Example/Example sentence
Energy		
Radioactive		
Avogadro's number		
Moles		
Proton		
Neutron		
Alpha		
Beta		
Gamma		
Negligible		
Imperative		
Asymmetric		

Tutorial 1 – Introduction - Some maths

Recall and reinforce your knowledge of algebra

- Objective 1: to manipulate and rearrange equations to make a different variable the subject.
- Objective 2: Substitute in numerical values to equations.
- Objective 3: Understand the importance of this in physics to extract quantitative information.

1.1 Some Algebra

Maths and science often requires an understanding of how to rearrange equations to work out particular values. For example, if we take the equation,

Distance = speed x time

or

$$d = s \times t \tag{1}$$

It might be necessary to rearrange the equation so that it is possible to find the speed of an object if the distance and time are already known. How would we do this? As a reminder, here is a worked example.

Let's imagine we want to know the average speed of a car in miles per hour (mph) given that it travelled 135 miles in 2.5 hours.

First of all, let's look at equation (1). In order to find the speed we want to make this letter the subject, that means to get this letter on one side of the equals sign and all the other letters on the other side. **Remember that if we do something to one side of an equation we must do exactly the same to the other.**

To get speed, *s*, on its own we need to move time, *t*, to the other side. We do this by performing the inverse (opposite) operation. Since we are multiplying initially we now need to divide. As it's the t we are trying to move over to the other side we divide the right-hand side by *t* then we must also divide the left side by *t as* follows,

$$d = s \times t,$$
$$d \div t = s \times t \div t,$$
$$\frac{d}{t} = s. \tag{2}$$

Happily, we notice that the *t* now appears on the left side only, and *s* is left alone on the right side. It is common to write equations with the subject on the left-hand side, although this is not absolutely necessary we will do this from now on. Equation 3 then becomes,

$$s = \frac{d}{t}. \tag{2}$$

This equation means exactly the same as equation 2, but the terms appear on opposite sides of the equals sign. This equation is correctly balanced; therefore, we can insert the values provided in the question to find the answer.

$$s = \frac{135}{2.5} = 54. \tag{3}$$

The average speed of the car was *s* = 54 mph.

When equations become more complicated and involve more terms, you need to think carefully about the order in which you perform the inverse operations. Look at the equation below,

$$y = \frac{x}{4} - 3. \tag{4}$$

How would you make *x* the subject? First you must add 3 to both sides,

$$y + 3 = \frac{x}{4} - 3 + 3,$$
$$y + 3 = \frac{x}{4} \tag{5}$$

Then we multiply both sides by 4,

$$4 \times (y + 3) = \frac{x}{4} \times 4,$$
$$4y + 12 = x. \tag{6}$$

I have inserted brackets to make it clear that you need to multiply every term on the left-hand side by 4.

<u>Rules to Remember:</u>

- It is often easiest when manipulating equations to perform addition and subtraction first to get the desired letter as subject.
- Then perform other operations such as multiplying, dividing,
- Finally do any necessary squaring or square rooting.

Let's try one more example, make y the subject of the following equation,

$$x = 3y^2 + 4. \tag{7}$$

What do we need to do first? Correct! Subtract 4 from both sides.

$$x - 4 = 3y^2 + 4 - 4.$$
$$x - 4 = 3y^2. \tag{8}$$

Then divide both sides by 3.

$$(x - 4) \div 3 = 3y^2 \div 3,$$
$$\frac{(x-4)}{3} = y^2. \tag{9}$$

Then square root both sides.

$$\sqrt{\frac{x-4}{3}} = \sqrt{y^2},$$
$$\sqrt{\frac{x-4}{3}} = y. \tag{10}$$

Now try to re-arrange a few equations yourself. In section 4 you are asked to practice inserting values into equations.

Section 1: Make x the subject of the equations below

a) $y = x + 1$ b) $y = x - 4$ c) $y = 1 - x$ d) $y = 2x$

e) $y = \frac{1}{2}x$ f) $y = \frac{x}{5}$ g) $y = \frac{2x}{3}$ h) $y = 2 - x$

Section 2: Make x the subject of the equations below

a) $y = 2x + 1$ b) $y = 3x - 1$ c) $y = 5x + 8$ d) $y = 1 - 2x$

e) $y = \frac{x+1}{2}$ f) $y = \frac{x-2}{4}$ g) $y = \frac{x}{2} + 5$ h) $y = \frac{x}{3} - 1$

h) $y = \frac{2x+5}{3}$ i) $y = \frac{3x-2}{8}$ i) $y = \frac{2x}{3} + 3$ j) $y = \frac{3x}{5} + 1$

Section 3: Make x the subject of the equation below

a) $y = x^2 + 5$ b) $y = 2x^2$ c) $y = 2x^2 + 1$ d) $y = 3x^2 - 4$

e) $y = \sqrt{x} + 1$ f) $y = \sqrt{x + 2}$ g) $y = \sqrt{2x} + 4$ h) $y = 2\sqrt{x + 1}$

Section 4: Answer each of the following. Show your working clearly. Give your answers to 3 sf where necessary.

a) $A = lw$. Find w when A = 8, $l = 10$
b) $A = \frac{(a+b)h}{2}$. Find b when $A = 10, h = 5, a = 1$
c) $D = st$. Find t when $D = 40, t = 11$
d) C = $2\pi r$. Find r when C = 300.
e) $V = l^3$. Find l when V = 50.
f) $l = 2t^2 + 11a$. Find t when $l = 121, a = -2$
g) $x^2 + y^2 = 1$. Find x when $y = -0.5$

Homework 1 – Answer on the pages provided

(1) Make y the subject of the following equations,
(a) $3x = 4y + 4$
(b) $x = x^2 + 4y - 16$
(c) $1/y = 1/(x+4)$

(2) For the equations above calculate the value of y if $x = 3$.

(3) Write down the equation that relates distance (d), speed (s) and time (t) making distance the subject.

(4) Rearrange the equation you have written in question (3) to make time the subject.

(5) Using Einstein's famous equation, $E = mc^2$, (look up the meaning of each term if you don't know them) find the energy equivalence of a 2 kg lump of matter.

(6)
(a) If a straight line goes through the coordinates (2,1) and (6,6) find the gradient, the y-intercept and the x-intercept.

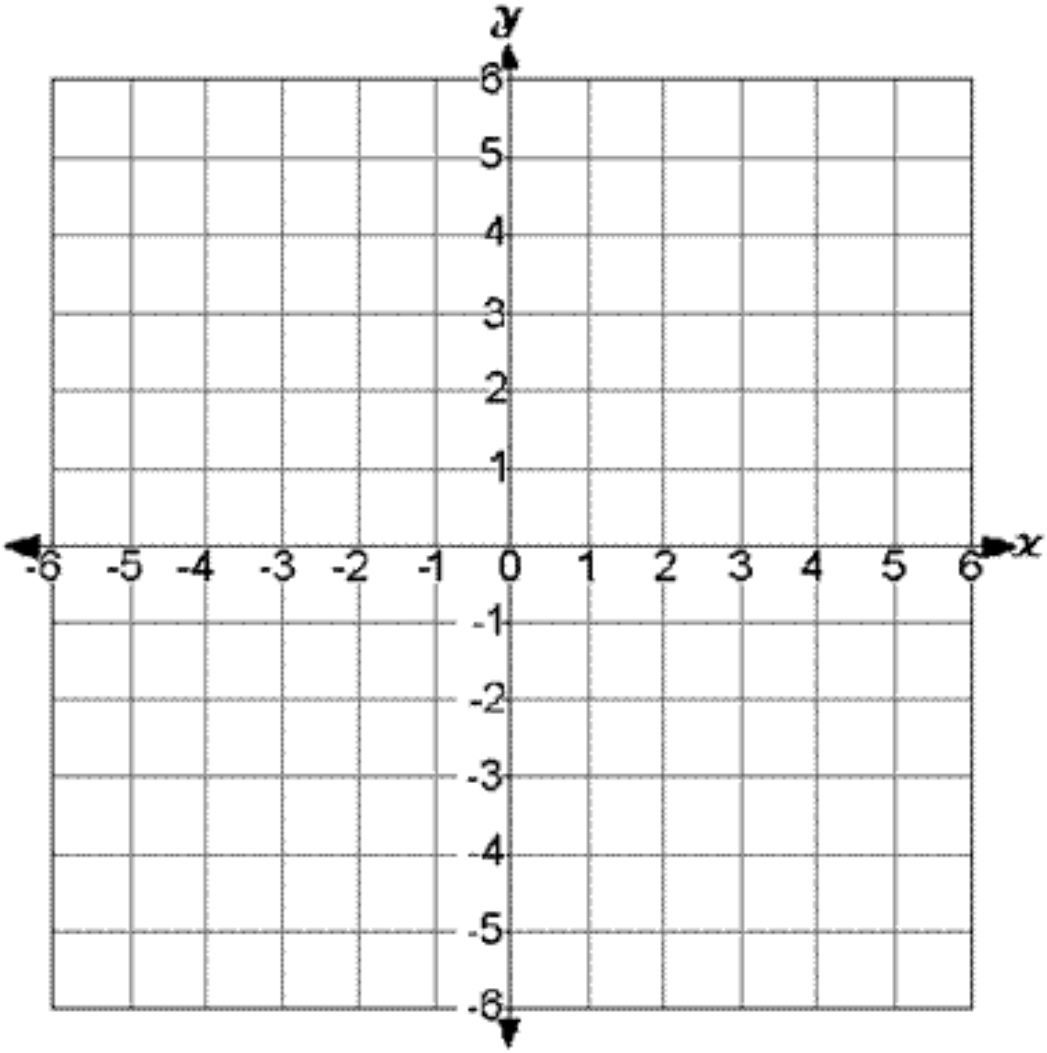
(b) Draw this line on the axes provided

(7) Write one sentence to describe,
(a) An alpha particle
(b) A beta particle
(c) A gamma ray

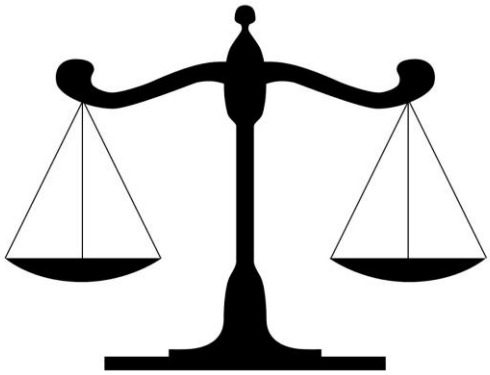
(8) What are the constituents of a nucleus? (Hint: look for two a two-word answer)

(9) <u>Without</u> using a calculator, find the value of,
(a) 3^3
(b) 4^2
(c) 5^3
(d) 2^{-2}

(10) Using a calculator, find the value of, (hint: can you remember what `e' means to a mathematician? Don't worry if you don't yet know.)
(a) 4.23^4
(b) 6.666^2
(c) e^2
(d) $e^{-3.3}$


y
6
5
4
3
2
1
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
x
-1
-2
-3
-4
-5
-6

Tutorial 2 – Units and Conversion



- Objective 1: Look at how to convert units in multiples of 10, e.g. metres to centimetres.
- Objective 2: Test your skills with more complex conversions e.g. between metric and imperial units.
- Objective 3: Understand the reasons for and how to perform conversions between units of mass and energy, a critical skill for the final assignment.

1.1 Useful Units

Scientists and mathematicians have to choose the most appropriate units for the quantity they are measuring. For example, if we were talking about distance or length measurements, the distance from home to school for example, then metres (m) or kilometres (km) would be appropriate. If we were talking about the width of a human hair then we might need the unit of micrometres (μm), which is 0.001 mm!

Pause for Thought Can you think of other measurements where different units are appropriate? Write them down!

Rearranging equations is a really important skill when it comes to converting between units. Let's take the example of distance, how many metres are in 1 km? You probably know that the answer is 1000 m. But if we look more carefully at how you change between units because it can become confusing when the differences are not multiples of 10. We can write that 1 kilometre is equal to 1000 m as an equation,

$$1\ \mathrm{km} = 1000\ \mathrm{m}. \tag{11}$$

Now we can use this to work out how many metres is equal to 35.54 km for example.

$$1\ \mathrm{km} = 1000\ \mathrm{m}.$$

$$\times\ 35.54 \qquad\qquad \times\ 35.54$$

$$35.53\ \mathrm{km} = 35{,}530\ \mathrm{m}. \tag{13}$$

Remember that whatever we do to one side we do to the other. Here we needed to multiply both sides by 35.53.

Let's look at another example of converting units. Britain is quite an unusual country in terms of the units it uses. Think about how you would describe your height to someone else, you will probably tell them in feet

and inches, for example you might say you are 5 feet 9 inches tall (or however tall you are!) However, when you are in school you will mostly use millimetres, centimetres and metres to measure lengths. In addition, you might describe your weight in stones and pounds, alternatively you could say it in kilograms. In either case it is useful to know how to convert between these units.

Below are some useful units to keep in mind for everyday life.

Length	**1 inch**	≈	**2.5 centimetres**
	1 foot	≈	**30 centimetres**
	1 mile	≈	1.6 kilometres
	5 miles	≈	**8 kilometres**
Capacity	1 pint	≈	570 millilitres
	1 gallon	≈	**4.5 litres**
	$1\frac{3}{4}$ pints	≈	**1 litre**
Weight	1 pound	≈	450 grams
	2.2 pounds	≈	**1 kilogram**

Challenge: From the information above can you work out how many inches are in 1 mile?

1.2 Units for our Space Mission

For our space mission, we will be using energy released from radioactive nuclei. Each nucleus is very small; in fact, the weight of each radioactive nucleus is approximately 5×10^{-25} kg (or 0.0000000000000000000000005 kg!) The energy each nucleus releases at one time is also really very small when compared to human scale, typically around 1×10^{-13} J. As you can probably appreciate these units are not very useful when considering nuclei. Instead for energy we use units of electron volts or even Mega electron volts (MeV), which is 1 million electronvolts, and for mass we use MeV/c^2. But what do these units mean?

An electron volt is defined by the amount of energy that an electron would gain when moving in an electric potential equal to 1 volt. It's not too important for us to dwell on what this means at the moment, but what is important is that it is equal to 1×10^{-19} J, a really small amount of energy. Let's write this as an equation again,

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}. \tag{12}$$

What about that strange unit of mass, MeV/c^2? Well this comes from that famous equation that a certain Albert Einstein came up with, $E = mc^2$. This equation tells us that energy, E, is equal to mass, m, multiplied by the speed of light squared, c^2. So, mass and energy are effectively different forms of the same thing, amazing isn't it? You might still be slightly confused, but if we use our rearranging skills from the previous section hopefully things will start to become clear. What happens if we divide both sides by c^2?

$$\begin{aligned} E &= mc^2, \\ E \div c^2 &= mc^2 \div c^2, \\ \frac{E}{c^2} &= m. \end{aligned} \tag{13}$$

The last line tells us that mass is equivalent to energy divided by the speed of light squared. Recalling that for nuclei we use the units of MeV for energy, MeV/c^2 is a unit of mass, which turns out to be very appropriate unit for nuclear masses! You will see the units of eV, MeV and MeV/c^2 in many areas of physics such as quantum physics, nuclear physics, optical physics and many more. Let's see a couple of example problems.

When you are looking at a traffic light that is on red, the energy of the light passing into your eye is about 3.2×10^{-19} J, when it is on amber it is 3.4×10^{-19} J, and when it is on green it is 4×10^{-19} J. What are these energies in eV?

Let's do the first one together, how much energy does the red light have in electron volts? Look again at our equivalence relation,

$$1.6 \times 10^{-19}\,\mathrm{J} = 1\,\mathrm{eV}\,. \tag{14}$$

Which is the same as equation 13 but the numbers written on opposite sides of the equals sign. We want to know is how many electron volts are equal to 3.2×10^{-19} J. We need to find the number to multiply both sides of equation 15; we do this by expressing our problem as an equation,

$$1.6 \times 10^{-19} \times a = 3.2 \times 10^{-19}\,, \tag{15}$$

That is to say that something multiplied by 1.6×10^{-19} gives us 3.2×10^{-19}, where I have labelled the missing value as *a*. We find *a* by rearranging as we have done in previous problems (I'll leave you to check this), giving *a = 2*. We can now multiply both sides of equation 15 by 2 to give the correct answer in eV.

$$\begin{array}{ccc} & 1.6 \times 10^{-19}\,\mathrm{J} = 1\,\mathrm{eV}\,. & \\ \times\,2 & & \times\,2 \\ & 3.2 \times 10^{-19}\,\mathrm{J} = 2\,\mathrm{eV}. & \end{array} \tag{17}$$

So, 3.2×10^{-19} J is equivalent to 2 eV. Try the other two colours yourself.

Challenge: A proton weighs 938.28 MeV/c^2, what is this in kg?

Hint: First convert MeV to eV, and don't forget to divide by c^2.

Homework – 2

1

a) Rearrange the equation PV = nRT to make P the subject.
b) Rearrange the equation C = Q/V to make Q the subject.
c) Rearrange the equation ρ = M/V to make V the subject.

2

a) The formula to convert degrees Celsius (C) to degrees Fahrenheit (F) is given by the equation, F = 9/5 C + 32. Use this equation to calculate how hot in degrees Celsius it is when it is 42 degrees Fahrenheit.
b) The formula to convert Celsius to Kelvin (K) is C = K – 273. Use this equation to calculate how hot in degrees Kelvin it is when it is 20 degrees Celsius.

3

a) What is the definition of π?
b) What is the value of π to 4 decimal places?
c) Can π be written as an exact fraction?
d) What do we call numbers that cannot be written as an exact fraction?
e) Are there any other numbers that cannot be written as an exact fraction that appear in maths and physics? If so list them.

4

You may be familiar with the unit of energy known as the Joule, J. It is the amount of energy used in exerting 1 N of force over a distance of 1 m. There are other units of energy, often used in physics for very small quantities, such as the amount of energy in individual atoms and nuclei; we call this unit the electronvolt (eV). 1 eV is equal to 1.6×10^{-19} J, a really tiny amount of energy.

a) Use the information above to write the value in eV of:
   i. 2 eV
   ii. 32 eV
   iii. 76 keV
   iv. 44 MeV

b) Now calculate the energy in Joules of:
   i. 3.2×10^{-19} J
   ii. 9.4×10^{-19} J
   iii. 13×10^{-17} J
   iv. 1

5 [Challenge]

For a solid object floating in water the height of the object above the water is given by the following equation,

$$H_{total}\left(1 - \frac{r_{object}}{r_{fluid}}\right) = H_{above}$$

The symbols in the equation have the following meanings:

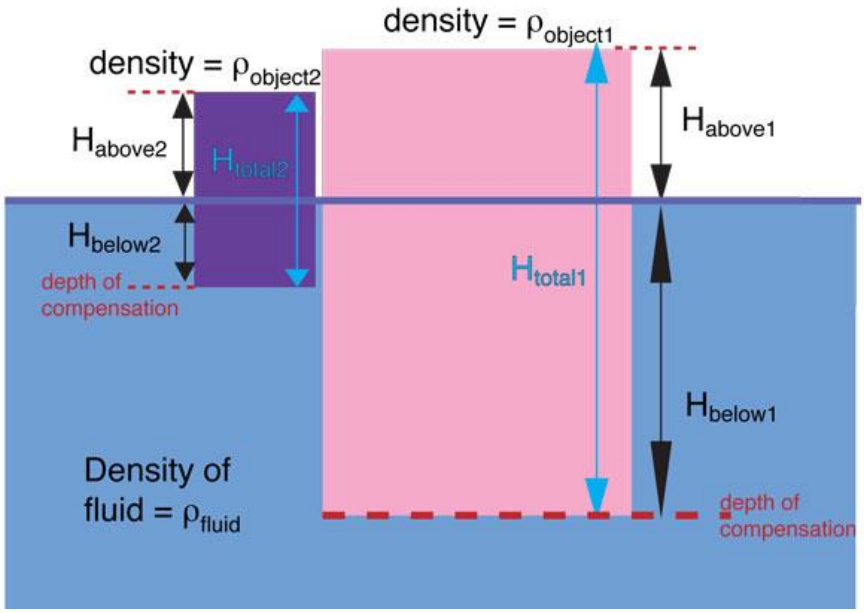
H_{above} Is the height of the object above the water.

r_{object} Is the density of the object.

r_{fluid} Is the density of the fluid.

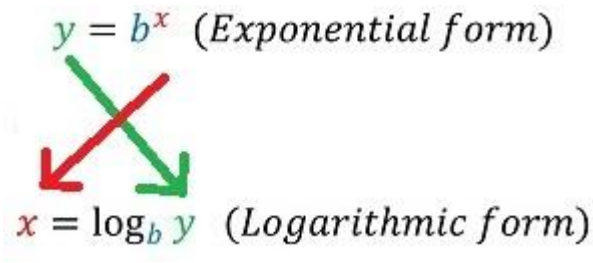
H_{total} Is the total height of the object.

The image below illustrates two objects with different densities floating in a fluid.



a) Rearrange the equation given above to make the density of the fluid the subject.
b) What is the density of the fluid if the total height of the object floating in the fluid is 29.1 km, the height above the fluid is 4.5 km and the density of the object is 2.79 g?

Tutorial 3 – Exponentials and Logarithms



- Objective 1: What are exponentials?
- Objective 2: What are logarithms?
- Objective 3: Switch between exponential form and logarithmic form.
- Objective 4: Introduce e and the natural logarithm, ln.

3.1 Exponentials

In order for you to complete your final assignment you will need to become familiar with exponentials and logarithms. Before we delve straight into exponentials, let's first start with some revision. In primary school when you first learned about multiplication, you were probably taught to think of it as 'lots of' as in 4 x 5 means 4 lots of 5. You could write 4 lots of 5 as,

$$5 + 5 + 5 + 5 = 20, \tag{16}$$

Which works well for small numbers, but as the numbers get larger this approach can become quite cumbersome and time consuming. A more concise approach is to use multiplication,

$$5 + 5 + 5 + 5 = 4 \times 5 = 20. \tag{17}$$

Exponentials can be thought of as a similar progression, if we have a series of numbers multiplied together we can write this in a more concise form. If we have a number multiplied by itself several times, it would be convenient if there was a concise way to write this? Well luckily there is, let's take the sum 5 x 5 x 5 x 5, a mathematician or a scientist would write this is 5^4. That just tells us that 5 has been multiplied by itself four times. Equally they would write 3 x 3 x 3 as 3^3, that's just 3 multiplied by itself 3 times. We call the number being multiplied the **base** and the number of times it is to be multiplied the **exponent**.

See if you can attempt the following problems based on what we've just discussed…

Number	Base	Exponent	Expanded Notation	Standard Notation
2^3	2		2 x 2 x 2	8
3^2		2		9
5^4			5 x 5 x 5 x 5	
	6	2		
	8		8 x 8 x 8	
			9x9x9x9x9x9	
10^3				
	7			49
	2			16
		3		27

3.2 Logarithms

There is another way to express exponentials and that is in the form of a logarithm. This is best understood by looking at another example. We know that 2 raised to the power 4 equals 16,

$$2^4 = 16 \tag{18}$$

Now suppose someone asked us "What power to we need to raise 2 by to get the number 16?" The answer to that question would be 4. We express this in mathematics as,

$$\log_2(16) = 4 \tag{19}$$

These are two equivalent expressions! They mean the same thing, they are just different ways of writing it.

$$2^4 = 16 \iff \log_2(16) = 4$$

This can be generalised so that this can be done with any numbers. It is summarised as follows,

$$\log_b(a) = c \iff b^c = a$$

- b is the base,
- c is the exponent, and
- a is the power.

Pause for thought: In much the same way that multiplication and division are inverse operations; exponentials and logarithms are also the opposite operation.

Let's look at a few more examples so that we are really clear,

Logarithmic form		Exponential form
$\log_2(8) = 3$	$\iff$	$2^3 = 8$
$\log_3(81) = 4$	$\iff$	$3^4 = 81$
$\log_5(25) = 2$	$\iff$	$5^2 = 25$

Now armed with this information, attempt some problems on your own,

1. Rewrite the following in Logarithmic Form:

1) $2^3 = 8$	2) $7^2 = 49$
3) $5^3 = 125$	4) $0.5^3 = 0.125$
5) $2^{-2} = \frac{1}{4}$	6) $64^{\frac{1}{2}} = 8$
7) $10^3 = 1000$	8) $\sqrt{36} = 6$
9) $2^x = 7$	10) $3^{x+5} = 2$

2. Write the following in Exponential Form:

1) $\log_3 9 = 2$	2) $\log_5 125 = 3$
3) $\log_{25} 5 = \frac{1}{2}$	4) $\log_3 \frac{1}{3} = -1$
5) $\log_7 x = 2$	6) $\log_9 x = \frac{1}{2}$
7) $\log_5 y = 3$	

Once you get the hang of rearranging between logarithmic form and exponential form, you can begin to solve equations to find a missing value. If we have two out of the three components (base, exponent and power) then we can find the other. For example,

$$\log_2(16) = x. \tag{20}$$

Now you probably remember that we have already looked at a similar equation earlier in this section, but how would you find the missing value if it is not given? Well remember what we mean when we write a logarithm. The question is asking how many times do we need to multiply 2 by itself to get 16. Practicing what we did in the last section, we could rewrite this equation as,

$$2^x = 16. \tag{21}$$

So you can begin to test your answers, you know that 2 x 2 = 4, 2 x 2 x 2 = 8 and finally 2 x 2 x 2 x 2 = 16. So, there is our answer, we need to multiply 2 by itself 4 times to get 16, or

$$\log_2(16) = 4. \tag{22}$$

This is a relatively straightforward example and you can find the answer to problems like this without a calculator. Try these next few questions, find the values of x, y and z.

1) $\text{Log}_4(16) = x$
2) $\text{Log}_{10}(1000) = y$
3) $\text{Log}_6(216) = z$

It does get slightly more complicated when the base or the power is missing and we do not need to focus on how to do that here. If you feel up to the challenge have a go at the problems on the next page.

Challenge

1. Solve for ‘x’

1) $\log_3 x = 2$	2) $\log_2 (x+3) = 4$
3) $\log_5 (x+50) = 3$	4) $\log_3 (2x-3) = 2$
5) $\log_x 3 = 2$	6) $\log_x 25 = -2$
7) $\log_x 7 = \frac{1}{2}$	8) $\log_{\sqrt{x-2}} 5 = 2$
9) $\log_{\sqrt{2x-1}} 3 = 2$	

3.2 Natural Logarithms and *e*

One last thing to mention in this section is about *e* and the natural logarithm, which mathematicians write as 'ln'. Recall that in previous sections and homework tasks we have briefly mentioned *e*. It is a special number that appears frequently in maths and science, in the same way that π does. Hopefully you know that π is approximately equal to 3.14, but actually it has an infinite number of digits after the decimal place π = 3.14159265... and goes on forever. Similarly, *e* does the same,

$$e = 2.71828... \tag{23}$$

Because this number is so special and features so regularly, it has its own symbol (like π), and will have its own button on your calculator so you can call upon it easily.

Even though this number is special, it still behaves like other numbers when it comes to logarithms and exponentials. For example

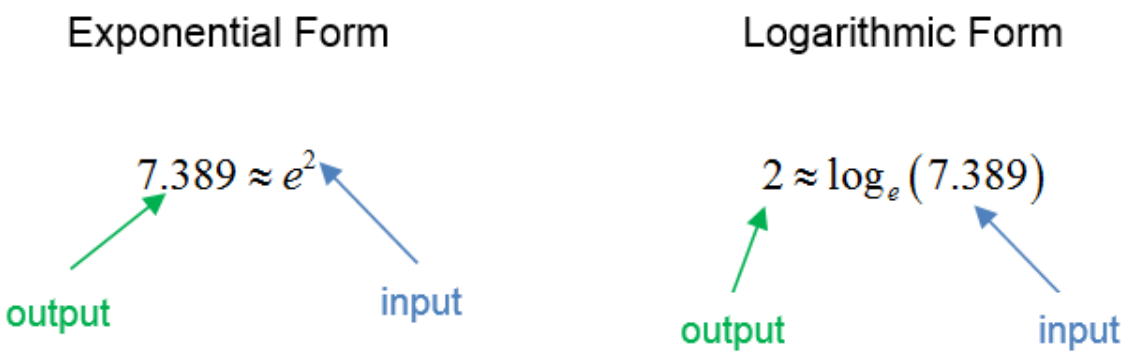
$$\log_e(e) = 1. \tag{24}$$

This is just like writing,

$$\log_{2.71828}(2.71828) = 1. \tag{25}$$

Because *e* is special and has its own symbol, we also give the log with base *e* a special term too, ln.

These functions are, just like with all other logs and exponentials, the inverse or opposite of each other.



$$\log_e(7.389) = \ln(7.389) = 2. \tag{26}$$

Because *e* is an irrational number, (can you remember what an irrational number is?) you are not expected to do many calculations with it in your head; you will most often use a calculator. For example, use a calculator to find *x* in the following problems.

1) x = 3ln(4)

2) x =12ln(84.5)

3) x = $e^{3.4}$

Homework 3- Logarithms and exponentials

1 Calculate the following exponentials showing your working (try not to use a calculator, except for part e)!

a) 8^2 b) 4^4 c) 5^{-3} d) 3^6 e) e^{-2}

2 Write these numbers in decimal form

a) 1×10^4 b) 2.2×10^2 c) 1.6×10^{-3}

3 Calculate the following logarithms without using a calculator!

a) $\log_4 64$ b) $\log_6 216$ c) $\log_4 10$ d) $\log_3 1/243$ e) $\ln e^3$

4 Convert the following between exponential form and logarithm form.

a) $(1/5)^x = y$ b) $\log_a 15/16 = x$ c) $b^a = 123$ d) $\ln(4) = x$

5 Find the value of t in each case

a) $24 = 6e^{3t}$ b) $0.5 = e^{-4t}$ c) $3 = e^{-0.5t}$

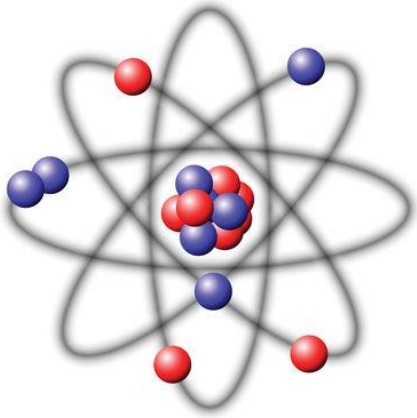
Extension – you may need to recall some skills from previous tutorials.

6 Make t the subject

$N = N_0 e^{-kt}$

Tutorial 4 – Radiation part I

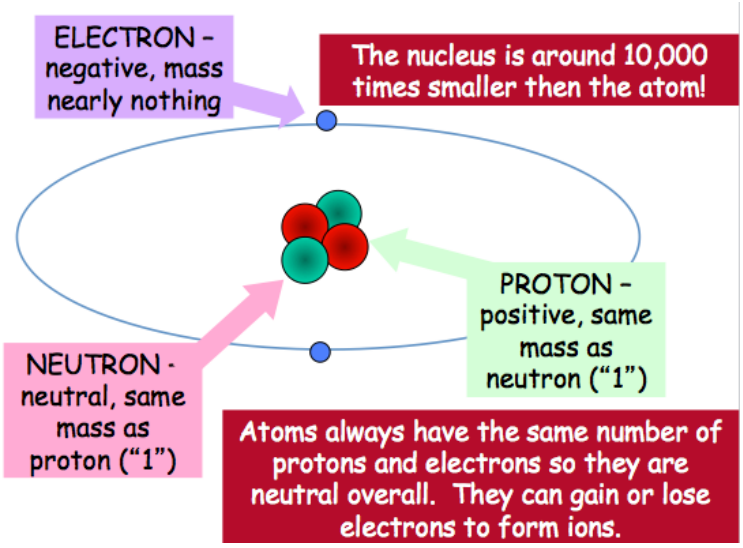
A look inside the atom!



- Objective 1: Introduce the basic concept of a planetary model of the atom.
- Objective 2: Look at the constituents of a nucleus.
- Objective 3: Look at the periodic table/chart of nuclides.
- Objective 4: What are N, Z and A?
- Objective 4: Why would it become unstable (in terms of balancing forces)?
- Objective 5: Calculate the number of molecules of an element using Avogadro's number.

4.1 The planetary model

To choose the most appropriate radioactive source to power the electronics on our space mission, we need to have a good understanding of how atoms and nuclei work. We also need to understand about radioactivity and how we can harness the resulting energy that is released. The picture shown below illustrates what we mean by the planetary model of the atom, and it has been used for around 100 years to describe atomic nuclei. The model describes a small dense nucleus made up of protons and neutrons surrounded by orbiting electrons. The charge of a proton is positive, the charge on an electron is equal in strength but is opposite in sign and neutrons are uncharged. It is important to realise that if we had a microscope powerful enough to actually enable us to see inside an atom it probably would not look like this attractive model. The important thing to remember is that this model works for describing the properties of many atoms and nuclei and the effects we observe around us!



4.2 What makes an element?

Elements make up (almost) everything around us, certainly everything that you can see. The chair that you are sat on, the ink forming the letters you are reading and even the eyes you are using to read with are all made up of many different elements. But what is an element? The answer to this is simply the number of **protons** that are inside the nucleus, nothing more, and nothing less. If I have a nucleus with one proton in it, that is one element, we call that element hydrogen. If we have a nucleus with 2 protons in it, that is a different element and we call that helium. We carry on adding protons to our nucleus until we reach about 82 protons, and we call that element lead, the heaviest stable element. If we keep on adding protons to our nucleus they become unstable, because in our universe, the laws of physics don't allow for more than 82 protons to remain in a nucleus forever.

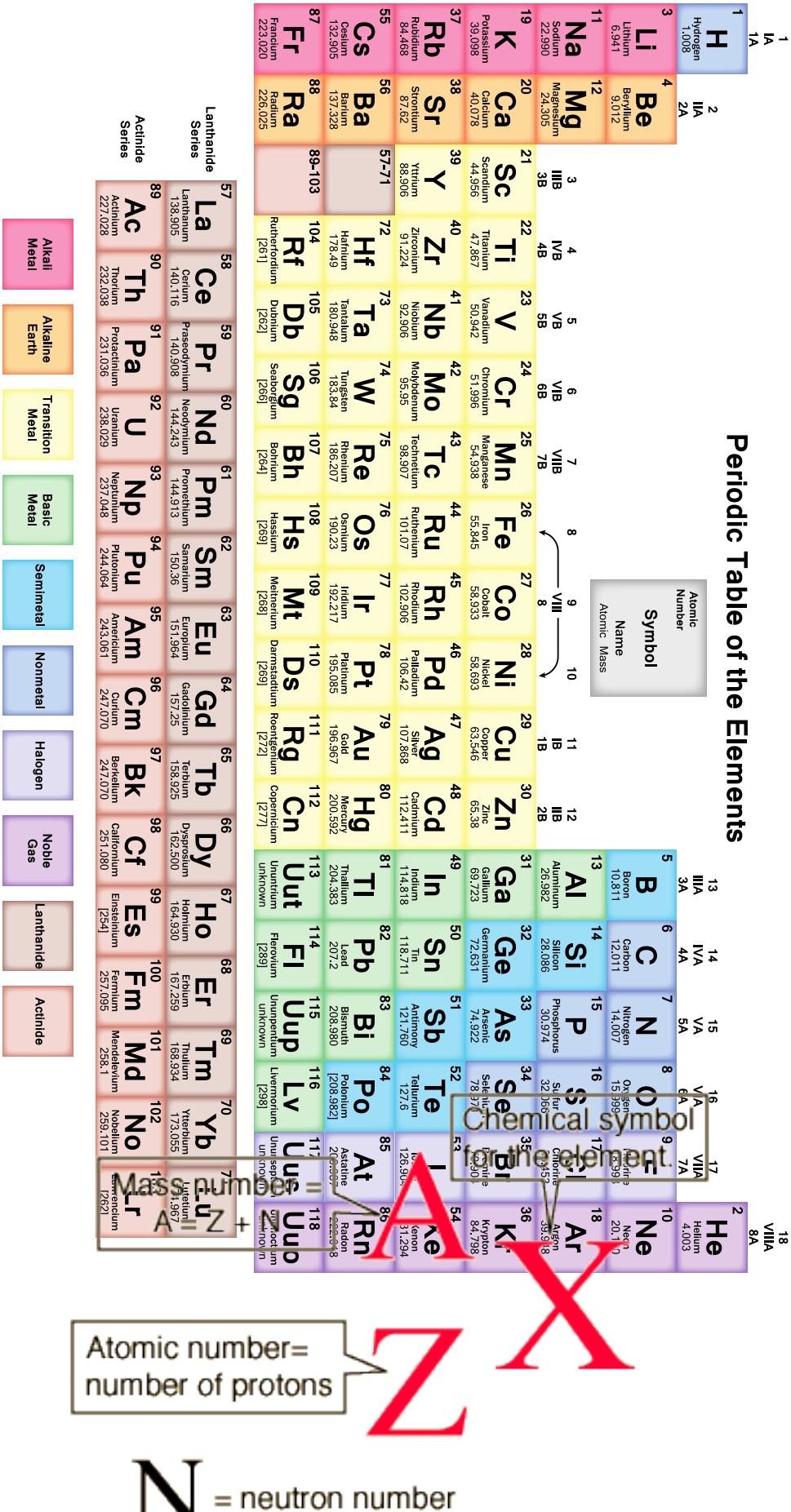
As you might be aware all the known elements have been written down in order, starting with 1 proton and going all the way up through lead and into some really exotic nuclei, that even the very best scientists know very little about. This way of writing the elements is known as the periodic table and is included on the next page. It is a nice way to find any element and lots of different properties about them.

4.3 The importance of the neutron

At the moment, you are probably thinking we've forgotten about something. What about the neutrons? And you'd be correct! The neutrons live inside the nucleus alongside the protons and play an important role. Crucially they neutrons don't change the element. For example, if we take the simplest element, hydrogen, almost all of the hydrogen in the universe is made up of 1 proton, and 1 electron and NO neutrons. It is possible however to find hydrogen with one proton and one neutron inside the nucleus. We call this an isotope of hydrogen, and we give it the name deuterium.

Pause for thought: Do you expect there to be a different number of electrons in deuterium compared with hydrogen?

In chemistry, it doesn't
make much difference
whether hydrogen
has no neutrons or
one neutron, it
doesn't really
change anything in
the chemical
world. But if we delve
deeper and look at the
properties of the
nucleus, it makes a
huge difference
how many neutrons
are present. In physics
we refer to nuclei in a
way in which it is



clear exactly how many neutrons and protons are present. We call this way of writing elements (and isotopes) nuclear notation and we write as shown below.

Let's look at an example. Life on earth is depends heavily upon carbon, in fact humans and animals are known as carbon based life forms. The isotope of carbon that is most abundant in our bodies has 6 protons and 6 neutrons, or in nuclear notation this would be written,

$$^{12}_{6}C \tag{27}$$

Our bodies also contain a very small amount of an isotope of carbon that has 8 neutrons, how would you write this in nuclear notation?

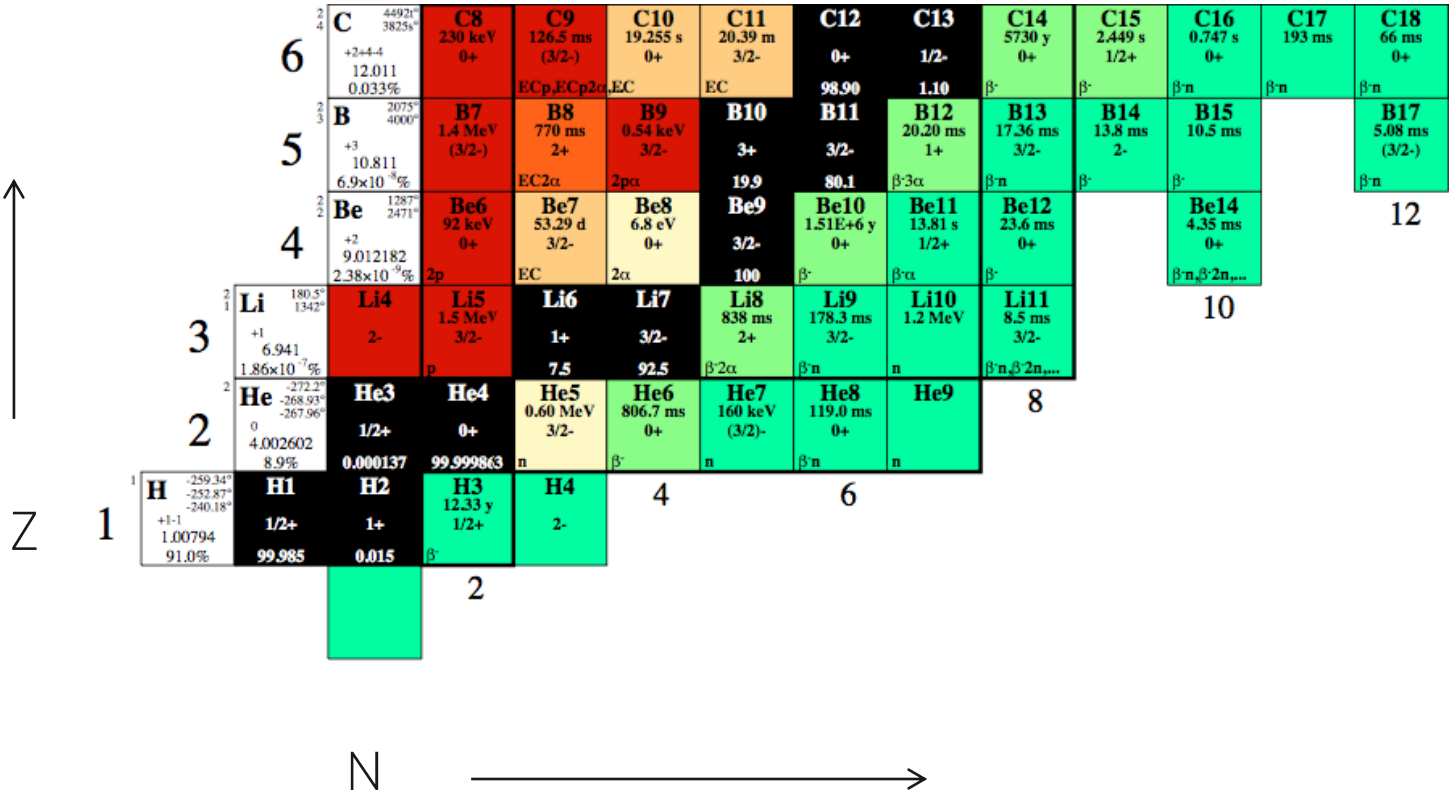
Aside: The technique of carbon dating relies on the presence of this small amount of carbon with 8 neutrons. This isotope of carbon is actually radioactive and so the amount present in the body will change over time. This allows for estimating the age of an object.

It is often common in physics to omit the proton number so $^{12}_{6}C$ would get shortened to ^{12}C. Where it is assumed that you can find out the proton number if required. Have a look at the following questions; write down the numbers of protons, neutrons and the total mass number for the following isotopes (refer back to the periodic table on page 34 to find the element name and hence the proton number).

	A	Z	N
^{238}Pu			
^{241}Am			
^{250}Cf			
^{244}Cm			
^{227}Ac			

4.4 The chart of nuclides

As mentioned in the previous sections, only the number of protons in the nucleus determines an element. But changing the neutron number DOES change the properties of each nucleus and we call nuclei with the same number of protons but different numbers of neutrons isotopes. Although the periodic table is good for referring to any element, we need a more comprehensive picture to show each individual isotope. We call this picture the chart of the nuclides. This chart contains all the known isotopes, which happens to be over 3000! Due to the large number of known isotopes we can't include an entire version here, and so only a small section is shown. To access the entire chart, follow this link: https://www.nndc.bnl.gov/chart/



It may seem confusing at first but it is actually a really concise way of depicting a huge amount of information. By moving right one square you add one neutron and by moving up one square you add one proton. The number of nucleons (protons and neutrons) determines almost everything about each nucleus from its size and shape to whether it is stable or lives for only a fraction of a second. And most of that information is displayed too. The section shown of the nuclear chart shown above contains just the first 6 elements and already there are over 50 known isotopes. We will return to the nuclear chart later after discussing radioactivity in more detail.

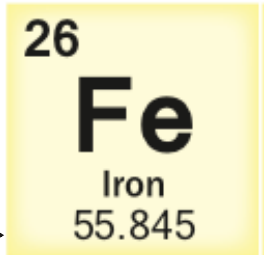
4.5 Calculating how numbers of atoms

To calculate the energy released from a lump of radioactive material we must be able to calculate how many atoms of that radioactive isotope we have. The problem is each atom is very tiny, so tiny that you could fit more than 10,000 lined up side-by-side across the width of a human hair. The number of atoms around us is therefore enormous. How many carbon atoms do you imagine are in a human body? 1 million, 1 billion… 1 trillion? But how could we possibly begin to count each individual nucleus!? Fortunately, we don't need to (Phew!) In order to find the number of atoms we first need to find the number of moles of a substance. Moles are defined as follows,

$$\text{moles} = \frac{\text{mass of substance}}{\text{molar mass}} \tag{28}$$

If a particular isotope is given, e.g. ^{208}Pb, the molar mass in grams is just the mass number, which is equal to the total number of protons and neutrons. The molar mass of an element that can contain a range of isotopes is found by reading the mass number at the bottom of each square in the periodic table.

Mass number this number is in grams! →



Question, what is the molar mass of ^{244}Cm?

Point to remember: Every mole contains the same number of constituent particles and that number of particles happens to be, 6.02×10^{23}, a truly huge number! This number is called Avogadro's number and is given the symbol N_A.

That means 1 mole of Iron contains the same number as 1 mole of carbon which contains the same number of particles as 1 mole of uranium and so on! Let's see an example of how to find the number of atoms in a sample. If we have a 2 kg lump of Iron, how many iron atoms do I have?

First, we need to calculate the number of moles,

$$\text{moles} = \frac{\text{mass of substance}}{\text{molar mass}} = \frac{2000}{55.845} = 35.813. \tag{29}$$

In equation 29 we have remembered to convert kilograms to grams in the numerator. Next, we need to find how many atoms this corresponds to, given that 1 mole contains 6.20×10^{23} atoms.

$$\begin{array}{ccc} & 1\,N_A = 6.02 \times 10^{23} & \\ \times\ 35.813 & & \times\ 35.813 \\ & 35.813\,N_A = 2.156 \times 10^{25}. & \end{array} \tag{30}$$

The number of atoms in 2 kg of iron is 2.156×10^{25}. That's a lot of atoms!

Of course, we could work the other way too. If we know the number of atoms we have we can find the number of moles. Try the following problems

1) How many atoms are in a 3.56 g sample of Cu? Is this a mole of Cu?

2) How man moles are in 3.28×10^{23} Na atoms in salt (NaCl)?

Homework 4 – Title

(You will need to refer back to the periodic table or table of nuclides if you don't know the chemical symbols)

1) What is the molar mass of $^{238}_{92}\mathrm{U}$?

2) How many atoms of $^{235}\mathrm{U}$ would you need to make 4.5 moles?

3)
   a. Using the section of the nuclear chart above, how many isotopes of lithium have been discovered?
   b. Using nuclear notation which isotope of lithium has a half-life of 0.173 seconds?

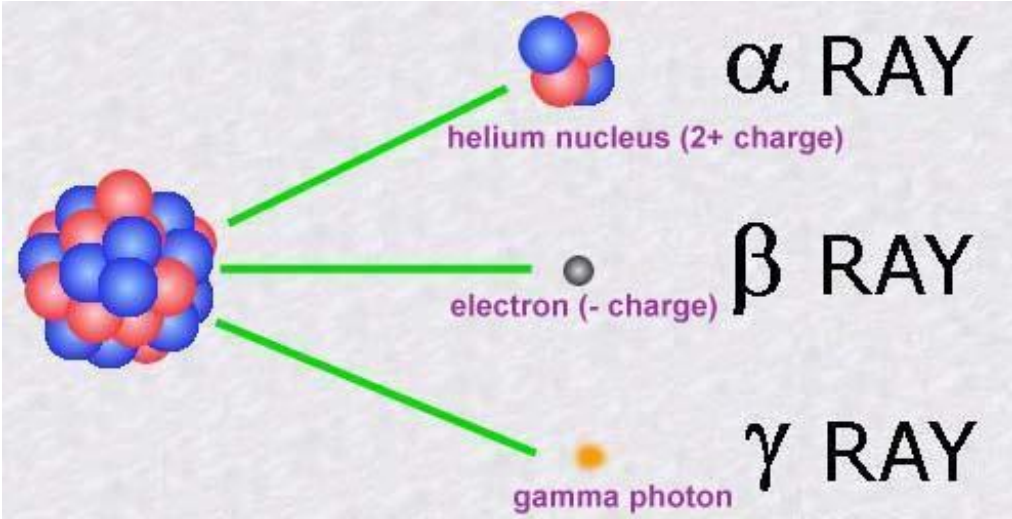
4) How do we refer to two nuclei with the same number of protons but different numbers of neutrons?

5) What defines an element?

6) How many electrons does an atom of $^{16}_{8}\mathrm{O}$ have?

7) What is the mass of 9.67×10^{24} atoms of $^{227}_{89}\mathrm{Ac}$?

Tutorial 5 – Radiation part II



Radioactivity, what is it?

- Objective 1: Introduce alpha, beta and gamma and their properties.
- Objective 2: Understand decay chains.
- Objective 3: Learn how to use the radioactive decay law.
- Objective 4: Be aware of spontaneous fission and how it releases neutrons.

5.1 Radioactivity

So far, we have discussed how elements are formed when more nucleons (neutrons and protons) are added to the nucleus. As we add more protons an equal number of electrons are added and this determines the chemical properties. But if too many neutrons or protons appear in the nucleus the forces binding the nucleons together become unbalanced and the nucleus becomes unstable. We call nuclei that are unstable radioactive. But what happens to radioactive nuclei? They make every attempt to get to a state that is stable! They can do this in several different ways depending on how many nucleons they need to get rid of, in the following sections we will explore the most common types of decay.

Point to remember: Nuclear decay is a random statistical process (perhaps the most random process in nature!) It is absolutely impossible for anyone to predict exactly when a nucleus will decay. The only thing that can be said is that there is a probability that a nucleus will decay. The more unstable a nucleus is, the higher the probability it will decay.

5.1.1 Alpha decay

Alpha decay is most common in large nuclei; uranium, plutonium, thorium and many other heavy nuclei are all most likely to decay by emitting alpha particles. An alpha particle is actually a preformed helium nucleus within a much larger nucleus. When an atom undergoes alpha decay two neutrons and two protons leave the nucleus simultaneously and as a result it is an efficient way of getting rid of 4 unwanted nucleons in one go, which is why it is common in heavy nuclei that are trying to get rid of both protons and neutrons. In physics, we write an alpha decay in the following way,

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He} + \mathrm{Energy}. \tag{30} $$

This means that some unknown nucleus X decays to some other nucleus Y, where Y has two fewer protons and two fewer neutrons, therefore it has four fewer nucleons overall.

5.1.2 Beta decay

For the majority of light to medium mass nuclei the most common decay method is beta decay. There are two different types of beta decay, one that occurs in isotopes that have too many neutrons called beta minus (β^-) decay, and another that occurs in isotopes that have too many protons, which is called beta plus (β^+) decay. Let's look at each process more closely.

Beta minus

Some nuclei have too many neutrons, and the easiest way for a nucleus to get rid of 1 (or more) neutron(s) is by turning it into a proton, in this process an electron is also emitted from the nucleus in order to conserve overall charge as well as another particle called an electron neutrino (actually an anti-electron neutrino but this isn't important here). We write the general beta decay equation as follows,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \overline{\nu_e} \tag{31} $$

Where a nucleus with too many neutrons, X, turns into another, Y, that has 1 extra proton and 1 fewer neutron, hence the total nucleon number stays the same. This way the forces inside the nucleus become more balanced.

Beta plus

If a nucleus contains too many protons it is necessary to get rid of 1 (or more) proton(s) in order to make the forces balance. However, instead of emitting an electron an anti-electron also known as a positron is emitted this time in order to conserve overall charge. In addition, an electron neutrino is emitted (again no need to worry about this here). In a very similar way to beta minus decay the process can be written,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e \tag{32} $$

In this instance some nucleus, X, turns into a nucleus, Y, with 1 fewer proton and 1 extra neutron.

5.1.3 Gamma decay

Gamma emission often follows alpha or beta (plus or minus) decay. The other types of decay leave the resulting nucleus in an excited state, even if the proton and neutron number are now balanced the nucleons themselves are likely to have excess energy. The most efficient way to remove this excess energy is release an electromagnetic wave, which we call a gamma ray. As you probably know visible light, radio waves, microwaves, ultraviolet light, x-rays are all electromagnetic waves the only difference between them is their wavelength. Microwaves and radio waves have quite long wavelength (centimetres or shorter wavelengths ($< 10^{-11}$m) meaning they have much greater energies. Nuclei will continue emitting gamma rays until all the nucleons are in their lowest energy state; we call this state the ground state. We can write a gamma emission as follows,

$$ {}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma \tag{33} $$

Where the asterisk denotes an excited version of nucleus X, and on the right-hand side is the same nucleus in its ground state, having emitted a gamma ray, γ. Crucially there is no change in the nucleons that comprise the nucleus as a result of gamma decay.

5.1.4 Spontaneous fission

A more rare and exotic type of radioactive decay is spontaneous fission. Certain very heavy species have such unstable nuclei that it is sometimes energetically favourable to them to split into two asymmetric

nuclei these are called the fission products. This process is very dramatic and causes multiple neutrons to be ejected as well as a large amount of energy. It is the fission process that provides all the energy to power generators in nuclear power plants. Spontaneous fission can be written,

$$ {}^{A}_{Z}X \rightarrow {}^{A1}_{Z1}Y + {}^{A2}_{Z2}Z + \left(N - (N1 + N2)\right) \times {}^{1}_{0}n \tag{34} $$

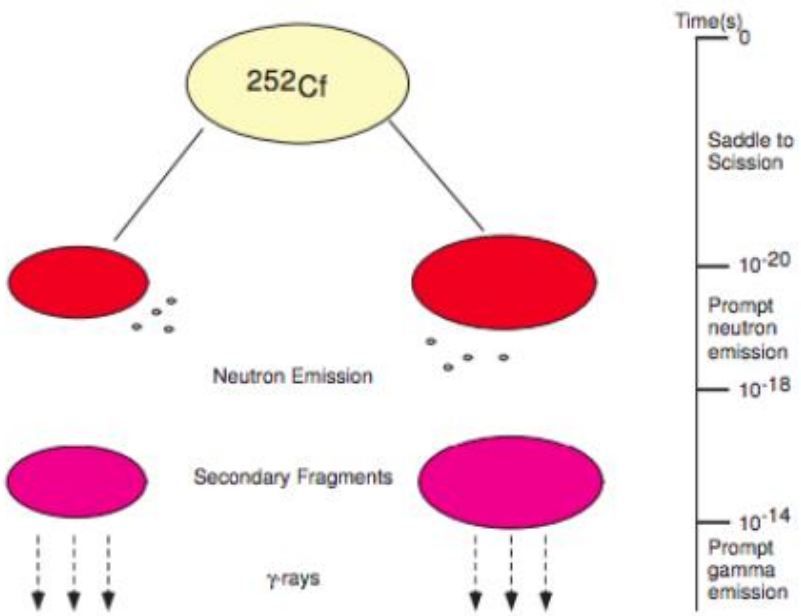
Where N is the number of neutrons in the original nucleus, N1 and N2 are the number of neutrons in the fission products and (N-(N1+N2)) is the number of neutrons that are ejected.

Side note: Each time a nucleus undergoes fission it does not necessarily split into the same two nuclei, but can split into a few different pairs with a given probability. These pairs are always asymmetric, one heavier nucleus (around A = 150) and one lighter nucleus (around A=100). This is not directly important for the work here but is worth knowing.

This might be better understood if we look at an example. Take ^{252}Cf, there is approximately a 3% chance that it will decay via spontaneous fission, and some of the time it will turn into a ${}^{102}_{40}\mathrm{Zr}$ and ${}^{148}_{58}\mathrm{Ce}$. But if you count up the number of neutrons in these two fission fragments you get 2 fewer than you would expect, that's because these neutrons are ejected from the nuclei. We can write this following equation 34 as,

$$ {}^{252}_{98}\mathrm{Cf} \rightarrow {}^{102}_{40}\mathrm{Zr} + {}^{148}_{58}\mathrm{Ce} + 2 \times {}^{1}_{0}n \tag{35} $$

If we look at the time scale of the fission process it is incredibly short, here is a pictorial representation of what happens when a nucleus undergoes nuclear fission.



Question:

a) Can you show that the above equation 35 is true?

b) If one of the fission fragments is found to be zirconium, can the other fission fragment be a different element than Ce?

5.2 The universal radioactive decay law

All the decay methods discussed above can and do happen at random within a nucleus. Sometimes multiple decay routes compete with each other. Looking again at the example of ^{252}Cf, 97% of the time it will alpha decay to ^{248}Cm, and 3% of the time it will spontaneously fission to different pairs of medium mass nuclei. We cannot predict exactly when each process will happen, but we can say for a large sample of ^{252}Cf that approximately 3% of the decays seen will be spontaneous fission. Regardless of the type of decay taking place, the number of radioactive nuclei in a sample will change over time. The way in which it changes is described by the radioactive decay law, which is written as follows,

$$N = N_0 e^{-\lambda t} \tag{36}$$

Hopefully after the previous tutorials and homework assignments you are becoming familiar with the form of this equation. It states that if at a given time there are N_0 of a given radioactive nuclei at a later time, t, there will be N nuclei. The number of nuclei remaining will depend upon λ, which is known as the decay constant it has units of 1/time or time^{-1}. The decay constant is specific to each radioactive substance, so a sample of ^{235}U will have a different decay constant to ^{252}Cf and so on.

Questions:

1) If there are two samples of radioactive nuclei with decay constants λ_1 and λ_2 and $\lambda_2 > \lambda_1$, which radioactive sample will decrease faster?

2) The decay constant for ^{90}Y is 0.01 hrs^{-1}. If I have 6.4 x 10^{22} nuclei at a given time, how many will I have after 1 hour?

3) What fraction of a radioactive sample with a decay constant of 1 minute^{-1} will be left after 2 minutes? Give your answers as a decimal.

This equation can be rearranged to work out the decay constant for a particular sample if the other parameters are known. Or if the decay constant is known and the original sample size and final sample size are known we can find the amount of time that has elapsed.

Have a look at these slightly more involved problems:

4) Using the radioactive decay law shown in equation 36, attempt the following problems (remember how to rearrange an equation with

(a) The time taken for a sample of the radioactive substance strontium 90 to decay to half of its initial mass is 28 years. Show that the value of λ is approximately 0.024755 yrs^{-1}.

(b) A sample of strontium 90 has a mass of 1 gram. Assuming this mass has resulted from radioactive decay, use the model to find the mass this sample would have had 100 years ago. Give your answer to three significant figures.

Something you may be familiar with is the half-life of a radioactive sample. It is defined as the period of time it takes for half the sample to have decayed. It is related to the decay constant and can be expressed as,

$$t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda} \tag{37}$$

Where $t_{1/2}$ is the half-life of a collection of radioactive nuclei. Rearranging the universal radioactive decay law and using the definition of half-life will yield the formula in equation 37. It might be worth trying this, as this will be useful in the final assignment.

Now try this question

5)

a) The half-life of ^{99m}Tc is 6 hours, what is the decay constant?
b) After how much time will 1/16th of the radioisotope remain?

5.3 The energy released in radioactive decay

The consequence of a nucleus decaying is that a large amount of energy, relative to the mass of the nucleus, is released. It transpires that the mass of the parent nucleus is always slightly larger than the sum of the masses of the particles emitted in the decay process. This missing mass has been converted directly into energy one again following Einstein's famous $E = mc^2$ equation. But what sort of energies are involved in nuclear decay? Well every alpha decay releases about 5 MeV of energy (that's 5 million eV), and spontaneous fission can release up to around 200 MeV. Remember that 1 eV is equivalent to 1.6×10^{-19}J, so individually each decay only releases a very small amount of energy. But also consider that in the real world each radioactive sample could be grams or even kilograms, which means there would be billions and billions of atoms, all together releasing a lot of energy. See if you can attempt the following problem:

Question: How much energy is released from the alpha decays from 1 gram of ^{229}Th in one day (24 hours)?

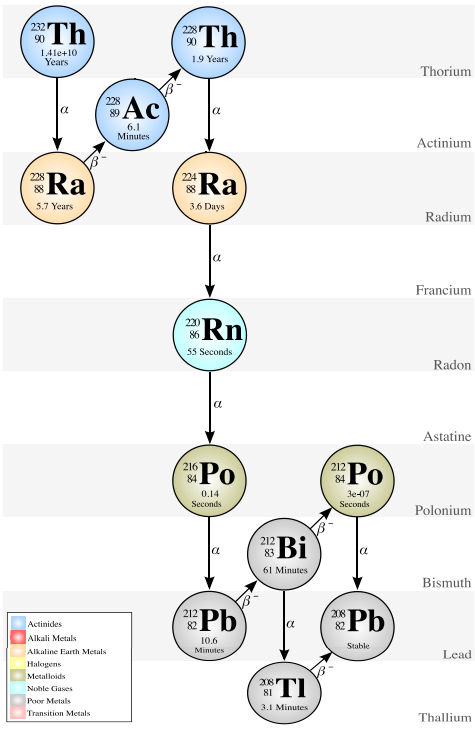
The half-life of ^{229}Th is 7932 years
The energy released per decay is 5.17 MeV

In nuclear reactors we utilise the fission process to generate heat. It turns out around 3 kg of nuclear fuel will generate 3 billion joules every second! That's a huge amount of energy. For comparison, to get the same amount of energy from coal you would need 10,600,000 kg!

5.4 Mapping radioactive decay using the nuclear chart

As each nucleus decays it creates a new one. For the example discussed above, when a ^{252}Cf alpha decays a new ^{248}Cm nucleus is formed. This new nucleus is also unstable and will eventually decay too; in fact, each nucleus will decay until it reaches something stable. We call this path from original radioactive nuclei to final stable nuclei the decay chain and we can map it on the nuclear chart. Let's look at the example of ^{232}Th, which eventually decays to ^{208}Pb via many alpha and beta decays as shown below.

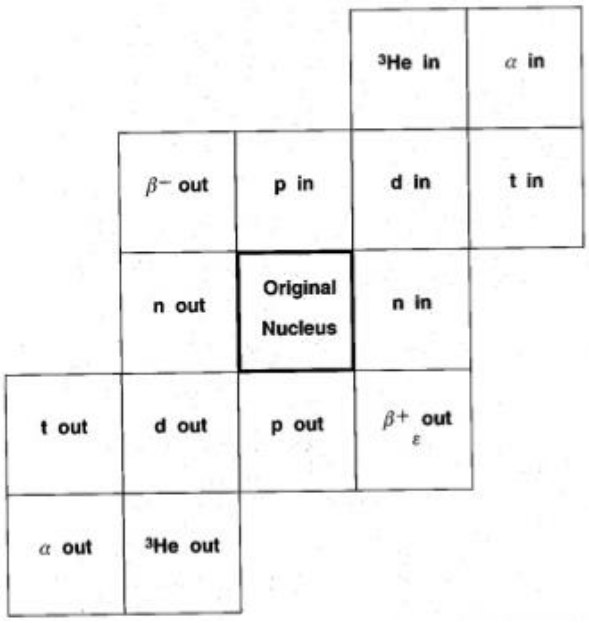
We can use the nuclear chart to work out where we will end up if we take start with a radioactive isotope. Using the same example of ^{232}Th, which alpha decays 100% of the time as indicated by the nuclear chart,



we lose two protons (move 2 spaces left) and two neutrons (move 2 spaces down) and arrive at ^{228}Ra, which beta minus decays, so we lose a neutron and gain a proton (move 1 space left and 1 up) and arrive at ^{228}Ac, and so on. The beginning of the decay path has been mapped out using red boxes in the figure below.

226Th 30.57 M α: 100.00%	227Th 18.697 D α: 100.00%	228Th 1.9125 Y α: 100.00% 2O: 1E-11 0%	229Th 7932 Y α: 100.00%	230Th 7.54E+4 Y α: 100.00% 24Ne: 6E-11 0%	231Th 25.52 H β-: 100.00% α: 4E-11%	232Th 1.40E+10 Y 100% α: 100.00% SF: 1.1E-9%	233Th 21.83 M β-: 100.00%	234Th 24.10 D β-: 100.00%
225Ac 10.0 D α: 100.00% 14C: 4E-12%	226Ac 29.37 H β-: 83.00% ε: 17.00%	227Ac 21.772 Y β-: 98.62% α: 1.38%	228Ac 6.15 H β-: 100.00%	229Ac 62.7 M β-: 100.00%	230Ac 122 S β-: 100.00% EF: 1.2E-6%	231Ac 7.5 M β-: 100.00%	232Ac 119 S β-: 100.00%	233Ac 145 S β-: 100.00%
224Ra 3.6319 D α: 100.00% 14C: 4.0E-9%	225Ra 14.9 D β-: 100.00%	226Ra 1600 Y α: 100.00% 14C: 3.2E-9%	227Ra 42.2 M β-: 100.00%	228Ra 5.75 Y β-: 100.00%	229Ra 4.0 M β-	230Ra 93 M β-: 100.00%	231Ra 104 S β-: 100.00%	232Ra 4.2 M β-: 100.00%
223Fr 22.00 M β-: 99.99% α: 6.0E-3%	224Fr 3.33 M β-: 100.00%	225Fr 3.95 M β-: 100.00%	226Fr 49 S β-: 100.00%	227Fr 2.47 M β-: 100.00%	228Fr 38 S β- < 100.00%	229Fr 50.2 S β-: 100.00%	230Fr 19.1 S β-: 100.00%	231Fr 17.6 S β-: 100.00%
222Rn 3.8235 D α: 100.00%	223Rn 24.3 M β-: 100.00%	224Rn 107 M β-: 100.00%	225Rn 4.66 M β-: 100.00%	226Rn 7.4 M β-: 100.00%	227Rn 20.2 S β-: 100.00%	228Rn 65 S β-: 100.00%	229Rn 12.0 S β-: 100.00%	230Rn >300 NS β-: 100.00%

When you use the nuclear chart to map different decay paths, you can use the following rules to help you locate which your nuclei will decay to. There are additional routes for other decays not discussed here, but you only really need to concern yourself with alpha, beta plus and beta minus. Using the figure below we see that alpha decay moves 2 left and 2 down, beta minus moves 1 left and 1 up, and beta plus moves 1 right and 1 down.



Final Assignment – A suitable radioactive power supply

The final assignment will consist of two sections; the first section will be questions that you are required to find numerical answers to, while the second section will be to write a short essay. Please see below for details.

Deep space missions run by organisations such as NASA often require spacecraft to leave the earth's atmosphere for many years. This poses a great problem: how is it possible to power all on-board electronics for such a long period of time? The method often employed is to use the energy released from radioactive materials. For years, ^{238}Pu has been used as a reliable source of heat energy, which is converted to electrical energy via generators. The attributes that make it such a good candidate are its half-life of 87.7 years (not too short and not too long) and the fact that it emits very few gamma rays that could damage equipment over time. In addition, its daughter nucleus (the nucleus it decays into) is ^{234}U, which has a much longer half-life of 2.46 x 10^5 years, meaning that very few subsequent decays will occur in a 20-year mission and additional radiation will be negligible.

If we have such a good source of energy, what's the problem? Well ^{238}Pu is running out - and fast! A 20-year mission requires around 2 kg of ^{238}Pu and the world supply is currently around 6 kg. It is therefore imperative that we find a replacement source. It is your task to assess the attributes of 4 different radioactive sources and decide which is the best candidate to be used in place of ^{238}Pu. The sources are ^{241}Am (241-americium), ^{250}Cf (250-californium), ^{244}Cm (244-curium) and ^{227}Ac (227-actinium). You will need to calculate the mass required of each radioactive sample that will provide the same amount of energy as ^{238}Pu. But, crucially, to choose the best replacement you need to find which of the three sources produces the least harmful radiation (gamma rays in this case). What you have learned in the series of tutorials should provide you with the tools to answer all these questions. Make sure you refer back to worked problems and homework sheets if you need a reminder of certain topics.

Part 1

1 Calculate the number of decays that will occur in a 2 kg sample of ^{238}Pu.

a) First calculate the number of atoms in a 2 kg sample of pure ^{238}Pu. [3]

b) Calculate the lifetime, λ, of ^{238}Pu given that it has a half-life of 87.7 years, (Use $\lambda = \ln(2)/t_{1/2}$). [3]

Bonus – can you derive the equation for the lifetime from $N = N_0e^{-\lambda t}$? [5]
(Remember the definition of half-life: the time it takes for half the sample to decay).

c) Use your answer to part b) to calculate the number of ^{238}Pu atoms remaining after 20 years using the formula $N = N_0e^{-\lambda t}$. [3]

d) From your answers to a) and c) calculate how many decays occurred in the 20-year period? [2]

e) Each alpha particle is emitted with an energy of 5.593 MeV. What is this energy in Joules? (Remember 1 MeV = 1×10^6 eV) [2]

f) Using your answers to parts d) and e) calculate the amount of energy in Joules that is released from the ^{238}Pu sample in 20 years. [4]

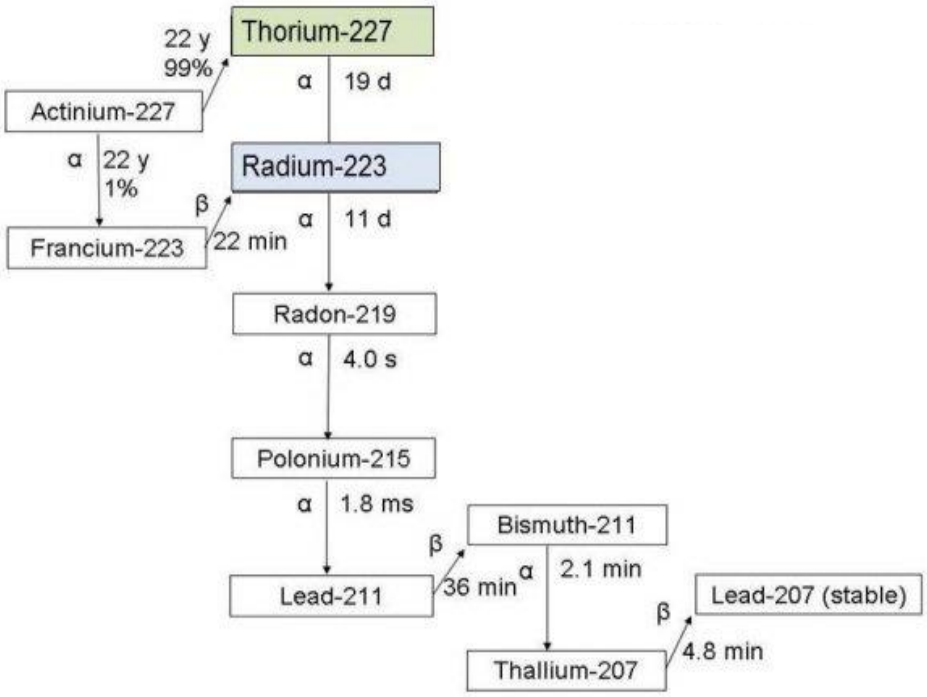
2 Now you need to compare the amount of energy for each of the other radioactive samples. Repeat parts a) to e) in question 1 but replacing ^{238}Pu with ^{241}Am, ^{250}Cf and ^{244}Cm So in total you will have the find the amount of energy that 4 different 2 kg radioactive samples will release in 20 years. All information needed for each radioactive element is given below. **Note – you do NOT need to calculate the energy released from ^{227}Ac, it is given as 25 times larger than ^{238}Pu.**

Table 1: Important information regarding the radioactive isotopes you need to answer questions 1 and 2.

	Half-life (years)	Spontaneous fission (%)	Alpha energy (MeV)	Gamma emissions (keV)	Daughter nucleus	Half-life of Daughter (years)
^{238}Pu	87.7	1.9×10^{-7}	5.593	N/A	^{234}U	2.5×10^{5}
^{250}Cf	13.08	0.08	6.03	N/A	^{246}Cm	4706
^{241}Am	432	4×10^{-10}	5.485	N/A	^{237}Np	2.1×10^{7}
^{244}Cm	18.1	1.4×10^{-4}	5.804	N/A	^{240}Pu	6561

^{227}Ac is slightly different as it mainly decays via beta emission, and its decay chain is given below. Look carefully at the radiation that comes from Lead-211 and Thallium-207. Use the link https://www.nndc.bnl.gov/chart/ to find more about these nuclei.

Figure 1: Decay chain for ^{227}Ac, this source provides 25 times the amount of energy as ^{238}Pu. The decay scheme is shown above.



Part 2

From your answers to part 1 and the information given above, pick the best candidate radioactive isotope to replace ^{238}Pu. Write your answer in the form of an essay that is no more than 500 words.
[20]

The structure of the essay should look something like this:

Introduction: Introduce the reader to the problem: why is this investigation important? What are you trying to find?

Main body of the essay: Should compare the different radioactive sources in terms of their energy output, the types of radiation each gives off, the amount of spontaneous fission for each source and the half-life of any daughter nuclei.

Conclusion: Summarise by choosing the best of the 4 potential replacements and justify your answer.

Tutorial 6 – Feedback tutorial

What is the Purpose of Tutorial 6?

- To receive feedback on final assignments.
- To share examples of best practice with the other pupils in your group.
- To write targets for improvement in school lessons.
- To reflect on the programme including what was enjoyed and what was challenging.

Final assignment feedback

What I did well…	What I could have improved on…
•	•
•	•
•	•

My target for future work is…

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
• • •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Appendix 1 – Referencing correctly

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

Why should I reference?

Referencing is important in your work for the following reasons:

- It gives credit to the authors of any sources you have referred to or been influenced by.
- It supports the arguments you make in your assignments.
- It demonstrates the variety of sources you have used.
- It helps to prevent you losing marks, or failing, due to plagiarism.

When should I use a reference?

You should use a reference when you:

- Quote directly from another source.
- Summarise or rephrase another piece of work.
- Include a specific statistic or fact from a source.

How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. **A marker in your assignment**: After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].
2. **Bibliography**: This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

    a. **Websites** – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

    E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

    b. **Books** – Author, date published, title of book (in italics), pages where the information came from.

    E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

    c. **Articles** – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

    E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

Appendix 2 – Using the VLE

VLE username	
VLE password	

Please remember the following key details…

- You are able log into the VLE either through the link on our website (www.thebrilliantclub.org) or going directly to the VLE site at (https://portal.thebrilliantclub.org/sign-in).
- Please update your profile with your full name and email address- this will allow you to retrieve forgotten passwords or usernames
- If you forget your log-in details you can request them to be emailed to you by clicking the link on the VLE home page. (If you are still having problems you can email: schools@thebrilliantclub.org)

What is the VLE?

The VLE is a virtual learning environment for all pupils on Uni Pathways it is used for:

- messaging your tutor
- submitting homework
- submitting your final assignment
- accessing resources for your tutorials
- finding out more information about university and careers

How should I use the VLE?

The VLE is a professional academic environment in which pupils are able to message their PhD Tutor. Here are a few things to consider:

- Ensure you keep a professional tone in the messages you send to your tutors.
- Ensure you always reply to your tutors in a timely manner.
- Thank your tutor for the effort they are putting in to give you your feedback etc.
- Submit all homework to your tutor on time.

IMPORTANT: Final assignment

- When you submit your final assignment, please remember that you need to do so through the 'My Activities' tab and not as an attachment to a message.

Notes

Notes

Notes

UNI PATHWAYS

Course Title

Key Stage 4 Programme

researchersinschools.org