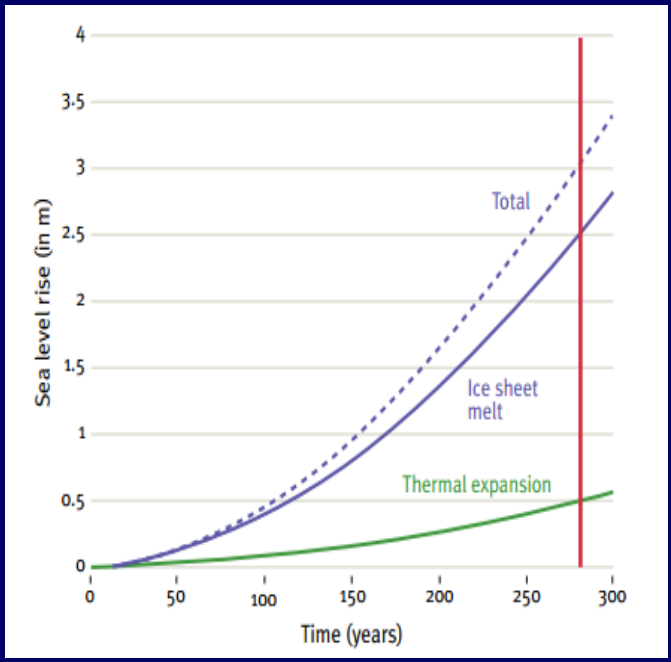
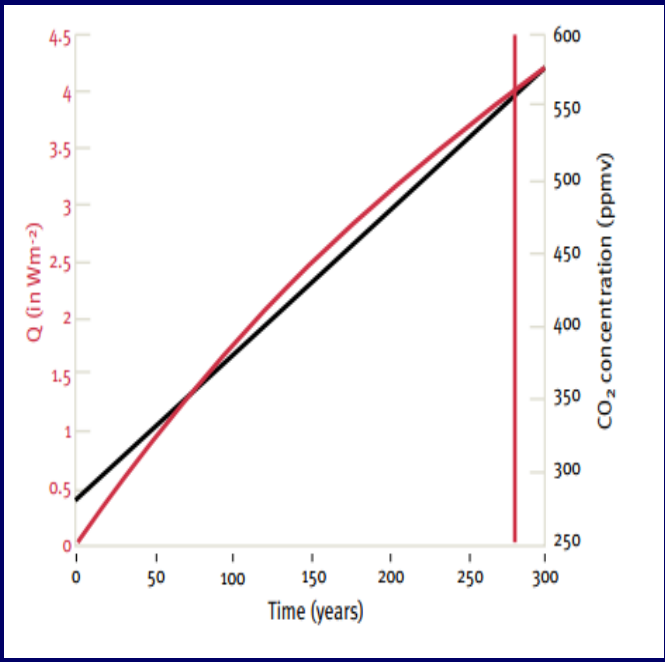
Towards Understanding the Relationship Between Sea Surface Temperature and Sea Surface Height in a Warming Climate: A Mathematical and Statistical Approach.

Key Stage 4 Programme – 2016-2017





$$m_o c_o \Delta T_o(t) = A_o \int_0^t Q(t')dt'$$

$$\Delta h_{ice}(t) = \frac{\alpha_T}{\rho_o c_o} \int_0^t Q(t')dt'$$

Teacher Name: Dr. Kowalski

Pupil Name_____

Timetable and Assignment Submission

Fill this in with your teacher.

Timetable – Tutorials

Tutorial	Date	Time	Location
1.Sea surface temperature anomalies and trends.			
2. A simple model of sea surface temperature			
3. A simple linear and non-linear model of sea surface height.			
4. Modelling in EXCEL (part 1)			
5. Modelling in EXCEL (part 2)			
6. Feedback			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1		
Tutorial 2		
Tutorial 3		
Tutorial 4		
Tutorial 5		

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after 4.00pm on Put date here	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

KS4 Programme 2016-17 – Pupil Feedback Report

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-Level
2:1	60-69	Performing to a good standard at A-Level
2:2	50-59	Performing to an excellent standard at current key stage
3rd	40-49	Performing to a good standard at current key stage
Working towards a pass	0-39	Performing below a good standard at current key stage
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Name of RIS Teacher			
Title of Assignment			
Name of Pupil			
Name of School			
ORIGINAL MARK / 100		**FINAL MARK / 100**	
DEDUCTED MARKS		**FINAL GRADE**	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Example

Learning Feedback Comment 1 - ***Enter Key Learning Priority Here***	
What you did in relation to this Key Learning Priority	**How you could improve in the future**
Learning Feedback Comment 2 – ***Enter Key Learning Priority Here***	
What you did in relation to this Key Learning Priority	**How you could improve in the future**
Learning Feedback Comment 3 – ***Enter Key Learning Priority Here***	
What you did in relation to this Key Learning Priority	**How you could improve in the future**
Resilience Comment	
How you showed learning resilience during the course	**How you could build learning resilience in the future**

Contents

Course Rationale Page 5
Mark Scheme Page 6
Glossary of Keywords Page 7
Tutorial 1 Page 9
Tutorial 1 Homework – Baseline Test Page 11
Tutorial 2 Page 12
- Assignment 2 Page 14

Tutorial 3 Page 15
- Assignment 3 Page 18

Tutorial 4 Page 19
- Assignment 4 – Draft of Final Assignment Page 20

Tutorial 5 Page 21
- Assignment 5 – Final Assignment Page 22

Tutorial 6 Page 23
Appendix 1: Derivation of Sea Surface Temperature Model Page 24
Appendix 2: University Application Guidance Page 25

Course Rationale

There is currently great interest in how the earth's climate system will respond to the threat of ever increasing concentrations of carbon dioxide (CO_2) in the atmosphere. Two of the most important climatic variables that are directly affected by CO_2 are sea surface temperature and sea surface height.

The oceans have a tremendous capacity to store heat. However, some of this heat is lost to the atmosphere, thereby influencing weather and climate. The changes in the amount of heat stored in the ocean are partly determined by changes in the sea surface temperature. Sea surface temperatures have been increasing since the advent of the industrial era and it is widely anticipated that sea surface temperatures will continue to increase, subsequently bringing about significant changes to the earth's climate.

It is well known that water expands when heated. For the case of the ocean, this expansion of water leads to an increase in the sea surface height. Thus, the sea surface height must be directly related to sea surface temperature. Like sea surface temperature sea surface height has also been increasing since the advent of the industrial era, and it is possible that some coastal regions may cease to exist.

The above statements are not sufficient for, for example, governments who need to put measures in place for tackling the threat of global warming. So what can be regarded as sufficient? Mathematics has played a crucial role in our understanding of how the world works. In particular, mathematics can be used to derive theoretical relationships between sea surface temperature and sea surface height using the laws of Physics, and these models can be explored and used to estimate the amount by which sea surface height changes due to changes in sea surface temperature. Finally, by analysing observed data using statistical techniques one can also verify the theoretical models.

The goal of this course is to provide pupils with an introduction into how mathematical models and statistics can be effectively used to model sea surface temperature and height variability in a warming climate. The students will derive and explore simple mathematical models of these variables and investigate the models using observed data. This is one approach taken by climate scientists today. Some of the mathematical techniques and statistical techniques that will be used are ones that the pupils have seen, such as solving equations, rearranging formulae and calculating the mean. In addition, pupils will develop their research skills and critical thinking and analyse data using statistical techniques in Excel.

Course Map

Mathematical Models.
Linear vs Non-Linear Models
Laws of Physics, Mass Continuity, Density, Thermal Expansion.
Relationship between sea surface height and sea surface temperature
Recurrence Relations/Iterative Processes
Statistics – Mean, Anomalies, Line of Best Fit (Regression), Trends.

Mark Scheme

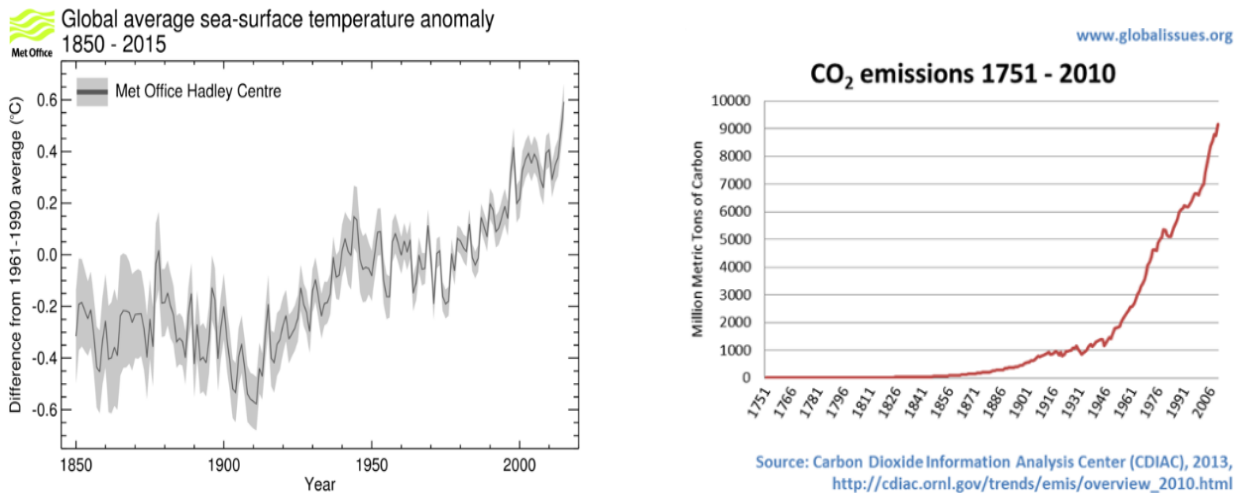
Key Skill	1st	2.2
Communication	• Your writing is coherent and is organised with a logical structure. • Your explanations are clear and concise, and your conclusions are well justified. • Your spelling, grammar and punctuation are almost faultless. • Your writing is of an appropriate style for a scientific report and you correctly use subject-specific vocabulary.	• At times your writing is poorly organised and lacks logical structure. • Your explanations are not always clear and concise, and your conclusions are always well justified. • Your spelling, grammar and punctuation are weak in places. • Your style is inconsistent and your use of subject-specific vocabulary is not always appropriate.
Mathematical/ Statistical Skills	• You can decompose a time series into components representing natural and anthropogenic forcing using standard mathematical techniques and regression in EXCEL. • You have a sound understanding of these methods, how to apply them and what all the terms in the resulting equations represent. • You are able to manipulate higher level formulas. • You have a deep understanding of the results from various plots.	• You can decompose a time series into components representing natural and anthropogenic forcing using standard mathematical techniques. • You can manipulate simple formulas. • You can compare plots and draw simple conclusions.
Understanding of Key Concepts	• You know and understand all the key concepts and can explain them to your fellow peers in a clear and concise way.	• You know the key concepts.

Glossary of Keywords (To be filled out by students during tutorials)

Word	Definition	In a sentence…

Tutorial 1 – What is a Sea Surface Temperature Anomaly? What is a Trend? How can we construct a Simple Model for a Time Series of Anomalies?

Figure (1)



Left panel: time series of global average sea-surface temperature taken from Met Office.
Right panel: time series of CO2 emissions 1751-2010 taken from Carbon Dioxide Information Analysis Centre.

Key Questions

- What is time series?
- What is an anomaly?
- What is a trend?
- Visual analysis of sea surface temperature time and CO2 time series (figure [1]) – can you spot a trend in the sea surface temperature time series? Can you see a relationship between sea surface temperature and CO2? What causes temperature changes around the CO2 trend.
- How can we separate a trend from other factors using simple mathematics?
- What do we mean by natural variability and anthropogenic forcing?

Purpose

Students will know:

- What a time series is.
- What an anomaly is and how to calculate anomalies.
- What a trend is, how to fit a linear trend by deriving an equation of for a line that best represents the trend in a time series (line of best fit). (NOTE IT IS NOT NECESSARY TO DERIVE AN EQUATION FOR THE TREND LINE SINCE STUDENTS CAN READ VALUES OFF THE LINE OF BEST FIT)
- How to separate the global warming trend from natural variability in a data set.
- How to reconstruct the time series by adding the components corresponding to global warming trend and natural variability.

Part 1: Time Series

A time series of a variable X is a series of values of X obtained at successive times, with each value (usually) recorded at equally space time intervals.

- ***Activity 1***

  Students will be given a time series of sea surface temperature and will be asked to identify how often the variable has been sampled (time interval) and when it is sampled (i.e. each month, year, day?), and plot it with axis clearly labelled.

Part 2: Climate Anomalies

If the variables in the climate system were to remain constant at all times our weather and climate would not change. However, in the real world climatic variables such as ocean temperature, atmospheric temperature, solar radiation and winds vary with time. In order to assign meaning to these variations, there must exist some average value for each variable that can be compared with the value of each variable at any point in time. The difference between the variable and its mean value is known as an 'anomaly'. It is defined mathematically as follows:

Let X denote a variable in the climate system at any point in time and $\bar{X}$ its average. The anomaly, which is represented by X', is

$$X' = X - \bar{X}.$$

- **Sea Surface Temperature Anomalies**

  Let $\boldsymbol{T}$ denote the sea surface temperature and $\bar{\boldsymbol{T}}$ the average. The sea surface temperature anomaly is

$$T' = T - \bar{T}.$$

- ***Activity 2***

a) Students will be given two data sets of sea surface temperature taken from the pre-industrial era (no CO2 trend) and will calculate the time series of anomalies and plot them with axis clearly labelled.
b) Students will discuss the possible factors that cause the deviations from the mean temperature.

Part 3: Trend

A trend is a pattern in a data set. The most simple trend line that can be fitted to data is a straight line (linear).

- ***Activity 3***

  Students will be given two data sets of sea surface temperature anomalies taken from after the start of the industrial era. The mean sea surface temperatures for each data set will be specified. Students will be asked to draw a line of best fit for each data set that represents the trend, and derive an equation for the line. The trend line will represent the effect of CO2 on sea surface temperature and will be denoted by T_A'. Students will plot the time series with axis clearly labelled.

Part4: Separating the Sea Surface Temperature Signal into Natural and Human Induced (Anthropogenic) Components

- ***Activity 4***

  In each data set from the previous task, students will subtract the trend line from the data to form a time series for deviations from the trend line and will be denoted by T_N'. Students will plot the time series T_N' with axis clearly labelled. It will be assumed that these deviations describe the component of the sea surface temperature signal that is related to natural variability and this will be justified later in the course.

Part 5: Constructing a Model of Sea Surface Temperature Based on Observations (Data)

- ***Activity 5***

  Using T_A' and T_N' students will reconstruct the time series of the temperature anomaly by calculating

$$T_A' + T_N'.$$

  This time series will be superimposed on the original time series. Students should find that there is an identical match between the data and their model.

Tutorial 1 – Baseline Test

The homework assignment for the first tutorial is a baseline test to see your initial level of attainment in this subject area. The assignment will test for some or all of the subject specific skills that are required later in the final assignment. However, it is shorter than the final assignment and is will be an introduction to the subject as well as a challenge!

Do not worry too much about doing 'well' or 'badly' on the baseline test, it takes into account the fact that you may not be familiar with the subject area. It is designed to help you and your PhD tutor identify where you are at the start of the programme and to help you measure your progress along the way.

In this assignment students will be asked to decompose a temperature anomaly time series into components related to anthropogenic and natural forcing, rearrange formulae and perform a simple research task in preparation for the next tutorial.

Assignment 1

Q1) What is

a) A time series?
b) An anomaly?
c) Anthropogenic forcing?
d) Natural variability?

Q2) You are given a data set of observed sea surface temperature anomalies.

a) Plot the data with axis clearly labelled.
b) How often has the data been sampled?
c) Derive the components of the time series that are related to anthropogenic and natural forcing.
d) Derive a formula for the total time series.
e) Plot the model time series to verify that is an identical match to that of the original data.

Q3) Research the natural factors that influence sea surface temperature.

Q4) Make P the subject of the formula *(Hint: you may have to factorise……)*

$$P = Q - aP$$

Q5) Show that $Z = m(K - hZ)$ can be rearranged and written in the form

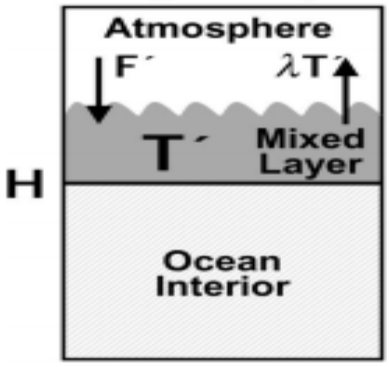
$$Z = AK,$$

where

$$A = \frac{m}{mh+1}.$$

Tutorial 2 – A Model of Sea Surface Temperature Variability

Figure (1)



Conceptual ocean-atmosphere system. Ocean temperature changes T' are confined to an upper layer of depth H known as the mixed layer. Temperatures are driven by forcing F' and damped back to the atmosphere at a rate $\lambda T'$. *(Figure taken from Deser et al. (2003)-Understanding the Persistence of Sea Surface Temperature Anomalies in Mid-Latitudes.)*

Key Questions

- What is the mixed layer depth?
- What is an ocean-atmosphere system?
- What is specific heat capacity?
- What is atmospheric feedback?
- What are the factors driving natural variability?

Purpose

Students will:

- Be introduced to a simple model of sea surface temperature variability based on figure (1). There is a derivation of this model in appendix (1).
- Fit the model to the data sets used in the previous tutorial.
- Determine if past sea surface temperatures are a good predictor of future sea surface temperatures.

Activity 1

a) Go through Q3) of the homework assignment. List the natural factors influencing sea surface temperature.
b) Go through Q4) and Q5) of the homework (similar algebraic techniques will be used in this tutorial)

Activity 2

Revision of recurrence relations and iterative processes (new to GCSE syllabus).

Activity 3 (in pairs)

Have a look at figure (1). Describe what is happening in this diagram.

Part 1: The model

The ocean is modelled as cuboid of fluid *(e.g. figure [1]).* The cuboid of fluid has a surface area, $\boldsymbol{A}$, and depth, $\boldsymbol{D}$. The changes in sea surface temperature are confined to the mixed layer $\boldsymbol{H}$, with $\boldsymbol{H} < \boldsymbol{D}$, and are determined by the anthropogenic forcing $\boldsymbol{F'_A}$ and forcing associated with natural atmospheric variability $\boldsymbol{F'_N}$, and damped to the atmosphere at a rate $\boldsymbol{\lambda T'}$. As derived in appendix (1) , the model describing changes in sea surface temperature is

$$T'_{i+1} = T'_i + \frac{F'_A + F'_N - \lambda T'_i}{\rho c H}. \qquad (1)$$

The other terms in the model are defined as follows:

- T'_i is the temperature at the start of the time interval.
- T'_{i+1} is the temperature at the end of a time interval.
- ρ is the ocean density.
- c is the specific heat capacity.

The variables in the model are T'_{i+1}, T'_i, F'_A and F'_N. The constants in the model are H, ρ, λ and c. (*Notice that equation (1) is very similar in form to a recurrence relation.)*

- ***Activity 4***

a) Show that equation (2) can be written in the form

$$T'_{i+1} = aT'_i + G'_A + G_N', \qquad (3)$$

where $a = 1 - \frac{\lambda}{\rho c H}$, $G'_A = \frac{F'_A}{\rho c\, H}$ and $G'_N = \frac{F'_A}{\rho c\, H}$.

b) What does a measure in equation (2)?

c) How does varying λ and H affect a, G'_A and G'_H?

Part 2: Fitting the Model to the Data

- ***Activity 5***

Take the data of the temperature anomalies taken from after the advent of the industrial era from tutorial 1. Recall from tutorial 1 that for each data set the time series of the temperature anomalies could be written as the sum of natural and anthropogenic variability

$$T'_A + T'_N.$$

a) In equation (3), which component corresponds to $\boldsymbol{T'_A}$?

b) Hence, which components correspond to $\boldsymbol{T'_N}$?

c) For the first data set, Met Office modelling work shows that $\boldsymbol{a = 0.4}$ and for the second $\boldsymbol{a = 0.7}$. Derive $\boldsymbol{G'_N}$ for both data sets.

d) On the set of axis provided plot $\boldsymbol{T'_i, aT'_i, aT'_i + G'_A, G'_A + G'_N}$ and $\boldsymbol{G'_N}$ for both data sets, labelling the axis clearly. Are past temperatures a good predictor of those in the future?

Assignment 2

Q1) You are given a data set of sea surface temperature anomalies from after the advent of the industrial era. From tutorial 2 the equation describing the evolution of sea surface temperature anomalies is

$$T'_{i+1} = aT'_i + G'_A + G'_N.$$

a) Describe each of the terms appearing on the right hand side of this equation.
b) For the data set derive G'_A. *(Draw a line of best fit and find the equation of this straight line.)*
c) For this particular data set, the Met Office found that $a = 0.7$. Derive G'_N.
d) Are past temperature anomalies are good predictor of those in the future? You may find it useful plotting aT'_i, $aT'_i + G'_A$, $G'_A + G'_N$ and G'_N.

Q2) *(We will be using this model in the next tutorial.)*

It can be shown that

$$\rho V' + V\rho' + V'\rho' = 0, \quad (1)$$

where ρ is the mean (constant) background density, ρ' the density anomaly, V the mean (constant) volume of the upper ocean and $V'$the volume anomaly.

a) Show that this equation can be rearranged to give

$$V' = \frac{-V\rho'}{\rho+\rho'} \qquad (2)$$

b) To a good approximation, density depends on temperature. A simple relation between the density and temperature anomaly is

$$\rho' = -\rho\alpha T', \qquad (3)$$

where α is the coefficient of thermal expansion. Substitute equation (3) into equation (2) and show that

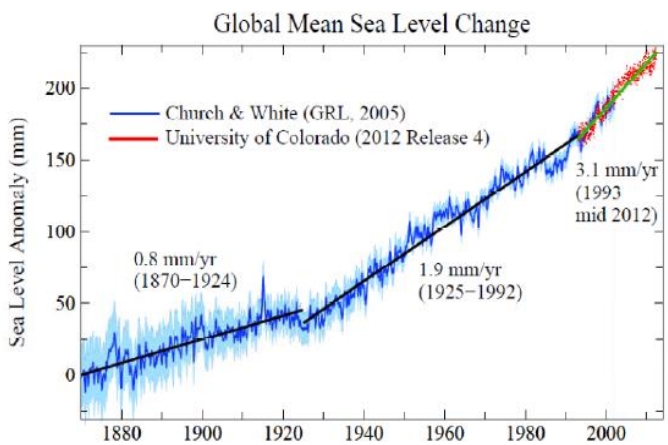
$$V' = \frac{V\alpha T'}{1-\alpha T'}. \qquad (4)$$

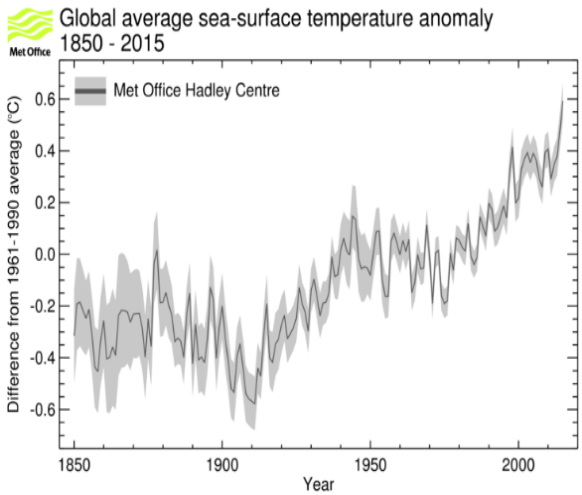
c) Given that $V = AH$, where A is the surface area of the ocean and H is the mixed layer depth, and $V' = Ah'$ where $h'$is the sea surface height, show that substituting V and V'into equation (4) gives

$$h' = \frac{H\alpha T'}{1-\alpha T'} \qquad (5)$$

Tutorial 3 – A Simple Linear and Non-Linear Model of Sea Surface Height

Figure (1).





Global mean sea level (sea surface height) and sea surface temperature anomaly. Notice that sea level has been increasing with temperature since 1900.

Key Questions

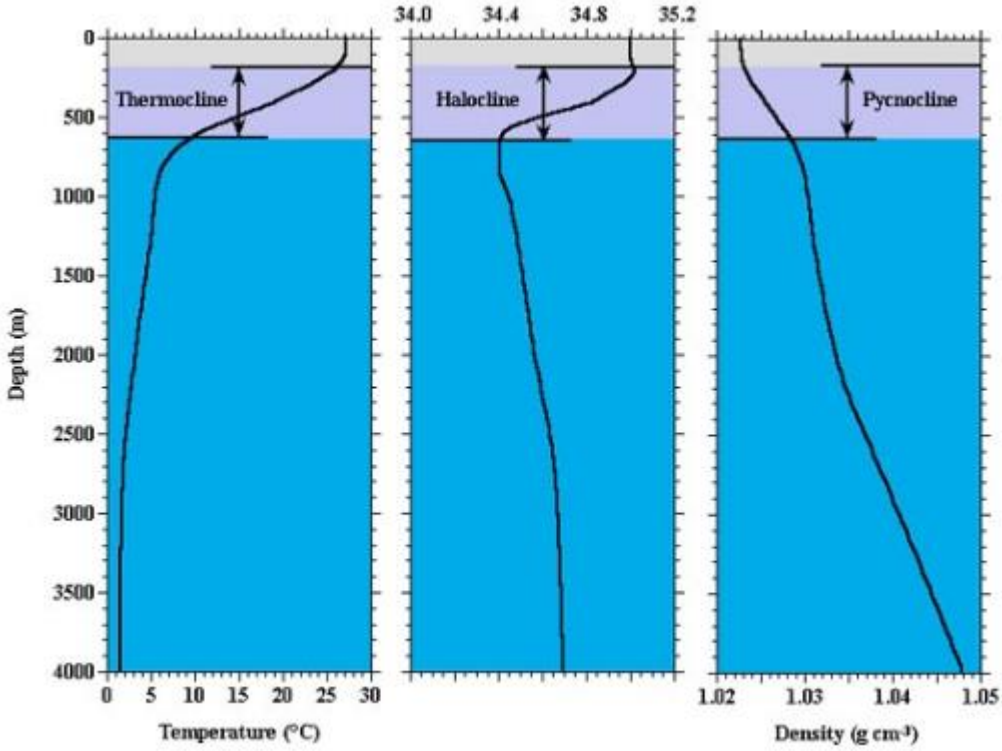
- What is a linear and non-linear model?
- Why does density (to an approximation that is useful) depend linearly on temperature?
- What is mass conservation?
- What does thermal expansion mean?

Purpose

Students will:

- Derive equation (2) from question (2) in the homework, and the relation $V' = Ah'$.
- Investigate the model of sea surface height from homework using the data sets from tutorials (1) and (2).

Part 1: Ocean Density



Density depends on salinity and temperature. In this course we are neglecting the effects of salinity.

Activity 1

- Describe how the ocean density and temperature vary with depth.
- Can you therefore explain why density is related to ocean temperature via

$$\rho' = -\rho\alpha T'$$

- Look at the vertical temperature profile. Where is the mixed layer depth?

Part 2: Mass Conservation

Brief lecture on mass conservation. Students will take notes.

Part 3: Derivation of Equations from Assignment (2).

Activity 2

- Derive $\rho V' + V\rho' + V'\rho' = 0$ using mass conservation principle.
- Derive $V' = A\,h'$.

Part 4: Investigating the Effect of Sea Surface Temperature on Sea Surface Height

Activity 3

Students will take data sets of sea surface temperature from previous tutorials and substitute the data points into the following equation for sea surface height derived in the homework but written as a recurrence relation

$$h'_{i+1} = \frac{H\alpha T'_{i+1}}{1-\alpha T'_{i+1}} \qquad (1)$$

and plot it with a data set of sea surface height. They will be asked if this model is a good fit to the data set.
Students will compare this model with the following simplification of the above equation

$$h'_{i+1} = H\alpha T'_{i+1} \qquad (2)$$

The difference between equation (2) and equation (1) is that equation (1) is linear and equation (2) is non-linear.

Key question: Is the linear model a good fit?

Activity 4

Using the linear model (equation [2]) and the model of sea surface temperature from tutorial (2)

$$T'_{i+1} = aT'_i + G'_A + G'_N$$

show that

$$h'_{i+1} = H\alpha a T'_i + H\alpha G'_A + H\alpha G'_N$$

Activity 5

Determine if past sea surface temperatures ($T_i{}'$) and the anthropogenic forcing (G'_A) can be used to predict sea surface height.

Assignment 3

Q1) You are given a data set of sea surface temperature and sea surface height. It is assumed that the equations describing sea surface temperature and sea surface height are

$$T'_{i+1} = aT'_i + G'_A$$

and

$$h'_{i+1} = H\alpha a T'_i + H\alpha G'_A.$$

Met Office modelling works suggests that appropriate values of H, α and a are $100m, 10^{-4}C^{-1}$ and 0.6 respectively.

a) For the sea surface temperature data use the equation for sea surface temperature and a to derive G'_A. *(Hint: draw a line of best fit as in previous homework and tutorials)*

(b) Now that you have derived G'_A and you know H, α, a and T'_i derive values for h'_{i+1} using the above model. Plot these modelled values of h'_{i+1} with the observed data values of the sea surface height. Does your model fit the data well? What does this suggest about the factors driving sea surface height variations?

Tutorial 4 – Modelling in EXCEL (Part 1)

Key Questions

- What is linear regression?

Purpose

Students will learn that

- Linear regression is a statistical method that essentially derives a line of best fit (i.e. not by hand-inspection).
- The previous tutorials were crucial to understanding what the routines for calculating linear regression models in EXCEL are doing and what the terms in a linear regression model represent.

Furthermore, they will

- Learn how to create a model of sea surface temperature in EXCEL using linear regression.
- Repeat the work of tutorial (1) using EXCEL. Students will use real Met Office Data.

Part 1: Separating the Natural Variability from the Anthropogenic Forcing in a Data Set of Sea Surface Temperature Anomalies

Activity 1:

- Using data set of sea surface temperatures, students will be shown how to fit a trend line in EXCEL to the data. This trend line will represent the component of the sea surface temperature that is related to the anthropogenic forcing.
- It will be shown how to remove the trend from the data. The part of the signal that is left will represent the natural variability.
- Students will be asked to perform this procedure on their own and write down bullet points on how it was done.

Part 2: Creating a Linear Model of Sea Surface Temperature Based on the Model from Tutorial 2

Activity 2:

- It will be shown by example how to fit a model of the form

$$T'_{i+1} = aT'_i + G'_A + G'_N$$

  in EXCEL.

- Students will be asked to perform this procedure on their own and write down bullet points on how it was done.
- Using their model, they will be asked to determine the extent to which aT_i' and G'_A influence T'_{i+1}.

Assignment 4: Draft of Final Assignment

Tutorial 5 – Modelling in EXCEL (Part 2)

Key Questions

- Is sea surface temperature a good predictor of sea surface height?

Purpose

Students will

- Use linear regression to derive a relation between sea surface height and sea surface temperature using real Met Office Data.
- Investigate the model by isolating factors.

A Statistical Model of Sea Surface Height

Activity:

- In the previous tutorial a linear regression model for sea surface temperature was derived in EXCEL. Use linear regression in EXCEL to derive a model between sea surface height and the linear regression model of sea surface temperature. *(Students will be asked to write down how they created the model in EXCEL.)*
- Plot your model sea surface height data against the Met Office sea surface height data and determine if it is a good model.
- To what extent do past sea surface temperatures, anthropogenic forcing and natural variability influence sea surface height?

Final Assignment

The Deadline is:

Leave blank to fill in later, or put your deadline in here when you know it.

Tutorial 6 – Feedback

What is the Purpose of Tutorial 6?

- Reflect on skills learned on the programme with a view to encouraging resilience
- Receive feedback on final assignment

What are the key questions?

- What strengths have I demonstrated in my work and what areas of development are there left for me to address?
- What steps do I need to take to improve my academic output?

Activity 1 – Marking your own

Using the mark scheme, identify three things you did well and three things you could improve.

What went well?

1.

2.

3.

What could be improved?

1.

2.

3.

Activity 2 – Responding to feedback

Read through your feedback sheet and highlight on your assignment where you received positive feedback (one colour) and areas for improvement (different colour). You might find it helpful to annotate this with a brief comment or two.

Activity 3 – Looking Forwards: Think, Pair Share

1. Think about how you can use this feedback in your future school work.
2. Talk to your partner about how you both plan to make changes to your school work in the future as result of this feedback
3. Share your ideas as a group.

Appendix 1 – Derivation of Sea Surface Temperature Model

Let us model the ocean as cuboid of fluid (*figure[1] in tutorial 2).* The cuboid of fluid has a surface area, A, and depth, D. The changes in sea surface temperature are confined to the mixed layer H, with $H < D$. The mass of the fluid in the mixed layer is given by

$$M = \rho V = \rho AH, \qquad (1)$$

where ρ is the density of the sea water.

Changes in sea surface temperature are described by the following formula

$$\Delta \mathrm{T}' = \frac{f'}{Mc}, \qquad (2)$$

where $\boldsymbol{f'}$ is the forcing, $\boldsymbol{M}$ is the mass and $\boldsymbol{c}$ the specific heat capacity. Substituting equation (1) into equation (2) yields

$$\boldsymbol{\Delta \mathrm{T}'} = \frac{\boldsymbol{f'}}{\boldsymbol{\rho AcH}}. \qquad (3)$$

Let us write $\boldsymbol{F'} = \boldsymbol{f'}/\boldsymbol{A}$ so that equation (3) can be written

$$\boldsymbol{\Delta \mathrm{T}'} = \frac{\boldsymbol{F}'}{\boldsymbol{\rho cH}}. \qquad (4)$$

Let $\boldsymbol{\Delta T'}$ denote the a change from a value $\boldsymbol{T'_i}$ to $\boldsymbol{T_{i+1}}$ so that

$$\boldsymbol{\Delta \mathrm{T}'} = \boldsymbol{T'_{i+1}} - \boldsymbol{T'_i}. \qquad (5)$$

Equation (4) can then be written

$$\boldsymbol{T'_{i+1}} - \boldsymbol{T'_i} = \frac{\boldsymbol{F'}}{\boldsymbol{\rho cH}} \qquad (6)$$

and it follows that

$$T'_{i+1} = \mathrm{T}'_{\mathrm{i}} + \frac{F'}{\rho cH}. \qquad (7)$$

We write $\boldsymbol{F}$ as

$$\boldsymbol{F'} = \boldsymbol{F'_A} + \boldsymbol{F'_N} \qquad (8)$$

where $\boldsymbol{F'_A}$ is the component that is associated with anthropogenic forcing (CO2) and $\boldsymbol{F'_N}$ the component associated with natural variability in the atmosphere. There must also be a loss of heat at the ocean's surface. The rate at which heat is lost is proportional to the temperature and we can therefore write this as $-\boldsymbol{\lambda T}$, where $\boldsymbol{\lambda}$ is the atmospheric feedback, which is the rate at which temperatures are damped to the atmosphere. Since the ocean has a large heat capacity and therefore has a strong memory of past temperatures, the temperatures being damped to the atmosphere are those created at an earlier time. In our model let us assume that these temperatures correspond to $\boldsymbol{T'_i}$ so that the heat lost to the atmosphere is $-\boldsymbol{\lambda T'_i}$. Including this term in equation (8) gives

$$\boldsymbol{F'} = \boldsymbol{F'_A} + \boldsymbol{F'_N} - \boldsymbol{\lambda T'_i} \qquad (9)$$

Finally, substituting this into equation (9) gives

$$T'_{i+1} = T'_i + \frac{F'_A + F'_N - \lambda T'_i}{\rho cH}. \qquad (10)$$

Appendix 2 – University Applications Guidance

For course choices and careers information:	www.brightknowledge.org.uk
For information and statistics on universities and course:	www.unistats.direct.gov.uk
For comprehensive information on universities, including rankings:	www.thecompleteuniversityguide.co.uk

PLEASE NOTE – The Researchers in Schools programme is designed to support the university applications process at your school. The work you complete for the programme should add to your schoolwork rather than detract from it. Please be aware of deadlines and try to manage your workload appropriately. If you are worried about the programme interfering with your schoolwork then please speak to your teacher.

Subject Specific Top Tips for Strengthening an application?

What ideas do you have for how students can prepare for university in your subject area or related discipline? Share top tips, reading lists, events, films, videos or podcasts that you think could help your students to succeed.

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.