Researchers in Schools

Big Bang Cosmology: *Expanding Horizons*

Key Stage 4 Programme

Pupil Name

Coursebook Designed by Dr C. S. Gallagher

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Uni Pathways launch event

Welcome to Uni Pathways! We are delighted to be able to launch the Uni Pathways programme with you. We hope that you are ready to embark on your Uni Pathways journey and that you enjoy the video.
You will be asked to pause the video at times to complete some work in this workbook, so make sure you have a pen / pencil to hand when you start the video.

By the end of the video, you will have

- Learnt about what studying at university means
- Learnt about some of the skills that you will develop during Uni Pathways
- Heard from pupils who have participated in Uni Pathways or The Scholars Programme (which is the same programme!)
- Heard from current university students talking about what life is like at university

If you are in school your teacher will play the video. If you are at home and logged in to a session with your teacher, your teacher will play the video and show it to you. If you are participating independently your teacher will email you the link to the video or the video file. There are opportunities for you to answer some questions, and you will be told when to pause the video to answer them.

Introduction to Uni Pathways

1. Write down what you think a supra-activity is

_____
_____
_____

2. Independent learning is

_____
_____
_____

3. Write down in your own words what resilience means, and come up with a different example to the one mentioned in the PowerPoint.

_____
_____
_____
_____

4. Why do you think analysis and evaluation skills are useful in your academic career?

_____
_____
_____
_____
_____

5. Why do you think analysis and evaluation skills are useful in life in general?

_____
_____
_____
_____
_____

6. What are the different types of learning that university students do?

Pupils' experience of Uni Pathways

You will now watch some pupils talk about their experience of learning a PhD topic and producing a final assignment. Some may refer to The Scholars Programme rather than Uni Pathways. As you heard in the introduction PowerPoint, The Scholars Programme is a very similar programme to Uni Pathways. Listen carefully and then respond to the questions below. Be prepared to share some of your responses with your peers.

1. How did the pupils describe talking about writing a long essay for their final assignment?

2. Write down something that one of the pupils mentioned was particularly interesting in their course.

3. What were some challenges that the pupils met?

Virtual campus tours

You will now go on a virtual tour of some universities! Once you have seen some of the university campuses, respond to the questions below.
Note down something that you liked out of any of the campus tours you saw. It may be a particular building, space, city etc.

1. Were there any similarities between the different university campuses? If so, what are the similarities?

2. If you had to pick one of those universities to go and visit in person, which one would it be and why?

3. Note down some of the societies that you could join at different universities

Meet university students!

You are about to virtually meet or hear from some current university students.
If you are virtually meeting them, think about what questions you would like to ask the students, share your questions with a partner and note those questions down in the space below.

If you are not meeting them but watching some videos that they have made, use the space below to note down what else you would like to find out about university. You can then ask your teacher during Uni Pathways tutorials!

Reflection

Congratulations on completing the launch of the Uni Pathways programme. Before you go, take some time to reflect on what you have learnt by answering the following questions:

1. Are there any aspects of university style learning that you would look forward to? If so, what are they?

2. What challenges do you think students face when learning at university? How would you try to overcome these challenges?

3. What skills do you hope to develop during your Uni Pathways course?

4. What part of Uni Pathways seems the most challenging for you?

5. What are you most looking forward to about Uni Pathways?

Timetable and Assignment Submission

Timetable – Tutorials

Tutorial	Date	Time	Location
1			
2			
3			
4			
5			
6 (Feedback)			
7 (Feedback)			

Timetable – Homework Assignments

Homework Assignment	Description	Due Date
Tutorial 1	Baseline Assignment Problem Sheet	
Tutorial 2	Written assignment on the CMB (200 words)	
Tutorial 3	Written assignment on dark matter (200 words)	
Tutorial 4	Written assignment on general relativity (200 words)	
Tutorial 5	Final Assignment: Scientific Report	

Assignment Submission – Lateness and Plagiarism

Lateness	
Submission after midnight on 9th August	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Course Rationale

This course will introduce you to **physical cosmology** – the science of our **entire universe** on the largest scales!

Where did the universe **come from**? Where is it **going**? What is it **made of**? Does the universe have a **shape**? How has that shape **changed through time**?

These are all examples of the kinds of questions that professional cosmologists try to answer. Whilst many models of the universe have been put forward (Monotheistic notions of heaven and hell, Norse Mythology, Greek Mythology, Flat Earthers...), the key difference with physical cosmology is that it tries to answer these questions using **scientific techniques**, mainly from **physics**, **mathematics**, and **astronomy**.

Our understanding of the universe and our place in it has changed beyond all recognition in the last 100 years due to the work of geniuses like Albert Einstein, Jocelyn Bell-Burnell and Stephen Hawking. Together, we will take a tour through the key scientific breakthroughs that underpin cosmology, from the theoretical model that our understanding is based on, to the observational evidence in the stars and galaxies on the night sky that supports that model.

You will learn why we think the Big Bang happened, what **dark matter** is and about the new mystery of **dark energy**. Together, we will build some basic models of the universe, and learn how to calculate the **age of the universe** using them. Then, we will compare the predictions of these different models to the **actual data taken by real astronomers** and see which model works best!

You will be lucky enough to get a taste of what real scientists at the cutting edge of their research fields do on a day-to-day basis, and learn what university and higher education is all about!

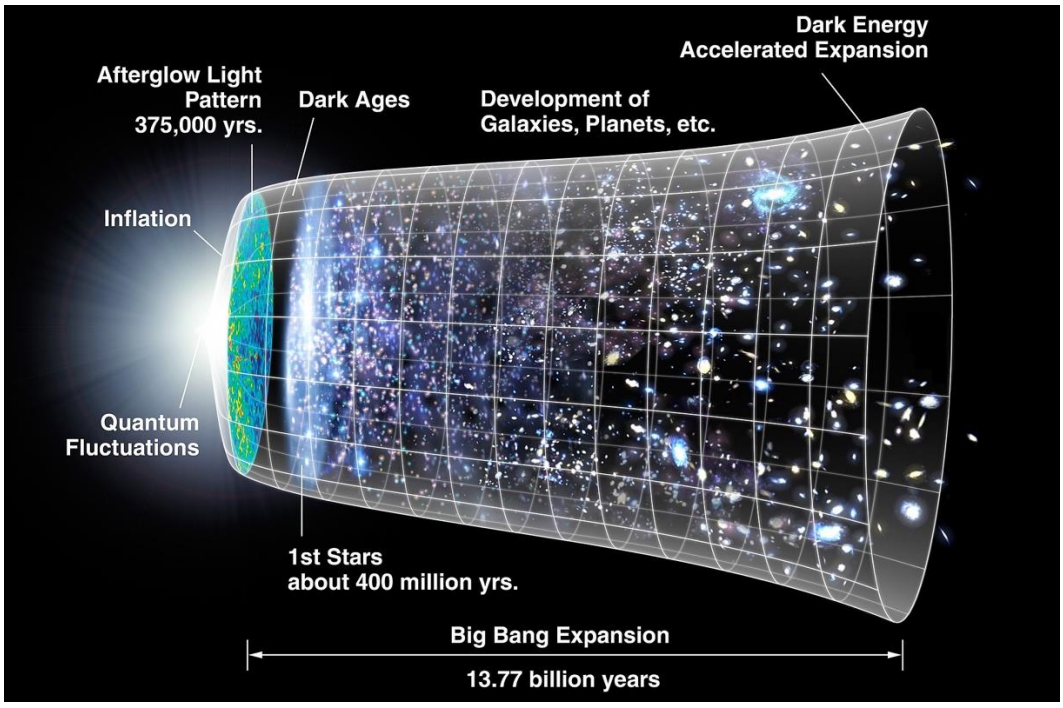

Figure 0.1 : A brief history of time

Uni Pathways Mark Scheme 2020 – science-based assignments

	Subject Knowledge	Critical Thinking	Written Communication
1st	The work shows a depth of knowledge and understanding of key concepts and scientific methods, through engaging with relevant sources. Knowledge is used to build and support highly effective scientific arguments and explanations.	Analyses key scientific evidence, arguments, and reasoning. Interprets meaning and makes connections. Identifies and critically evaluates key scientific arguments and evidence, deciding on their credibility, strength, and relative significance, drawing convincing conclusions.	The work has a coherent flow and is well structured. The writing style is appropriate; scientific language and key scientific terms are used accurately and effectively to support the arguments and explanations made. There are no, or very few, errors in spelling or grammar. Consistent referencing, appropriate paragraphing and use of correctly labelled tables and graphs matching the style taught in the course.
2:1	The work shows an understanding of key concepts and scientific methods, drawing on relevant sources. Knowledge is used to build and support effective scientific arguments and explanations.	Analyses relevant scientific evidence, arguments, and reasoning. Identifies and critically evaluates relevant scientific arguments and evidence, deciding on their credibility and strength, drawing reasonable conclusions. Shows some understanding of the relative value of evidence and arguments.	The work is well-structured. The writing style is appropriate; scientific language and key terms are used correctly. There are few errors in spelling or grammar. Mostly consistent referencing and use of tables and figures; matching the style taught in the course.
2:2	The work shows an understanding of key concepts and scientific methods, with no major misconceptions. Beginning to apply this knowledge to build and support effective scientific arguments and explanations.	Identifies and uses basic scientific evidence, arguments, and reasoning. Showing some understanding of the quality of scientific arguments and evidence. Not yet showing understanding of the relative value of evidence and arguments.	The work has some structure. The writing style can sometimes be informal; occasionally scientific language and key terms are not used when it would be appropriate to do so. There are some errors in grammar and spelling do not get in the way of communicating the content. Referencing has some consistency; matching the style taught in the course Limited use of tables and graphs.
3rd	Shows a developing understanding of key concepts	Beginning to analyse scientific evidence, arguments, and reasoning.	The grammar, spelling, style, and structure of the work need

	and scientific methods, with some misconceptions. Does not yet apply this knowledge to build and support scientific arguments and explanations.	Describes evidence and arguments, while not yet evaluating them.	improving in order to communicate ideas to the reader. Scientific language, key terms and references are not always used correctly. Limited, or no use of tables and graphs.

Baseline Assignment (problem-set): Pupil Feedback Report

Name of Pupil	
Name of School	
Name of RIS teacher	
Title of Assignment	

How your assignment is graded:

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Marks

OVERALL MARK / 100		FINAL MARK / 100 (including any deductions)	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Mark Breakdown and Feedback

Subject knowledge	
	mark
Critical thinking	
	mark
Written communication	
	mark

Problem Set	
Comment	*mark*

Final Assignment (essay based): Pupil Feedback Report

Name of Pupil	
Name of School	
Name of RIS teacher	
Title of Assignment	

How your assignment is graded:

Grade	Marks	What this means
1st	70+	Performing to an excellent standard at A-level
2:1	60-69	Performing to a good standard at A-level
2:2	50-59	Performing to an excellent standard at GCSE
3rd	40-49	Performing to a good standard at GCSE
Working towards a pass	0-39	Performing below a good standard at GCSE
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
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Marks

OVERALL MARK / 100		FINAL MARK / 100 (including any deductions)	
DEDUCTED MARKS		FINAL GRADE	

If marks have been deducted (e.g. late submission, plagiarism) the teacher should give an explanation in this section:

Mark Breakdown and Feedback

Subject knowledge	
	mark
Critical thinking	
	mark
Written communication	
	mark

Subject Vocabulary

Word	Definition	In a sentence
Galaxy	A **galaxy** is a collection of billions of stars that are all gravitationally bound together. In cosmology though, galaxies are the smallest objects we will consider - you can think of galaxies as being like cosmological atoms!	The **Milky Way** is a somewhat unremarkable medium sized spiral galaxy located in the **Local Group**, which also includes **Andromeda** and the **Triangulum** galaxy. It also happens to be our home.
Universe	**The universe is all of space and time, and everything in it!** Essentially, you can think of the universe as being the collection of **everyWHERE, everyWHEN and everyTHING** that ever happened, is happening, or ever will happen.	The Milky Way galaxy may seem very large to you, consisting of billions of stars like our Sun, but compared to the **size of the Universe**, it is **absolutely tiny**!
Spacetime	**Spacetime** is the mathematical construction we use to describe universes. You can think of any event as being locatable in the universe using 4 numbers: three spatial coordinates that tell you how to get there, and a time at which it happened. A spacetime is the **collection of all events**.	Previously, scientists thought that space and time were separate properties that were independent of one another, but Einstein's theory of **special relativity** showed conclusively that we must think of both simultaneously as being part of a greater whole: spacetime.
Cosmology	A "**cosmology**" is a framework for understanding the Universe. Cosmologies don't have to be scientific; for example, the ancient Vikings had their own cosmology, where they thought the universe was separated into three realms Asgard, Midgard and Utgard. **Physical cosmology** is concerned with the creation of a cosmology based on scientific understanding and evidence - mainly **physics**, **mathematics** and **astronomy**.	Scientific cosmology was born the 1920's as a result of the discovery of galaxies outside our own Milky Way in combination with the theoretical work of Albert Einstein on gravity.
Mathematical Model	A **mathematical model** is a mathematical description of a system that allows you to "evolve" a system forward in time. The idea is that if you are given all the numerical values of the variables that describe the system at a particular time, the model will then tell you what values these variables have at a different time.	If we are given the starting position and velocity of a ball with a fixed mass, and we know that the only force acting on the ball is gravity, we can use Newton's second law to build a simple mathematical model that will tell us the position and velocity of the ball at any future time.

Hubble flow	**The Hubble flow** is the geometric expansion of space itself on the largest scales.	Objects that are noticeably affected by the Hubble flow are usually separated by distances of 1 Mpc or larger.
Redshift	**Redshift** is a number that cosmologists use measure of how fast an object is moving away from us. The higher the redshift, the faster the object is moving away from us.	Light is a wave - due to the **Doppler effect**, light emitted by any object that is moving away from the observer will be **redshifted**, because the wavelength (which is related to the colour) of the emitted light is stretched!
Dark Matter	**Dark Matter** is matter that we know must exist because we have measured it's gravitational effects, yet we cannot see because it does not interact with light the way normal matter does. It is thought that dark matter makes up 85% of the total matter content of the universe.	Spiral galaxies are observed to rotate at a rate that is much faster than is possible due to the gravitational effects of the visible stars they contain. It is hypothesized that they **contain extra dark matter** which is responsible for the **additional gravitational force**.
General Relativity	**General Relativity** (mostly called "GR" by practitioners) is our best working theory of **gravity**. It was created by Albert Einstein in the 1920's and describes the effect of gravity as being caused by mass distorting the geometric properties of spacetime itself.	General relativity, is quite simply, the greatest and most beautiful idea anyone ever had in the entire history of human thought.
Dark Energy	"**Dark Energy**" doesn't actually mean anything at all! It really is nothing more of an expression of our own human ignorance as to the cause of the **accelerated expansion of the universe**. We're not even sure that it is caused by energy - as it would have to have extremely weird properties - like having a pressure that increases with volume!	Dark Energy is the most mysterious physical phenomenon ever measured.

Tutorial 1- The nature of things that are far away from the Earth…

"Ambition, idly vain; revenge and malice swell her train."

– Sir Roger Penrose, Nobel Prize winner 2020, quoting Rev. Thomas Penrose, cleric & poet, (1742-1779).

Figure 1.1 : How is the shape of the universe similar to the expansion of a balloon being filled with air?

By the end of this tutorial I will be able to:

- Describe cosmological distances using the unit "Mpc" (Mega-parsec).
- Describe some non-Euclidean geometries.
- Describe the Hubble flow and apply Hubble's law in some simple cases.

Starter Activity:

How far is it from where you're sat to the door? What about from where you're sat to Big Ben? How far is it to the centre of the Earth? What is the largest distance you can imagine?

Discuss these things with your partner. Between you, come up with an estimate for each distance *in metres*. Do you think "metres" is a sensible unit to measure these sorts of distances in?

Cosmological Distances

In cosmology, we will be concerned with ABSOLUTELY ENORMOUS distances. One common unit of extremely large distance that you may have heard of is the "light-year" – the distance that light travels in precisely one year.

Exercise:

If the speed of light 'c', is approximately $c = 300{,}000$ km s^{-1}, and

$$c = {}^{d}/_{t},$$

where 'd', is the distance travelled, and 't', is the time taken, calculate the length of one light-year in kilometres.

NB Although 'light-year' contains the word 'year', it is actually a unit of DISTANCE, not a unit of time!

One light year = _____ km
Although this is an absolutely enormous distance, it's not big enough for cosmology. To get even large, we must introduce a new unit – the 'parsec'.

One parsec is defined via the following construction:

Imagine a right-angled triangle, where the short side is the distance from the Earth to the Sun (this distance also has a special name – it's called *one astronomical unit*, or 1 AU, and it's approximately equal to 150 million kilometres).

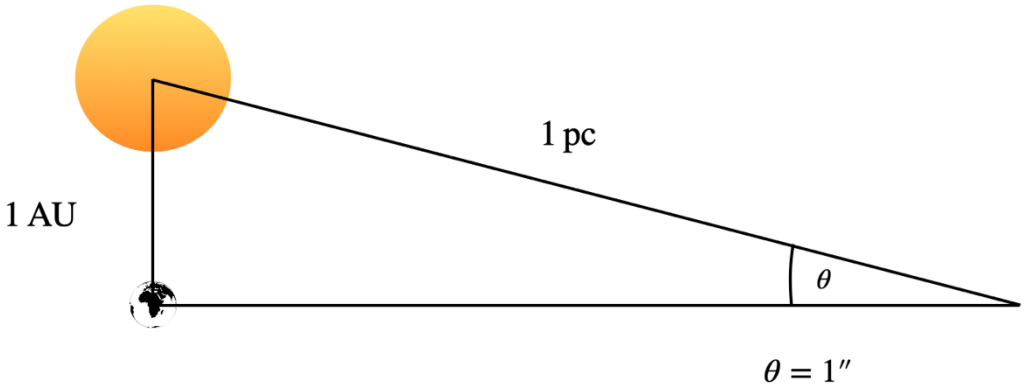


Figure 1.2 : Definition of a parsec

If the angle opposite to the Earth-Sun distance is a special angle, called '*one arcsecond*', where the symbol for arcseconds is $''$ and $1'' = {}^{1}/_{3600}\,^{\circ}$ (one arcsecond equals one thirty-six-hundreth of a degree!), then the hypotenuse of the triangle is equal to one parsec.

Using trigonometry, and the approximation $\sin\theta \sim \theta$, which is accurate for very small values of θ, astronomers can calculate that:

1 pc = approximately 301,800 billion km = 3.262 light-years.

NB 'parsec' is a unit of DISTANCE even though it contains the letters 'sec'. The letters 'sec' come from the 'arcsecond' angle. This is the mistake Han Solo famously made when he claimed that the Millenium Falcon 'made the Kessel run in under 12 parsecs'!

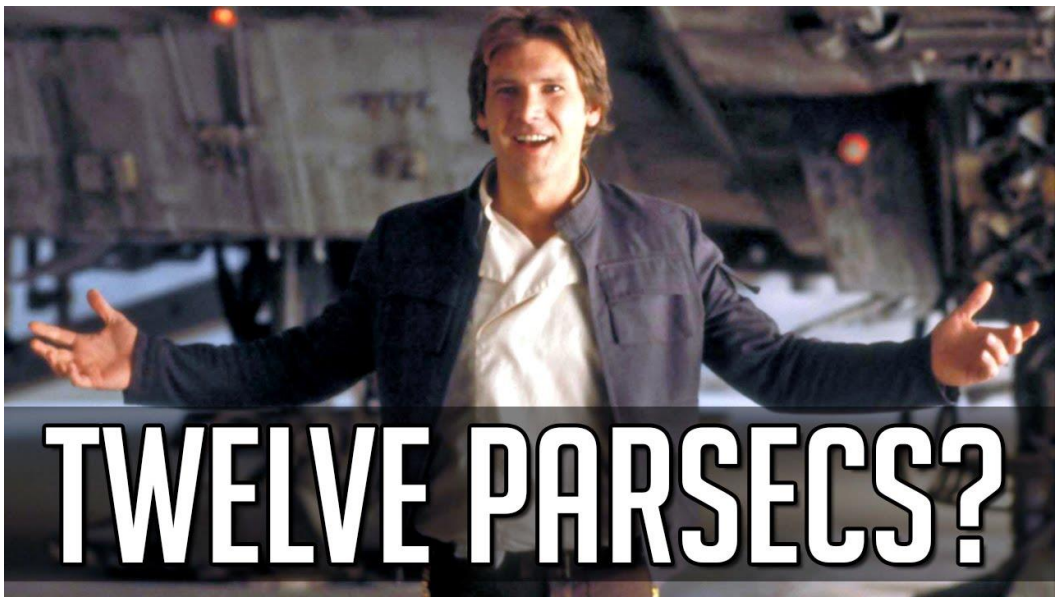


Figure 1.3 : Han Solo is not a very good astronomer…

But STILL….one parsec is not enough for a cosmologist.

Our very own Milky Way galaxy has a diameter of approximately 32.4 kpc, or 32,400 parsecs.

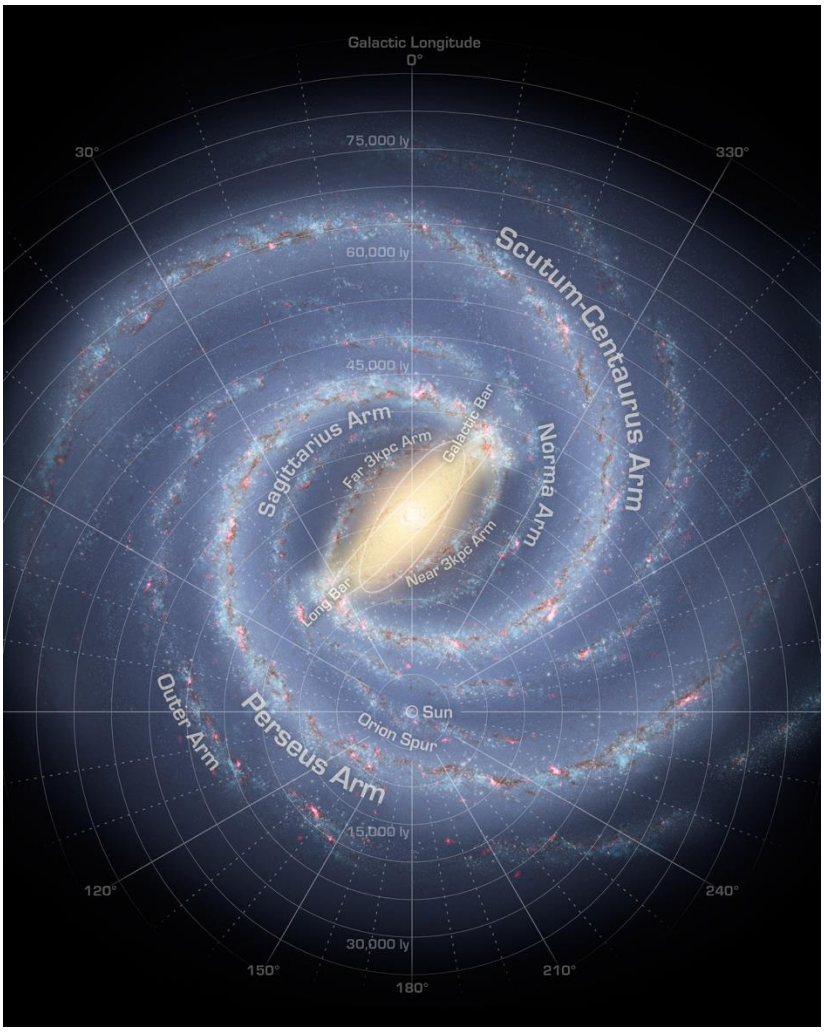


Figure 1.4 : Spitzer Space Telescope picture of the Milky Way – NASA.

To a cosmologist – an *entire galaxy* is comparable to an atom – it's the smallest object that a cosmologist would consider!

Cosmologists therefore like to measure their distances in a unit called the 'Megaparsec'.

One megaparsec is equal to precisely one million parsecs.

Exercise:

How far is 1 Mpc measured in metres?

1 Mpc = _____ m.

The Doppler Effect

The first key piece of physics we need to understand in order to start making sense of objects this far away is called 'The Doppler Effect'. It's actually much more familiar than you might think – you experience it whenever you hear a police car siren!

The Doppler effect occurs whenever something that emits or absorbs *waves* of any kind is *moving*. For sound waves – the *pitch* of the note you hear is related to a quantity called *the wavelength* – essentially how long the wave takes to repeat itself.

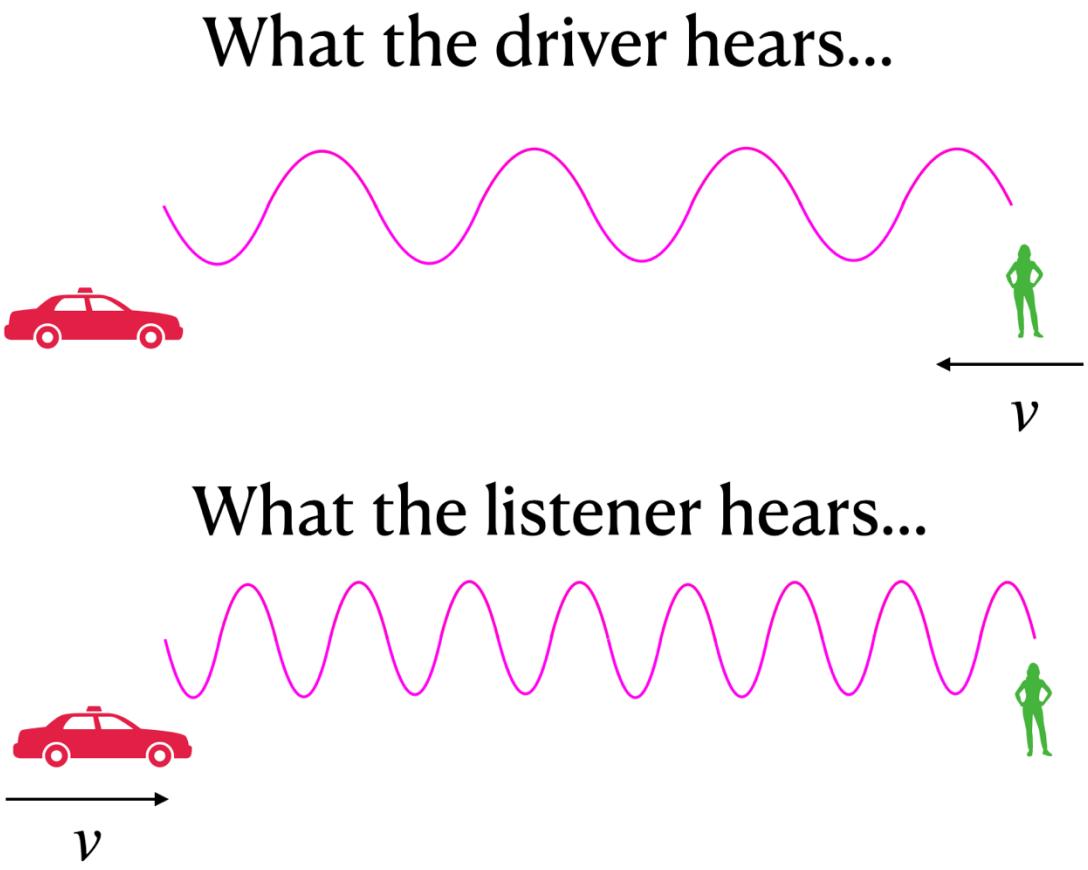
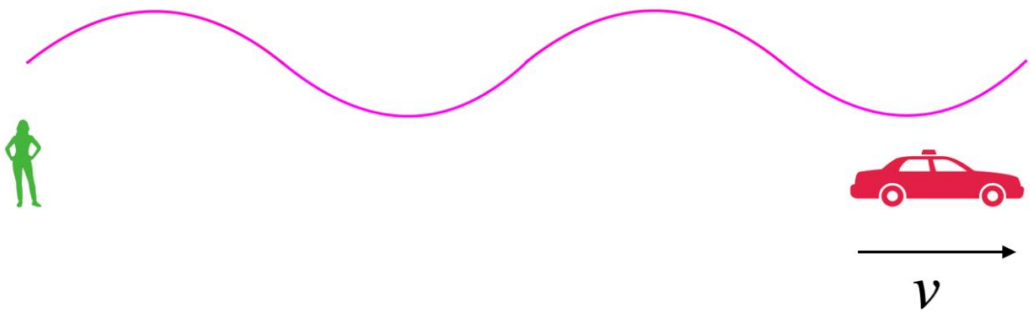


Figure 1.5 : Doppler Effect

What happens is that the driver and listener hear DIFFERENT pitches, depending on how fast the driver is driving! The driver always hears the same note, since they're moving at the same speed as the wave themselves – however the listener will hear a **higher note** – since from their perspective, the sound wave is **compressed** by the speed of the driver.

Furthermore, once the car passes the listener, and starts **moving away**, the sound wave will be **stretched**. Accordingly, the listener hears a **lower note.**



Light is also a wave, just like sound! However when you change the wavelength of light, you don't change its pitch – **you change its colour!** Longer wavelengths lead to **redder** colours, whilst shorter wavelengths lead to **bluer** colours!

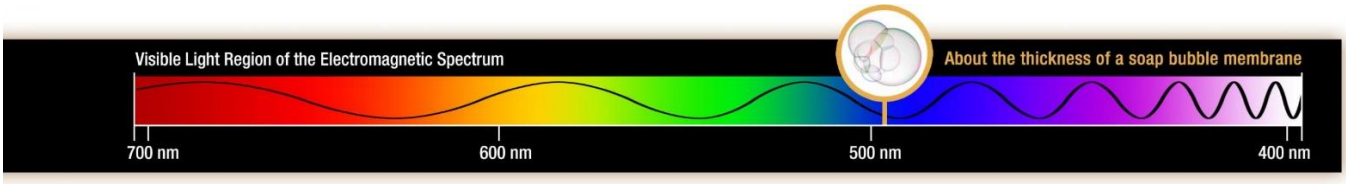


Figure 1.6 : The visible portion of the electromagnetic spectrum.

Therefore – light emitted by **moving objects** will be either **redshifted** (if the object is moving away from us) or **blueshifted** (if the object is moving towards us)!

Around 110 years ago, an American astronomer called Vesto Slipher figured he could measure the velocities of distant galaxies by trying to measure whether the light they were emitting was redshifted or blueshifted.

Slipher expected that he would find approximately as many distant galaxies that were redshifted as that were blueshifted. However, what he discovered (almost by accident) was that **every single galaxy** that he observed that was further away from Earth than about 1 Mpc was redshifted. Without fail. This observation is the birth of modern cosmology.

Even more strangely, it seemed that the further away the galaxies were, the MORE redshifted the light coming from them was…

Activity 1:

Write a short explanation of why the Doppler effect occurs. Explain if you would expect the following to be redshifted or blueshifted, and why.

1) A galaxy moving towards the Earth.
2) A galaxy moving away from the Earth.

What surprises you about Vesto Slipher's observations? Can you think of a way of explaining them?

A Detour into Flatland…

Slipher's bizarre observations seemed to defy all logic. How can it be possible that all objects beyond a certain distance are somehow flying away from the Earth, sometimes even at speeds **greater than the speed of light** (which is IMPOSSIBLE according to Maxwell's theory of electromagnetism)?

In order to understand this mystery, we need to develop a whole new framework for thinking about the universe – one that is based on **geometry**.

Euclid's Geometry

The geometry that all school children learn, the one where the area of a triangle is $A = \frac{1}{2}bh$, where the angles of a triangle add up to $180°$, and where parallel lines never meet, is the geometry that was discovered by the Ancient Greek mathematician, Euclid of Alexandria.

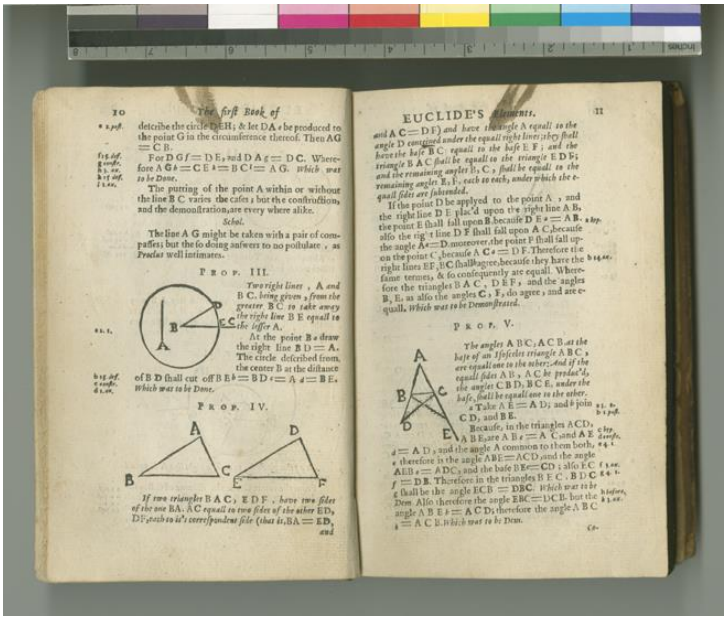


Figure 1.7 : Euclid's Elements

Euclid's Elements is a seminal text in the history of academic thought, and for a while it was considered an essential part of every schoolboy's education. The great American President Abraham Lincoln famously once said:

*"In the course of my law-reading I constantly came upon the word **demonstrate**. I thought, at first, that I understood its meaning, but soon became satisfied that I did not. Resolving to understand it better, I went to my father's house and staid there till I could give any propositions in the six books of Euclid at sight." – Abraham Lincoln*

What Lincoln was referring to was the idea of **mathematical proof**. Others had known the geometric formulae that Euclid wrote down in his book – but what was special about Euclid was that he endeavoured to rigorously prove everything he wrote down from only **five fundamental assumptions**.

These five fundamental assumptions are called the **axioms of Euclidean geometry**.

1. A line can be drawn from a point to any other point.
2. A finite line can be extended indefinitely.
3. A circle can be drawn, given a centre and a radius.
4. All right angles are ninety degrees.
5. Two parallel lines will never intersect one another.

They cannot be proved themselves – they are assumptions. However, using only these 5 assumptions, and the power of logic, Euclid was able to deduce a wonderful array of truths about geometry and its connection to numbers.

For a long time, people suspected that the fifth axiom, sometimes known as the "Parallel postulate" was not actually necessary – and that it could somehow be shown to be a consequence of the first four. Trying to prove the parallel postulate became something of a hobby project for many mathematicians over the next millennia, until to the great shock of the mathematical community, Omar Khayyam actually discovered that not only was it not possible prove the 5th postulate – but that it was not in fact necessary at all!

The consequence of this was **startling** – somehow **other geometries** beyond Euclid's geometry – geometries where two parallel lines do in fact meet, where the angles in a triangle **do not** add up to $180°$, and where all sorts of weird and wonderful things were possible!

The ideas of straight lines, right angles and circles still exist in these geometries – but many of the other geometric results we're used to using have to be modified, or even discarded entirely. Let's look at our first example of a **non-Euclidean geometry**:

Spherical Geometry

Imagine that you are an ant, confined to live on the surface of an enormous sphere. You are never allowed to leave the surface – in fact, to you, the idea of "leaving the surface of the sphere" does not even exist – your world is entirely 2 dimensional. To you, the world just looks flat – like a 2d grid. We call this perspective **Flatland**.

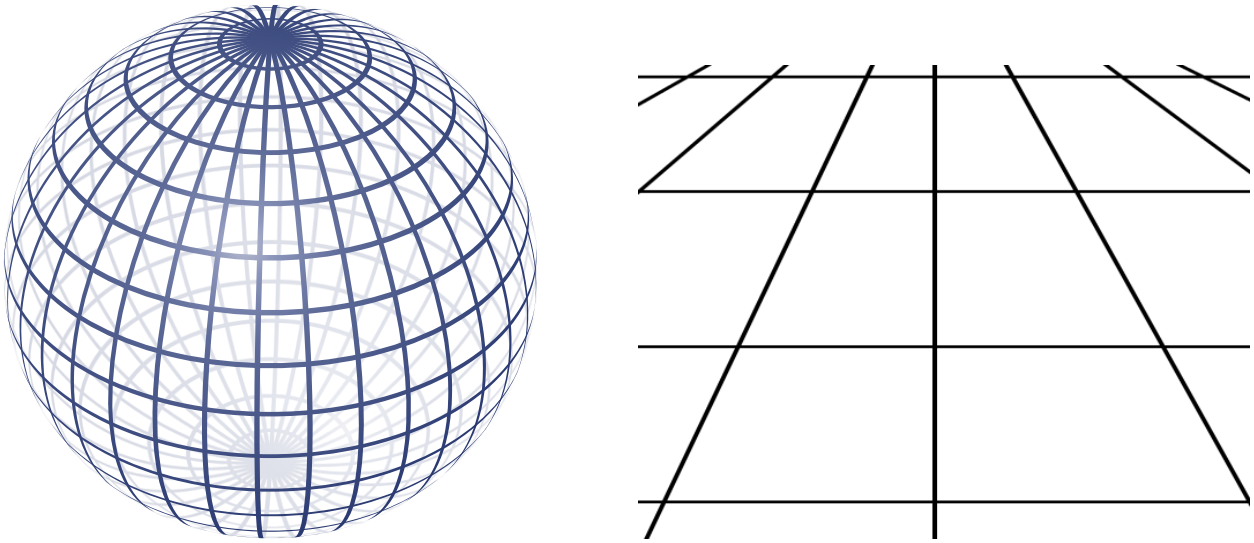


Figure 1.8 : Flatland

Now the question that your anty alter-ego wants to answer is:

If you can't actually see that flatland is spherical (since you can't leave the surface), is there any experiment that you can actually do to tell if you're living on the surface of a sphere or not?

It turns out that there is in fact a simple geometrical experiment you can do to tell – you simply measure the three angles in a triangle! If they add up to MORE than $180°$ - you're living on a sphere. If they add up to exactly $180°$ - you're living on a flat plane, and if they add up to LESS than $180°$ - you're living on something that looks a bit like a saddle shape!

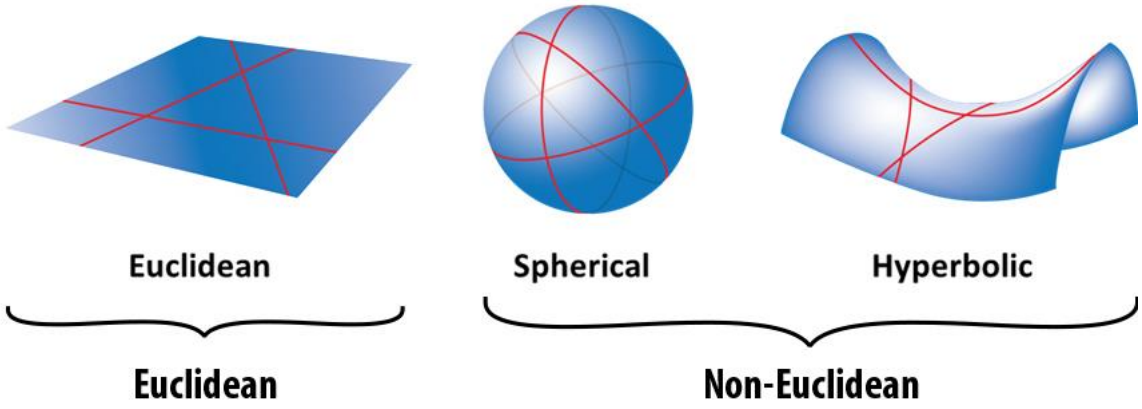


Figure 1.9 : Euclidean geometry vs Non-Euclidean geometry

Of course, people had known about spheres and saddle-shapes for centuries – but they had always imagined these shapes as somehow existing within a higher dimensional Euclidean geometry. For example, the surface of the sphere is 2 dimensional – because you only need two numbers to find any position on it (longitude and latitude) – but we imagine spheres as "living" in normal 3 dimensional Euclidean space. What was revolutionary was the idea that the higher dimensional space did not have to exist at all – these geometries could somehow exist "on their own".

What was even more radical though, was the suggestion that the geometry of the actual world we live in – our **universe** – might **not** be Euclidean. It was precisely this seemingly insane idea though, that turned out to be the key to understanding the mystery of the galaxy redshifts observed by Slipher…

Activity 2:

Imagine you lived on the surface of "Flatland". Which of the following geometries would you be able to tell apart, just by measuring lengths and angles very close to your current position?

1) A perfect, infinite flat plane.
2) A sphere.
3) A saddle-shape.
4) A cylinder.
5) A torus (donut shape).

Hubble Flow & Hubble's Law

Following the work of Slipher, the great astronomer Edwin Hubble spent years meticulously measuring the distances to the very distant redshifted galaxies that Slipher had observed.

Figure 1.10: Edwin Hubble

What Hubble discovered was that the **distances** of the far away galaxies were **directly proportional** to their **recession velocities** (the velocity with which they're moving away from the observer). The fact that all distant galaxies obey this is called **'Hubble flow'**. The relationship can be encapsulated very simply in what is now known as **Hubble's Law**:

$$v = H_0 d \ .$$

The quantity H_0 is called the Hubble constant, and it's the constant of proportionality in this relationship. We add the "0" subscript to signify that this refers to the Hubble constant *today* – in fact the Hubble constant is expected to vary with time (we will discuss this more later though), even though the timescale over which the change happens is so large that we could never feasibly measure it. Our best current estimate of the Hubble constant comes from measurements made by the Planck satellite, and is:

$$H_0 = 67.4 \text{ (km/s) Mpc}^{-1} \ ,$$

although this value is disputed by other teams who measure a slightly different value using different methods.

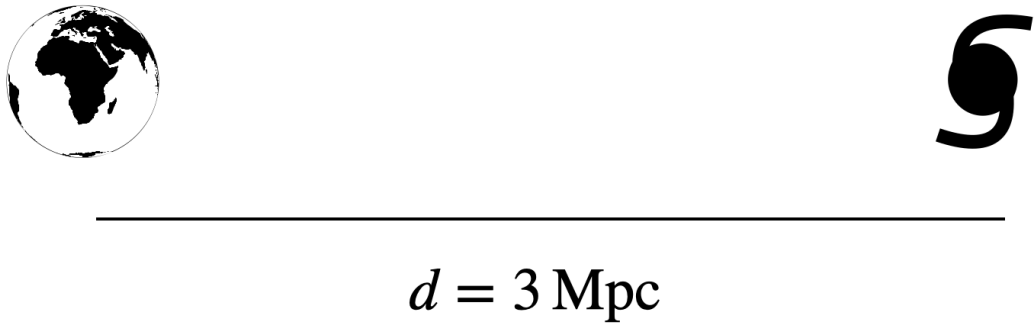
Why the wacky units??? What the hell is a "kilometer per second per megaparsec"?

It's often joked that astronomers like to use silly units – but actually this unit makes a lot of sense! If we measure the distance to a galaxy in megaparsecs, and the recession velocity in km/s, then this value of the Hubble constant tells us that that a galaxy that is exactly one megaparsec away will have a recession velocity of precisely 64.7 km/s!

It should be noted though that by converting km into Mpc, we could express the Hubble constant in units of 'per seconds' – indicating that the meaning of the parameter is something to do with **time**…

Exercise:

Let's do an example:



Consider the following scenario – suppose there is a galaxy that is 3 Mpc away from Planet Earth. What recession velocity would an astronomer expect to measure this galaxy having?

Strategy:

First let's write the relevant equation down – here it is Hubble's law, so we write

$$v = H_0 d$$

Now, we know that $d = 3$ Mpc, and $H_0 = 67.4$ (km/s) Mpc^{-1} , so we can substitute those numbers into Hubble's law – obtaining:

$$v = H_0 d = 67.4 \text{ (km/s) Mpc}^{-1} \times 3 \text{ Mpc}$$

$$= 67.4 \times 3 \text{ km/s}$$

$$= 202 \text{ km/s}$$

Therefore, we would estimate that the astronomer would measure a recession velocity of 202 km/s – meaning the galaxy would be flying away from the Earth at a speed of 202 kilometres every single second!

Connecting Redshift to Recession Velocity

Now Hubble's law connects recession velocity and distance – but what astronomers really measure is the redshift – not the actual recession velocity. However, fortunately there exists a simple way of calculating the redshift from the physics of the Doppler effect:

$$z = \frac{v}{c},$$

where z, is the redshift (a positive number that has no unit, telling us what factor the wave has been squished by), and c is the speed of light. So if you have the redshift – you can easily calculate the recession velocity – simply by multiplying by the speed of light!

Activity 3:

Use Hubble's law and the redshift formula to calculate the following:

1) The recession velocity of a star 0.5 Mpc away from Earth.
2) The recession velocity of a black hole 10 Mpc away from Earth.
3) The recession velocity of a supernova 100 Mpc away from Earth.
4) The redshift of a pulsar 1 Mpc away from Earth.
5) The redshift of a spiral galaxy 10 Mpc away from Earth.
6) The redshift of a globular cluster 100 Mpc away from Earth.
7) The distance away from Earth of a galaxy moving at one-tenth of the speed of light.
8) The redshift of a quasar 150 Mpc away from Earth.

Do we have to worry about Hubble flow when considering objects only 1 pc away from Earth? Why/why not?

Can you find out how far away a galaxy would have to be in order for its recession velocity to be *superliminal* (faster than the speed of light)? Does your answer disprove Einstein's theory that *nothing* can travel faster than the speed of light?

Baseline Assignment (Problem Set)

1. Figure out many centimetres are in 1 Mpc (one megaparsec).

2. Using your answer to question one, calculate your own height in megaparsecs!

3. Can you think of another example of the Doppler effect in everyday life beyond police car sirens?

4. Can you find a place on Earth's surface where you could walk 1km South, 1km East and then 1km North but still end up back where you started?

   How many of these places exist on Earth's surface?

   HINT:

   First try to think about the surface of a sphere and what straight lines and triangles look like on that surface.

   Then try find a triangle on the surface of the sphere that has three right angles! (I KNOW IT SOUNDS CRAZY!)

   Once you've found this triangle, see if you can position it somewhere on Earth's surface where the side lengths of the triangle can be 1km, and where the directions the sides point in can be South, East and North.

   Are there any places on Earth's surface where travelling 1km East would take you all the way around the world & back to where you started. (Think about the Equator and the Tropics of Cancer and Capricorn on the glove).

   If you found this question hard – good! It's a very challenging question that's designed to get you thinking hard about non-Euclidean geometry. It also has a very beautiful answer. University will challenge your mind – you will have to be prepared to sometimes struggle to figure out the answers – but it's worth it because you *learn how to think properly* – which is worth more than any knowledge!

5. A galaxy is moving away from Earth at 8% of the speed of light. Calculate the approximate distance of this galaxy from Earth in metres.

   HINT: Convert the speed of light into kilometres per second, then work out the speed of the galaxy in kilometres per second. You can then use Hubble's Law to work out the distance of the galaxy in megaparsecs. I will leave the final step up to you!

Success Criteria

- All answers not only show full working, but EXPLAIN exactly what is being done at each step, using words and diagrams.

- All written answers use correct spelling and grammar

- All sources of information correctly referenced.

- Extremely high levels of care taken in presentation. You should be actively PROUD of your work – it should be a thing of beauty that you cannot wait to hand in to your teacher.

Tutorial 2 - The Afterglow of Creation…

"I consider myself so lucky."

– Robert Woodrow Wilson, accidental co-discoverer of the Cosmic Microwave Background with Arno Penzias in 1964, and Nobel Prize winner in 1978. Initially Penzias and Wilson thought the strange microwave signals their instrument was detecting was caused by bird droppings…

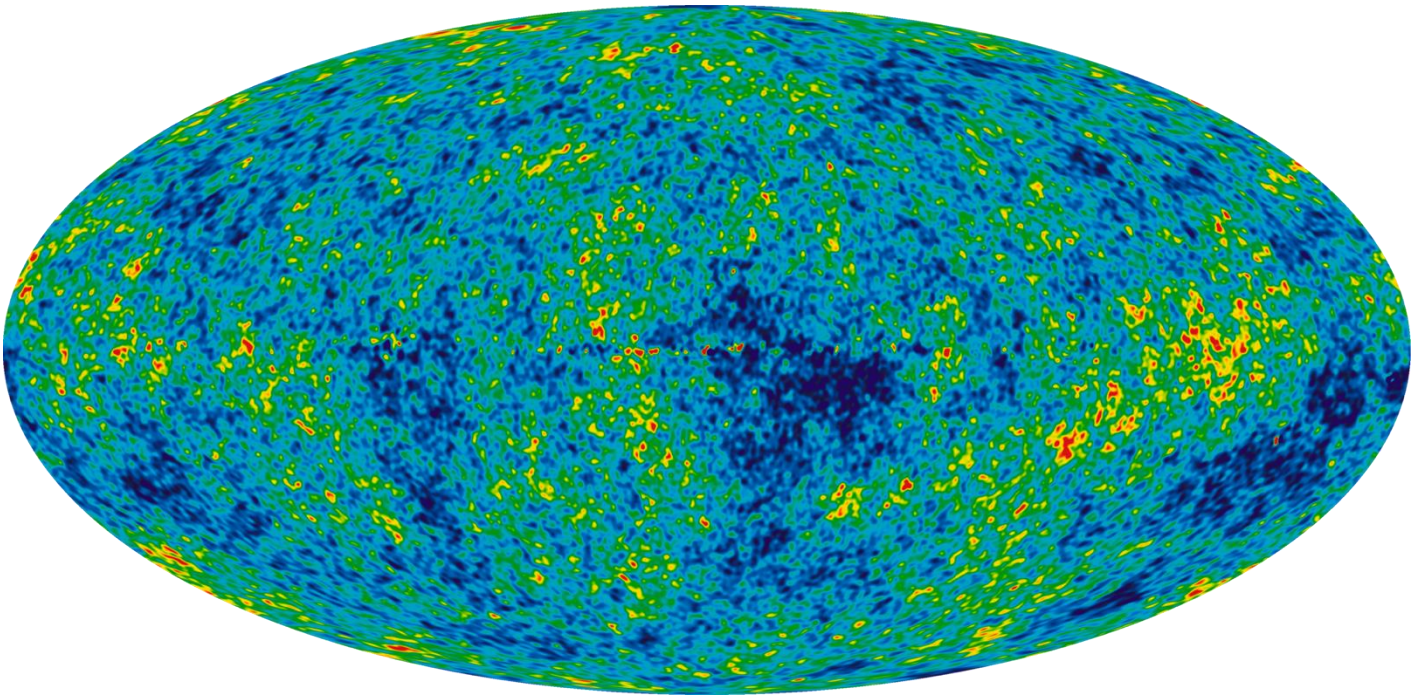


Figure 2.1 : The full sky map produced by the *Wilkinson Microwave Anisotropy Probe (WMAP)*, displaying the variation in the temperature of the Cosmic Microwave Background.

By the end of this tutorial I will be able to:

- Describe the nature of the expansion of the Universe.
- State the Cosmological Principle.
- Explain why tracing expansion of the universe backwards in time leads to the idea of the Big Bang.
- Describe in words what the Cosmic Microwave Background is and how it was formed.
- Describe and explain why the CMB is a key piece of evidence for the occurrence of the Big Bang.

Expanding Geometry & The Cosmological Principle

'But wait just one second!' you might say…

'In the last tutorial, we learned about two things that are totally unrelated – non-Euclidean geometries and the Hubble flow. What do these things have to do with each other???'

Fasten your seatbelts and let's find out!

To begin with, I'd like you to imagine the following Flatland scenario – an infinite grid of coordinates, that **expands over time**.

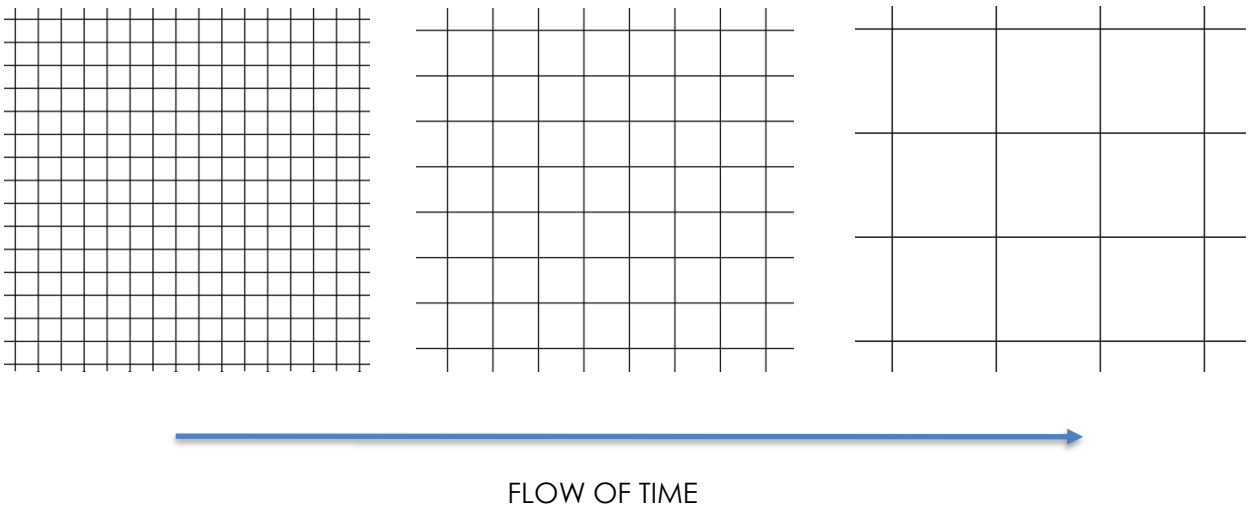


Figure 2.2 : Expanding Flatland

Imagine that your ant alter-ego is anchored to one of the intersections of the grid lines. Now imagine that you've got an anty friend who is also attached to an adjacent grid line intersection. As time goes by, you both remain absolutely still, yet somehow the **physical distance** between you increases!

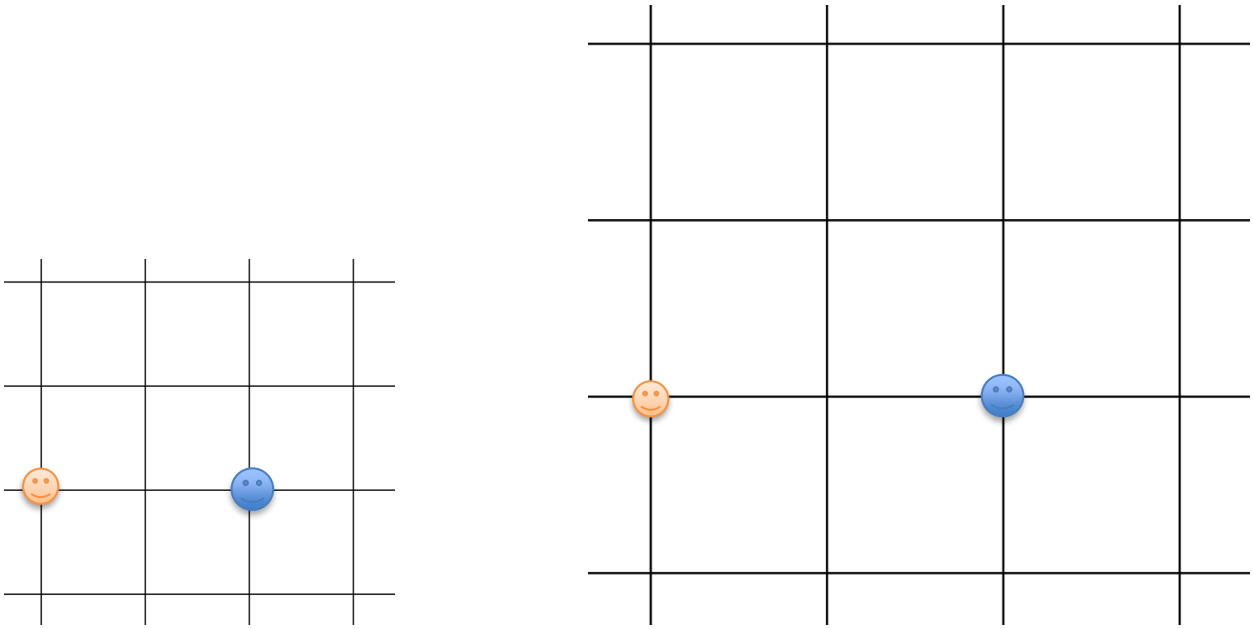


Figure 1.3 : Distance expansion

Both you and your anty friend perceive yourself to be completely still, but you observe each other to be moving away from you.

It was a key observation of the French scientist/priest Georges Henri Joseph Édouard Lemaître that the Hubble flow could be naturally explained if our universe was taken to have this type of **expanding geometry!**

Figure 1.4 : Georges Lemaître

According to Lemaître, if the universe had this type of geometry, then not only would observers on Earth see distant galaxies all being redshifted – **any observer anywhere** in the entire universe would see the same thing! Lemaître came to this conclusion simultaneously to Alexander Friedmann, Bob Robertson and Arthur Walker. The geometry is called the Friedmann- Lemaître -Robertson-Walker geometry (or "FLRW" for short!) in their honour.

This is certainly more pleasing than the alternative view that somehow the Milky Way just happens to be the centre of our infinitely large universe entirely by chance (although many astronomers believed this even up until the 1950s…), because it makes Hubble's law an inevitable result of geometry rather than just some incredible cosmic coincidence. As physicists, we like to believe that the laws of physics are the same wherever (and whenever!) you are – they wouldn't be much good otherwise!

This observation is in agreement with something called **the cosmological principle**.

The cosmological principle states:

*"The universe is **homogeneous** and **isotropic**."*

Or in more plain language – it looks the same whatever direction you look in, wherever you're looking from.

Obviously, this should be taken with a pinch of salt – London definitely doesn't look the same as Timbuktu – the idea is that once you look at very very large scales (100's of Mpc out), everything looks the same **on average**.

This actually tallies with our everyday experience looking at the night sky – if you were to randomly select patches of night sky and count the stars – you'd find that provided you picked a big enough patch, you'd on average tend to count the same number of stars!

Activity 1:

If the distances between objects is increasing due to the expansion of the universe, create a diagram or drawing indicating what would have happened in the *past – e.g. if time was to be* ***reversed****…?*

The Big Bang

As you may have guessed, Lemaître's model seems to somehow indicate that all the objects in the universe were much closer together at earlier times in the universe. In fact, if the *rate of expansion* is allowed to vary, it seems to indicate that there was a finite time the past where every single object in the entire universe was infinitely close to every single other object in the entire universe.

This special moment in time is referred to as the **Big Bang singularity**.

Whilst this theory of the universe is now thought to be the best description – it is deeply unsettling. Many astronomers were so uncomfortable thinking about the idea of a 'beginning of time' that they resorted to "Steady State" models of the universe, where the Milky Way was at the centre, and matter was constantly being created to "fill the space".

The idea of "before" the Big Bang singularity doesn't actually make any sense at all – if you take it that the Big Bang actually happened, it turns out that the laws of physics tell you that both space and time cease to exist at that precise point, and the answer to every question that you ask is "infinity". This situation is unfortunately a bit of a **paradox**.

My favourite way of thinking about this is the following:

The current laws of physics do tell us that the Big Bang happened – but it's also true that the laws of physics are **not yet complete**.

The physics that predicts the Big Bang comes from a theory called **general relativity** invented by Albert Einstein to describe gravity. General relativity tends to describe things that are extremely large (like black holes, or even entire universes).

On the other hand, once things begin to get really really close to each other – the effects of another theory come into play – the theory of **quantum mechanics**, which usually only describes incredibly tiny things like electrons, protons and atoms.

It therefore stands to reason that the correct physics to describe what's going on at the Big Bang should somehow include both general relativity and quantum mechanics. However, currently no-one has been able to figure out how to join the two theories together into a theory of **quantum gravity** – they seem to be incompatible with each other.

The most important goal of fundamental physics since the 1970s has been to discover the correct theory of quantum gravity. Perhaps such a theory could explain what's really going on at the moment of the Big Bang, or even it might tell us that there was no Big Bang after all…

Current theoretical physics research has a few ideas for quantum gravity – **string theory** and **loop quantum gravity** being the main candidates. These theories are fiendishly mathematically complicated however, and unfortunately neither have been able to make any actual scientific predictions yet to test with experiments – so we can't know if they're correct (or not).

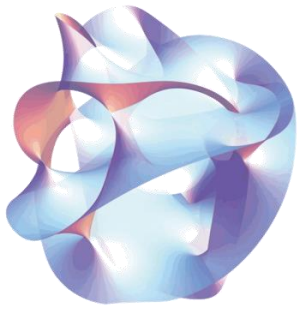


Figure 1.5 : A Calabi-Yau manifold – one of the weird and wonderful objects in string theory.

Evidence for the Big Bang – The Cosmic Microwave Background

Your basic intuition tells you that the more you crush lots and lots of objects together, the **hotter** things are going to become. This is exactly true for the universe too! In fact it provides some of the main evidence that led to scientists dropping Steady State models once and for all and settling on the Big Bang.

We can do some clever maths in an expanding geometry and prove a formula that relates the temperature now, T_0 to the temperature some time ago, T , when the universe was much smaller. The formula is usually given in terms of redshift as:

$$T = T_0 (1 + z) ,$$

where the redshift can be related to the *scale factor of the universe*, 'a', (the number you'd have to multiply every distance by to convert from 'now sizes' to 'back then sizes') by

$$a = \frac{1}{1+z} .$$

Back close to the Big Bang, everything was extremely hot – there was so much thermal energy that protons didn't have enough electrostatic attraction (plus charges of protons attracting minus charges of electrons) to actually form atoms.

Activity 2: Primordial Soup!

We are going to create our own primordial soup!

Let's do this as follows:

All but one of you are going to play the role of charged particles, moving around in the primordial soup, bouncing around off each other like dodgems! All of you will be confined to a 3m by 3m box.

Someone will play the role of a light beam, trying to make it through the box unscathed (without "interacting" with one of the charged particles).

Run 1

First time round - we'll run the experiment at low temperature!

This means the charged particles won't be moving around so much – so you have to stay still! Let's see if the light beam can move through the box without crashing into anyone!

Run 2

Now let's turn up the temperature!

The only rule is you **have** to move in a straight line. If you collide with someone else, you change direction and keep walking in a straight line. If you reach the edge of the box – reflect yourself off the boundary!

Let's see if the light beam can make it to the other side now!

When electrons and protons (the blue and red blobs in Figure 6a!) are zooming around free like this – they can interact with light waves – **changing their directions**. You can think of it as being like an extremely hot thick primordial soup!

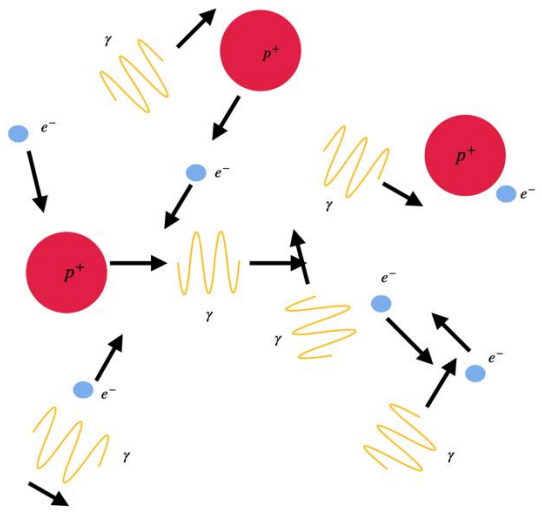


Figure 1.6a : Primordial soup!

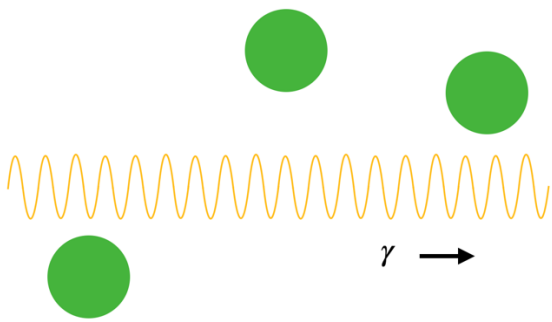


Figure 1.6b : After recombination!

Because the light waves are changing their directions so often, the universe effectively becomes opaque – but when the universe cools enough so that electrons can bind with protons in a process called **recombination** – the resulting atoms (the green blobs in Figure 6b!) effectively become **charge neutral**. Once this has happened, the probability of a light wave interacting with a charged particle dramatically decreases and light starts to travel in straight lines again unless it happens to collide with something like a star or a planet!

At this point, the universe effectively becomes transparent – as we look out into space, the furthest distance we can see is defined by the distance that a light wave emitted at the time of recombination has travelled to reach our eyes (or telescopes). This distance is known as the **horizon**.

We therefore expect that in a universe that is well described by the Big Bang theory, that there should be a **homogeneous** and **isotropic afterglow** made up of all the light that has travelled in a straight line directly from when the time of recombination happened until now.

This afterglow is exactly the same type of glow that happens when you heat up metal – it's called "blackbody radiation".

Figure 1.7 : Blackbody radiation.

The remarkable thing about blackbody radiation – is that you can tell the precise temperature of the object that emitted it, just by looking at the colour of the light. Now, if we know that the recombination process happens to occurs when the universe has a temperature of approximately $T = 3000\ K$ (to convert from Kelvin to Celsius simply use $0\ K = -273.15\ ℃$), and we know that the recombination process happened when the redshift was approximately $z = 1100$, then we can work out what the "temperature" of the same radiation is today using

$$\frac{T}{1+z} = T_0 = \frac{3000\ K}{1101} = 2.72\ K = -270.43℃\ .$$

It turns out that blackbodies with this temperature are predicted to radiate *microwaves* – light waves that have wavelengths so long that our eyes cannot detect them (though we use them to heat our food sometimes!).

Now it just so happens to be the case that in 1964, two lowly graduate students named Arno Penzias and Robert Wilson were on the roof of their university building trying to get a new type of telescope that observed the sky using microwaves rather than visible light to work. They couldn't get the results they were looking for because there seemed to be a low-level signal in every direction they pointed the telescope. Initially, they thought that their instrument was broken, or even that bird poop was causing the signal, until on the off chance, they decided to calculate the temperature of an object that might emit such a signal.

They found that the signal that looked the same in all directions had an exact temperature – of precisely $2.72\ K$, exactly as was predicted by the Big Bang theory.

Penzias and Wilson won the Nobel Prize for this discovery - the afterglow of creation – which we now refer to as the **cosmic microwave background** (or CMB for short).

Activity 3:

Calculate the temperature of the cosmic microwave background at the following redshifts:

1) z = 0.01
2) z = 0.1
3) z = 1
4) z = 10
5) z = 100
6) z = 1000

At z = 1100, the formula we gave for calculating the temperature of the cosmic microwave background stops working. Can you guess why?

Homework (Essay based)

Explain, in your own words, the significance of Penzias and Wilson's discovery of the cosmic microwave background radiation.

Success criteria

- You should write approximately 200 words.
- You should discuss how the discovery of the CMB changed how scientists think about our universe.
- You should explain what the different theories competing to explain the Hubble flow were.
- You should explain why the discovery of the CMB decisively favoured one theory over the other.
- You should discuss how modern cosmologists use tiny fluctuations in CMB to understand more about the contents of our universe.

Tutorial 3 - A Scanner Darkly…

"There are five dark matters and five lamps…"

– Abu Bakr, early companion of the Prophet and first Rashidun Caliph.

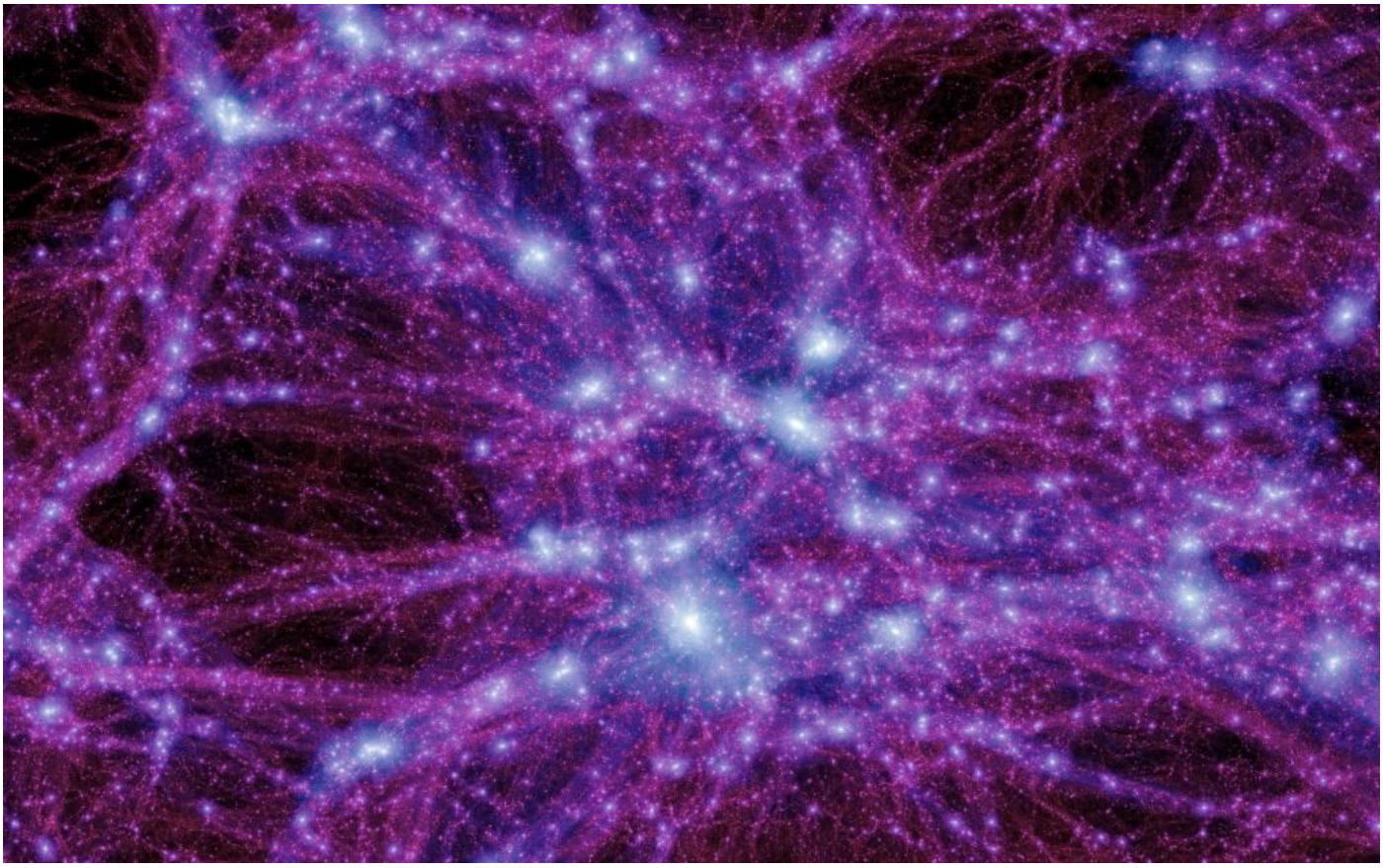


Figure 3.1 : An image produced by the *Millenium-II* cosmological simulation. The brighter parts of the image correspond to areas where the simulation predicts that dark matter will clump together more – although we cannot actually see the dark matter – so it definitely doesn't *look* like this!

By the end of this tutorial I will be able to:

- Describe at least one key piece of evidence for the existence of dark matter.
- Explain why dark matter is "dark".
- Explain what different dark matter candidates are (WIMPS, MACHOS, Primordial Black Holes, Modified Gravity).

The Trajectory of Uranus

Back in 1846 astronomers thought that the Solar System consisted of Mercury, Venus, Earth, Mars, Jupiter, Saturn and Uranus. A German astronomer named Johann Gottfried Galle was doing his nightly observations of the trajectory of Uranus.

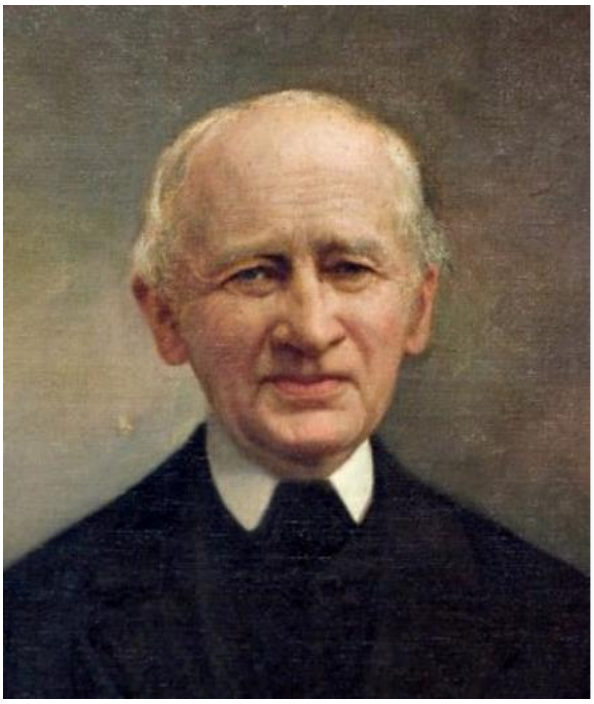


Figure 3.2 : Johann Gottfried Galle

Galle was using Newton's theory of gravity and the positions of the other planets to try and predict the position of Uranus. However for some reason, Uranus seemed to be particularly badly behaved that evening and he couldn't find it in the expected position.

Try to come up with a reason to explain why the perturbation of Uranus's position may have perplexed Galle.

Discuss with your partner.

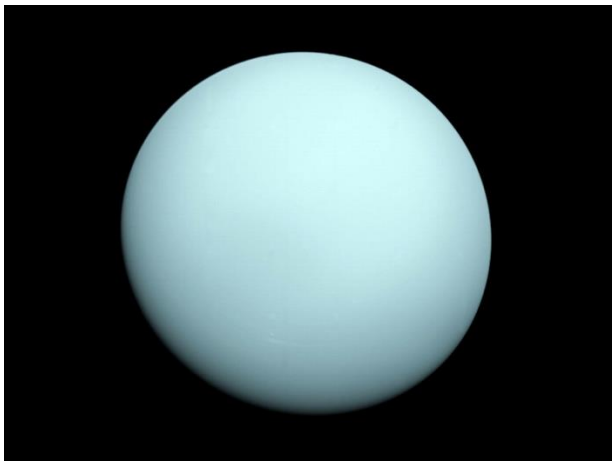


Figure 3.3 : Uranus

(Hint – think about the planet that *hasn't* been listed…it hadn't actually been discovered yet, so Galle wasn't aware of it!)

The perturbation of Uranus has shown us that even objects that we can't see can be detected through their *gravitational effects on other bodies*. Whilst the discovery of Neptune via this method in 1846 was considered a crowning jewel of 19th century astronomy and a huge success of celestial mechanics, this method of detection was to have even more astounding consequences in the 20th century.

Whilst the discovery of Neptune largely confirmed the predictions of previously existing physics, things were about to get much….much…weirder…

Spiral Galaxies and Rotation Curves

The astronomers Jan Hendrik Oort and Horace Babcock had begun investigating a particular type of galaxy called *spiral galaxies.*

Figure 3.4 : Messier 101 (sometimes known as the "Pinwheel galaxy")

The key thing about spiral galaxies is that they **rotate**.

Now – the laws of gravity tell us that the speed at which the galaxy rotates should be related to the sum of all the mass in the galaxy.

Oort and Babcock had been observing these galaxies and trying to figure out how heavy they were using two separate methods:

1. Assume that every star in the galaxy is approximately the same mass and brightness as the Sun. Then, simply measure how bright the galaxy is – translate that into an "equivalent number of Suns", then add up all the masses. Simple!

2. Measure how fast the "arms" of the galaxy are rotating – then using the laws of gravity to calculate how much mass there is. Also simple!

The problem however, was that the results of these methods did not agree.

And not only did they not agree – they failed to agree by a factor of 10.

Clearly this was an enormous problem for the physics community – what could be going wrong?

Initially, people suspected that Oort and Babcock's measurements and observations were incorrect and their work was ignored, however years of painstaking effort and meticulous research by the American astronomer Vera Rubin showed that not only were their observations correct – but that the conclusion was **inescapable** – there must somehow be more matter in the galaxies than we can see!

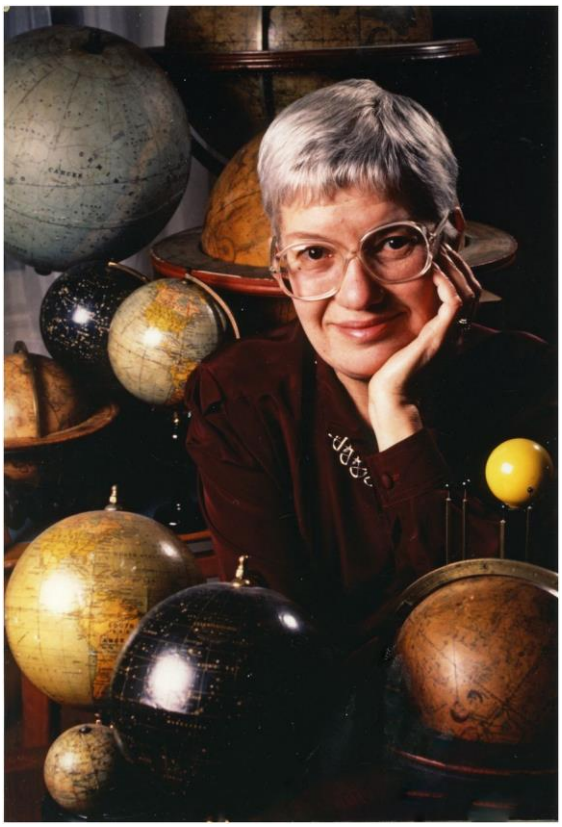


Figure 3.5 : Vera Rubin – legendary astronomer and discoverer of dark matter.

Rubin had discovered the phenomena that we now call **dark matter**.

Dark Matter

Nobody knows what **dark matter** actually even is – we just know that it has to be there because we can see its gravitational effects.

Initially people thought you could explain it away quite easily just by suggesting that there were more objects that didn't emit light…

But years and years of careful investigation into dark matter has shown us that the true nature of dark matter is far weirder than anyone could possibly imagine – and that if the dark matter is in fact matter at all – it must have incredibly strange properties.

It turns out that we can measure the existence of dark matter in other ways too. X-ray clusters, gravitational lensing and even the entire Universe itself have all been used to

measure the dark matter's gravitational effect in different ways – and miraculously all these different methods **all** predict the same proportion of dark matter to non-dark matter – approximately 90% dark matter to 10% visible matter.

Some people now even prefer to believe that dark matter is evidence that the theory of gravity is completely wrong! These people instead try to propose new theories called **modified gravity** that somehow account for the dark matter.

However these theories have their own difficulties – almost all of them fail to predict the correct properties of the Cosmic Microwave Background, and so until they manage to successfully overcome that hurdle, they will not be accepted. Many ambitious researchers live in hope of becoming the next Newton or Einstein though, and discovering their very own theory of gravity!

Main Activity:

Examine the different rotation curves for different mass profiles using the computer simulations. Which mass profile do you think matches observations best? Sketch the mass profile the program shows you.

Homework (Essay based)

Investigate some candidates for dark matter using your search engine:

You might find this article useful.

https://www.discovermagazine.com/the-sciences/what-is-dark-matter-made-of-these-are-the-top-candidates

Briefly discuss your findings. Which dark matter candidate do you think is most likely to solve the mystery? Why? What do you think about the modified gravity explanation?

If you find this homework hard to understand – don't worry, you're in good company! The cleverest scientists in the world have just as much idea as to what dark matter is as you do! Just try and do your best – see what you learn!

Success criteria

- You should write approximately 200 words.
- You should discuss some dark matter candidates (Hint – try googling "WIMPs", "MACHOs", "axions" and "primordial black holes". Look for non-technical explanations!)
- You should choose one of the candidates and try to explain your preference.
- You should discuss whether or not you believe the modified gravity explanation and why.
- Remember – there is no right answer to this homework! I just want to see that you have tried your hardest to discover something new and think for yourself – like a great scientist should!

Tutorial 4 - A Matter of Some Gravity…

"I have no special talents. I am only passionately curious."

– Albert Einstein, father of general relativity and *Time* Person of the Century.

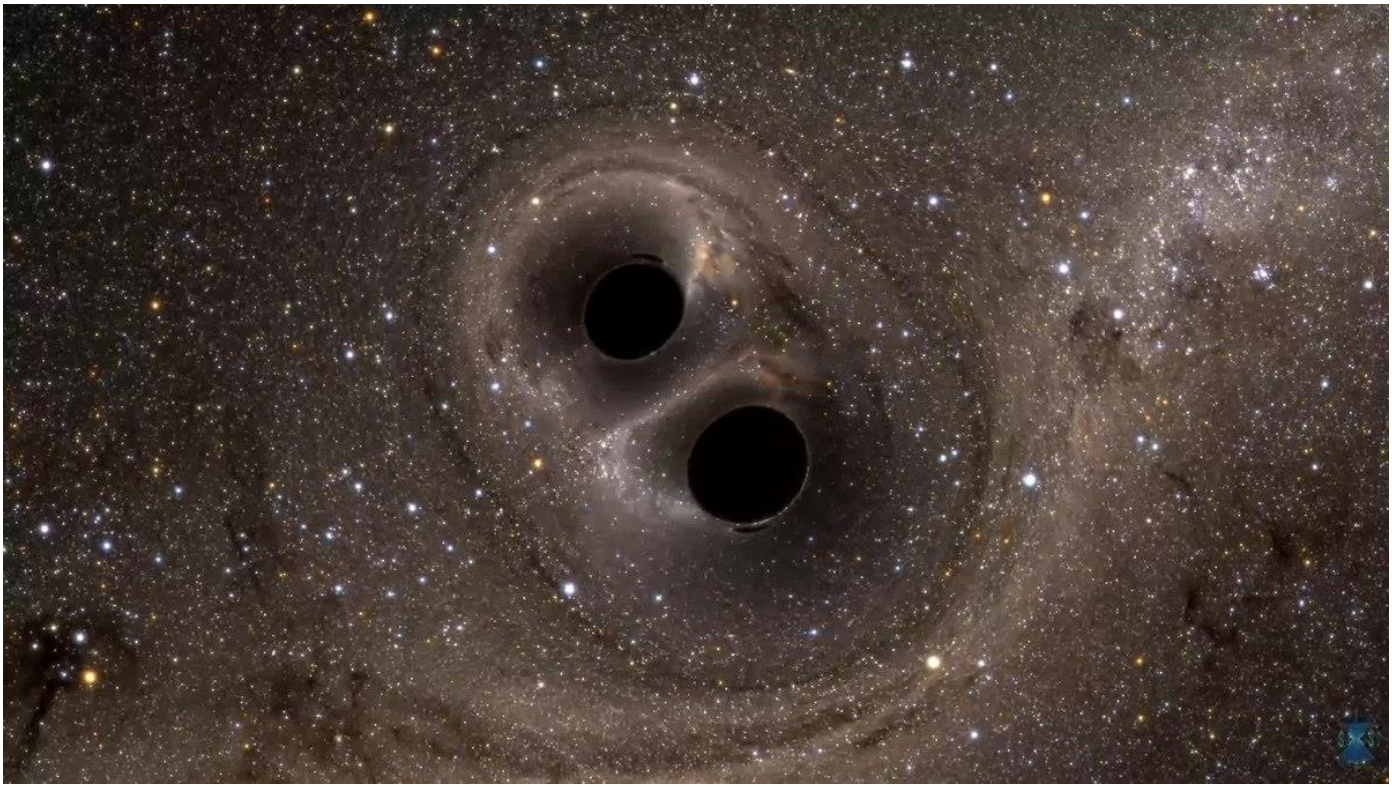


Figure 4.1 : A simulation of what the collision of two black holes might look like to an observer. Calculations were performed by numerically solving the Einstein field equations of general relativity on a computer. Notice how the black holes lens the light from the stars that are behind them!

By the end of this tutorial I will be able to:

- Describe in words how gravitational physics can produce mathematical models of the expansion history of the universe.
- Describe in words how the material contents of the universe affects the expansion history.
- Use the graphical output of a model universe to estimate the age of the universe.

General Relativity

General relativity is a notoriously difficult and technical subject, usually being first introduced at the very end of an undergraduate course in physics, or investigated in more depth at masters level. The mathematics that the theory is written is called *differential geometry* – the study of the properties of the curved surfaces we encountered earlier in the section on non-Euclidean geometry using advanced techniques from calculus.

Nevertheless, it is possible to give a pretty good flavour of what general relativity is all about without delving into the horrifying technicalities and instead focusing on the big ideas.

And when it comes to ideas, general relativity's central idea is one of the most radical and profound that has ever been put forward in science.

Spacetime

Before discussing general relativity, we must first discuss special relativity. James Clerk Maxwell's theory of electromagnetism predicts that the speed of light is a *universal constant* – i.e. it's value doesn't change, regardless of the motion of the observer or emitter!

This is a bit weird – and doesn't really fit with our common experience of how velocities work. Consider the following example:

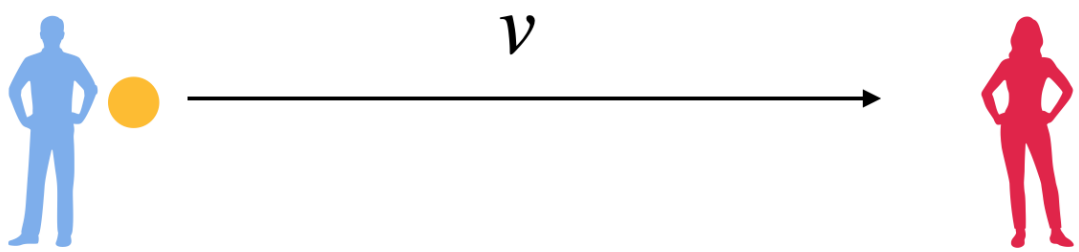


Figure 4.2 : Bob throws a ball to Alice.

Bob throws a tennis ball for Alice to catch at a speed v. Both Bob and Alice are standing still, so they both measure the speed of the tennis ball to be the same number, represented here by v. Everything so far so good!

However what happens if either Alice or Bob is *moving themselves*?

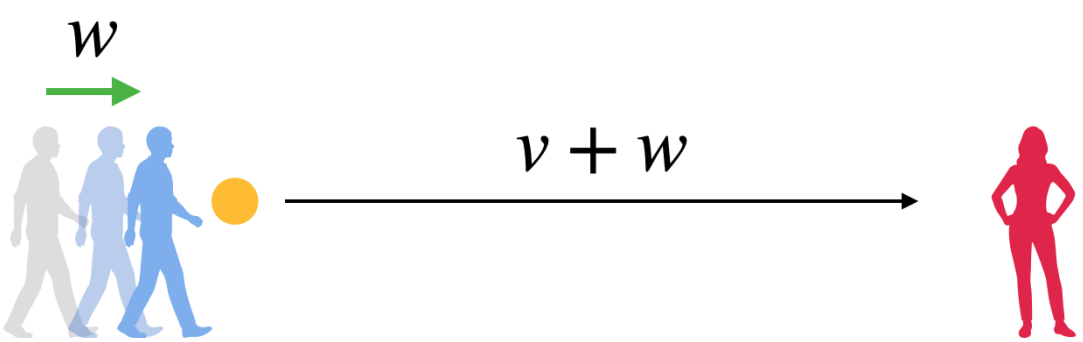


Figure 4.3 : Bob, who is now moving, throws a ball to Alice, who is still.

This time round, according to Alice, Bob is moving with a velocity w, and she measures the velocity of the ball she catches as being this velocity *plus the velocity that Bob chucks the*

ball at her with. We can say that Alice measures the velocity of the ball to be $v + w$. In contrast, from Bob's point of view, it is Alice that is moving towards him with a velocity of $-w$, and he sees the ball travelling at velocity of just v again.

This situation tallies up with our everyday experience – if we throw a tennis ball and 10 mph out of a car moving at 70 mph, we expect that someone standing still by the roadside will measure the velocity of the ball to be 80 mph.

However this is **not** what Maxwell's theory of electromagnetism predicted would happen when moving observers measure the speed of light!

Let's redo our previous experiment, but this time, Bob will shine a torch at Alice instead of throwing a tennis ball.

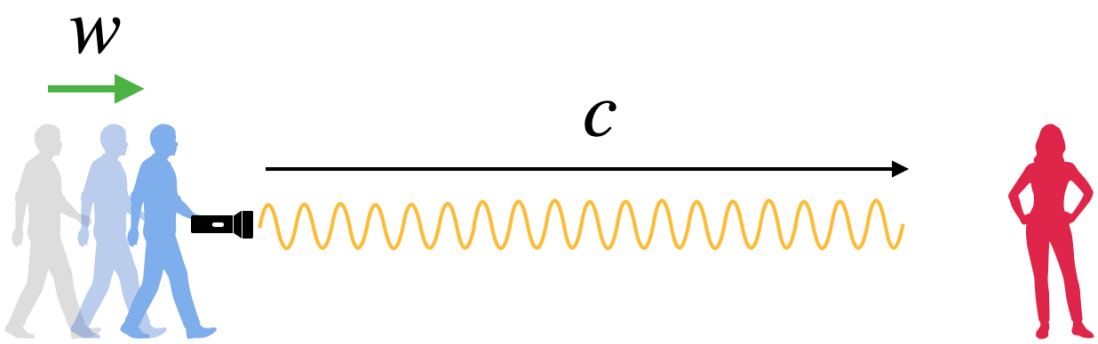


Figure 4.4 : Bob, whilst still moving, shines a torch at Alice.

What speed does Bob measure for the speed of light? The answer that Maxwell's theory tell us is that the answer is always $c \sim 3 \times 10^8$ m/s, *regardless of the motion of either the emitter or observer!* This means that Alice, unlike in the case where Bob threw the ball at her, will measure the SAME value for the speed of light!

Maxwell's theory predicted an **ultimate speed limit** for the universe. Naturally, there are some quite bizarre consequences to this, and people thought that the theory must be modified in someway to get rid of these weird effects. However it was Albert Einstein who possessed the physical insight to realise that in fact it was Maxwell's theory that was giving the correct answers to all the experiments, and that it was the rest of physics that needed adapt to the new theory.

Einstein modified Newton's laws of motion slightly to account for the constancy of the speed of light, and in doing so, came up with the theory of **special relativity**. The main consequence of special relativity was the idea that space and time could no longer be thought of as being separate things – they had to be joined together into a structure that was referred to as **spacetime**. Spacetime is simply a scientists way of saying that every event that has ever, is currently, or will ever happen can be located by four numbers, three spatial coordinates (x,y, and z) telling you where the event is relative to your current location, and one temporal coordinate telling you how long it was/will be until that event occurred.

In the same way as you can think of 3 dimensional space as being the collection of all the spatial points (x,y,z), we can think of spacetime as being the collection of (t,x,y,z) – i.e the collection of *all events*. Since there are four numbers – spacetime is 4 dimensional!

Unfortunately, 4 dimensional objects tend to be pretty hard to visualise, so usually we choose to ignore 2 spatial dimensions and focus on distance in one particular direction and time.

Let's have a think about what Alice (who is standing still) looks like in space, vs in spacetime:

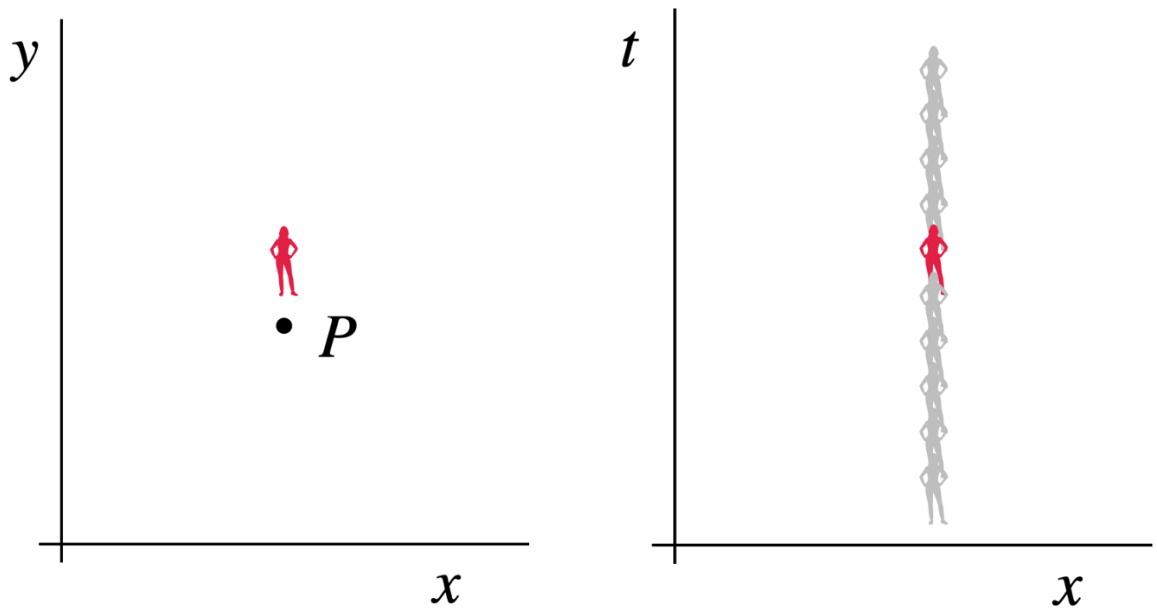


Figure 4.5 : Space vs spacetime

In the left panel of Figure 5 we can see Alice standing stationary at a point, labelled P in physical space. She stays at P, and as time advances everything looks the same. In the right panel, we see a **spacetime diagram** of the same situation. Although Alice stays on the same position on the x-axis, time advances on the t-axis, so Alice's trajectory through spacetime looks like a straight line! This line is called Alice's "**worldline**" – it represents her past, present and future!

Special relativity tells us that any object's worldline is straight, unless that object is acted on by a force (which would cause the object to accelerate, and hence change its position - curving its worldline).

Einstein's Elevator

Suppose you were in an elevator that was sealed off from the outside world. You decide to do an experiment to test how strong the gravitational field is, by dropping a tennis ball and measuring how long it takes to hit the floor. After you release the ball from your hand, it floats, motionless. What can you conclude?

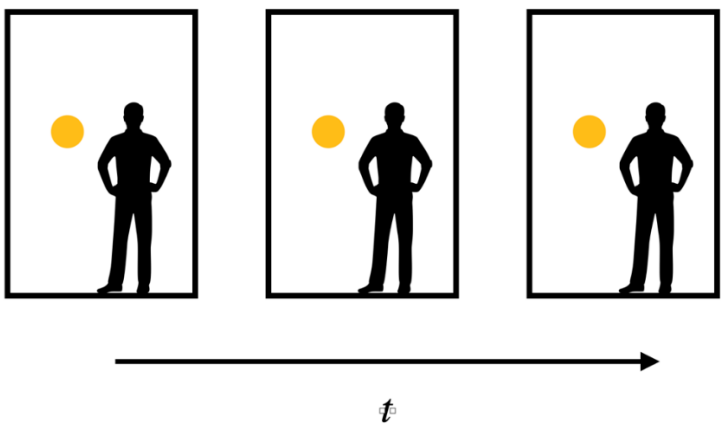


Figure 4.6a : Einstein's Elevator, Part 1.

Well, since the ball's velocity is zero from our perspective, either the elevator must be stationary, or it must be moving at **constant velocity**. This is the same as when you are moving at constant velocity in a car on the motorway – things still feel "normal" to you inside the car – you do not need any extra force exerted on you to keep you moving along with the car!

Let's say that we repeat the experiment, but this time, we measure the ball falling towards the floor. What can we conclude this time?

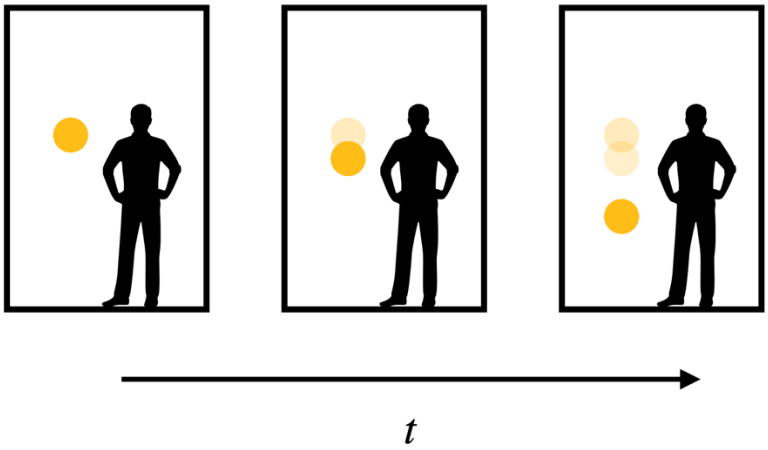


Figure 4.6b : Einstein's Elevator, Part 2

Well, naively, we might conclude that we were experiencing the effects of **gravity,** and that therefore we must be inside some gravitational field. However Einstein asks us to consider a different possibility: What if the **elevator was accelerating** in a direction opposite to the supposed direction of the gravitational pull?

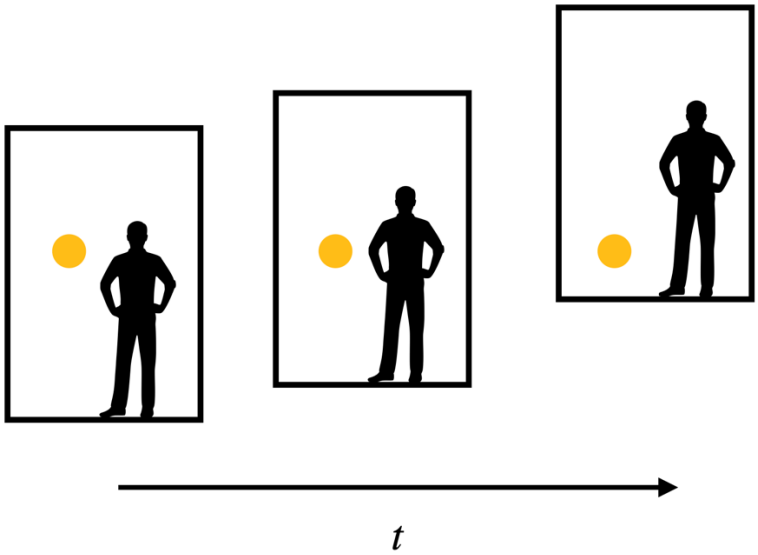


Figure 4.6c : Einstein's Elevator, Part 3

Obviously here, we would also measure the ball "falling" towards the floor (even though it is completely stationary from outside the elevator!). We would also **feel** the floor pushing up at our feet, in exactly the same way as we do in Earth's gravitational field!

Einstein's final question was the following – and it's a real head scrambler!

"Is there **any** scientific experiment that could be performed inside the elevator that would be able to tell the difference between these two situations?"

The answer, inescapably, is **no**.

And as any scientist worth his salt should conclude – if there is no experiment that could distinguish between two phenomena – **they must be the same phenomena**.

So what the hell is going on?!? You're telling me that **gravity = acceleration?!?**

Slow down, Soldier!

What we're really saying is that the curvature of the worldline seen by an accelerating observer can always be explained by the presence of a gravitational field and vice versa. But if all objects travel on straight lines through spacetime, how can we explain this?

The next, and truly amazing insight that Einstein had, was that if **spacetime itself was curved**, then "straight lines" themselves would cease to appear straight – just like how straight lines on the surface of a sphere are really segments of big circles!

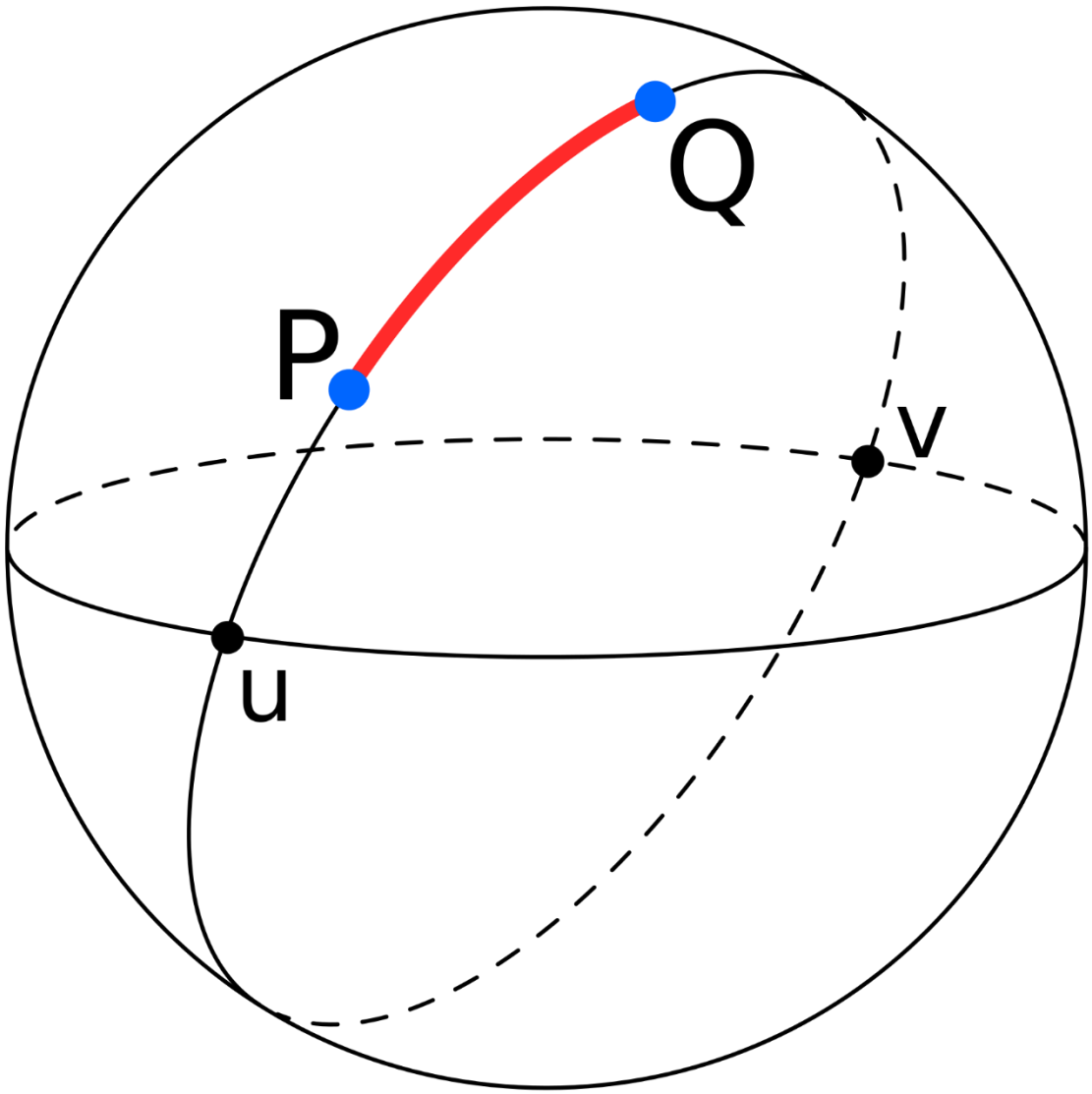


Figure 4.7 : Straight lines on a sphere

Therefore, an object that could **still seem to be accelerating**, due to the effects of curvature of spacetime – even though they were moving along what appeared to be a straight line in spacetime to them!

Einstein used this idea to **explain why gravity occurs**. According to Einstein, the presence of **mass** causes the **fabric of spacetime to warp**. This curvature in the fabric of spacetime gives rise to the effect that we perceive as "gravity". Since matter then travels along "straight lines" in this curved spacetime, we see it as the effect of a gravitational field.

In the words of the great John Archibald Wheeler:

*"Matter tells space how to **curve**. Space tells matter how to **move**."*

This remarkable statement really encapsulates the difference between Newtonian gravity and general relativity. Newtonian gravity just tells us that there is a gravitational force field that somehow transmits information instantaneously from place A to place B. It does not explain why or how this gravitational field came into being.

By contrast, general relativity provides an explanation as to why gravity exists (due to matter curving spacetime). Furthermore, in general relativity, changes in the gravitational field ripple through spacetime like waves on the surface of water – this eliminates the need for some kind of crazy "action at a distance" that seems to be baked into Newtonian gravity from the start.

The simplicity and power of this idea has led to scientists describing general relativity as one of the most beautiful scientific theories ever.

Activity: Losing Your Marbles!

We are going to investigate how curvature can affect the trajectories of particles by doing a short experiment:

We will stretch out a big bed sheet until it's taut – this sheet will represent our "spacetime".

In the middle, we will place a big watermelon (which we may or may not be able to eat at the end of the tutorial…). This watermelon will represent a big massive star – let's call it Watermelonis A.

We will then take some marbles, and investigate their trajectories through our spacetime.

What happens!? What will the effect of Watermelonis A be on our marbles?

Building a Model Universe

To actually make scientific predictions with general relativity, we need a way to calculate things with it. Einstein gave us a set of equations for doing this – called the *Einstein Field Equations:*

$$G_{\mu\upsilon} = \kappa T_{\mu\upsilon} \ .$$

It might not look like much – but wrapped up inside that short equation, there are actually 10 separate partial differential equations (a hard type of equation you wouldn't learn about until quite far into a university course!). If you wrote out every single term in these equations by hand, the result would fill an entire book!

The thing on the left hand side can be approximately understood as "curvature", whilst the thing on the right hand side ($T_{\mu\upsilon}$ – called the "stress energy tensor") should be understood as "matter". To make a GR model – you specify all the stuff that goes into $T_{\mu\upsilon}$ – essentially all the matter and energy in the universe! Then you solve the equations to calculate $G_{\mu\upsilon}$ – the curvature tensor. Once you've got the curvature tensor – you have a description of the curvature everywhere in your spacetime – essentially a **curvature map of the universe!** It is the study of these types of curvature maps that was done by cosmologists when they predicted the existence of a Big Bang! These curvature maps are our model universes.

Now, actually performing these calculations is, of course, far beyond the scope of this tutorial (go and do a physics degree to find out how it's done!), but fortunately, your handy Researchers In Schools Teacher has done all the calculations for you throughout his PhD – and has designed some computer programs that show you the output of the models!

Main Activity:

First, use the computer program to investigate the behaviour of the scale factor in a flat Universe filled with 100% dark matter. This model is called the *Einstein de-Sitter Universe* and was a very popular candidate for a long time. It is still used as basic model to test more complicated theories since it's quite easy to calculate results with.

Sketch your results:

What is the age of the universe in this model?

Second, try changing the parameters such that the universe is 100% full of radiation. This is called a "radiation-dominated universe".

Sketch your results here.

What is the age of the universe in this model?

Can you find a model with an age of the universe less than 5 billion years? Can you find a model where the universe has *another big bang in the future?* (This is more accurately called a Big Crunch!). What parameter values did you have to choose in order for these results to happen? Sketch your results below, making sure to note down the parameter values you chose.

Homework (Essay based)

Discuss the conceptual difference between general relativity and Newtonian gravity. What do you think physicists mean when they talk about theories being **beautiful**?

Success criteria

- You should write approximately 200 words.
- You should discuss how Newtonian gravity includes "spooky-action-at-a-distance", and how general relativity removes this.
- Explain how general relativity gives us an explanation for **why** gravity occurs & compare this to Newtonian gravity.
- Discuss the ideas of truth and beauty in the broader context of science.
- Discuss whether you think it's important that science be *beautiful*.

Tutorial 5 - A Modern Mystery…

"You want your mind to be boggled. That is a pleasure in and of itself."

– Saul Perlmutter, co-discoverer of the accelerated expansion of the universe with Adam Riess in 1998, and Nobel Prize winner in 2011.

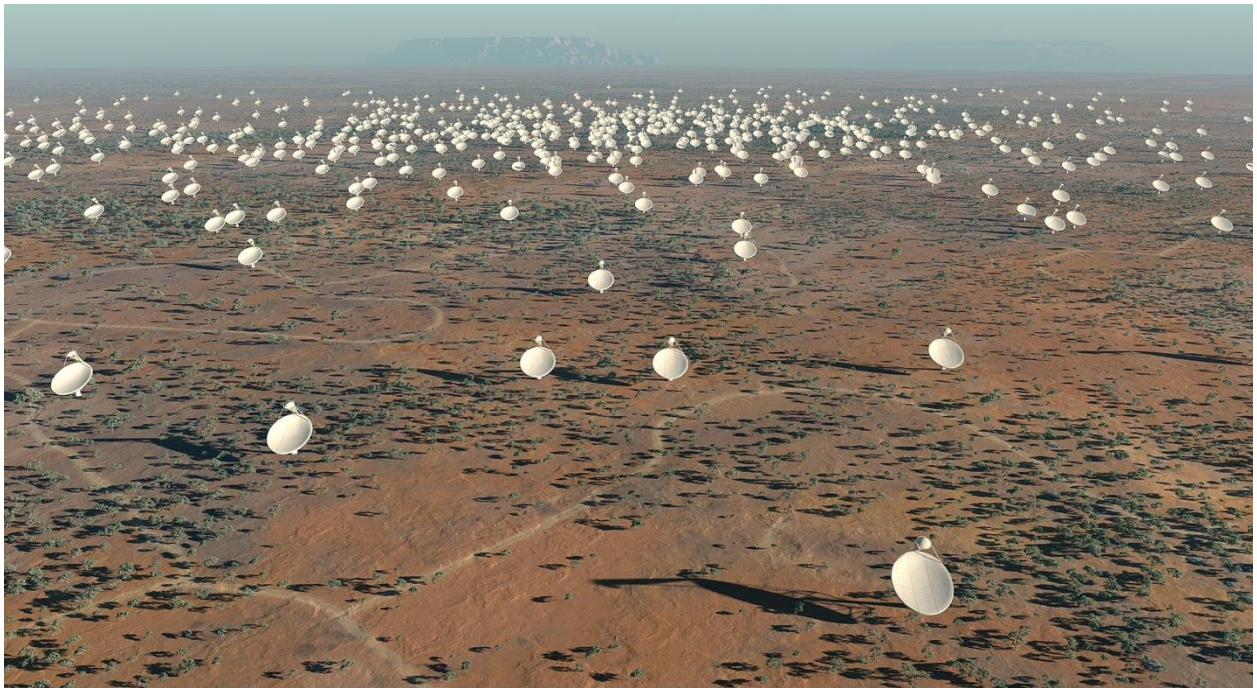


Figure 5.1 : The Square Kilometre Array (SKA) Telescope in South Africa. A network of interconnected dishes, all cooperating harmoniously as one to form the telescope with the largest baseline in the world.

By the end of this tutorial I will be able to:

- Explain why the Einstein-de Sitter/de Sitter models' predictions of the age of the universe are inconsistent with observations.
- Describe in words how the addition of a "cosmological constant" helps to alleviate this problem.
- Explain the meaning of the term "dark energy".
- Describe some proposed solutions to the dark energy problem.

Starter Activity

What is the obvious problem with the predictions of the Einstein de-Sitter model, given that we know there are some globular clusters with stars in them that are over 12 billion years old…?

The Problem of the Supernovae…

In the 1990s, two independent teams of astronomers were competing to try and observe distant collapsing stars called *supernovae*. A supernova is an **enormous stellar explosion** powered by **nuclear fusion**, which emits enough light during a brief moment in time to be comparable to the brightness of an **entire galaxy**.

Figure 5.2 : The galaxy NGC 4526, which is host to a Type 1a Supernova, SN 1994D (the bright spot at the bottom left).

There are several different ways a supernova can occur, but the projects (the Supernova Cosmology Project, led by Saul Perlmutter, and the High-z Supernova Search Team led by Brian Schmidt) were specifically interested in a type called Type 1a. Type 1a Supernovae occur when a white dwarf star gravitationally attracts enough matter from a companion star to exceed a certain mass limit – known as the *Chandrasekhar Limit* – predicted by the Nobel prize winning American-Indian mathematician and astrophysicist **Subrahmanyan Chandrasekhar**.

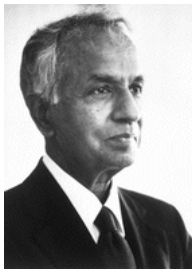


Figure 5.3 : Chandrasekhar, known affectionately as "Chandra" by his colleagues and students.

By carefully considering quantum mechanical and gravitational effects, Chandrasekhar was able to show that if a white-dwarf star became any heavier than around 1.44 solar masses, it would inevitably collapse under its own weight – as its "self-gravity" would become powerful enough to overcome the pressure forces keeping the structure of the star in place. The result will be either the formation of a neutron star, a black hole or a Type 1a supernova, depending on the mass of the collapsing star.

The Luminosity-Redshift Relation

Chandrasekhar's theory of supernovae indicates that all Type 1a supernovae should have extremely similar brightness – which makes them an exceptionally useful type of astronomical object to observe – since by measuring the amount of light that reaches Earth against the known theoretical brightness, we can precisely calculate their **luminosity distance**.

This idea is quite familiar to you – think about how a candle gets fainter the further you move away from it – by measuring how much fainter it is than its known brightness right up close, you can tell exactly how far away you must be from it!

Figure 5.4 : Consider how the intensity of the light from a candle decreases with distance…

Cosmologists refer to these Type 1a supernovae as **standard candles** – objects with known brightness that can be used to calculate distances. Since the redshifts could be measured easily using photometry (light-measuring techniques), the hope of the two teams was that they could use Type 1a supernovae to probe the *luminosity-redshift* relationship of the universe…

Hubble's law gives us the basic connection between distance and redshift, as discussed in Tutorial 1. Since the distance of a supernova can be measured by measuring its brightness,

it follows that there must be a connection between the luminosity of distant supernovae and their redshifts.

When looking at extremely large distances (as both the supernova teams were), different GR models predict slight modifications of this distance-redshift relation, and hence different luminosity-distance relations. By measuring the actual relationship between Type 1a supernovae's luminosities and their redshifts, the supernova teams hoped to try and learn about the material contents of the universe. They considered three separate models:

Model 1: A universe with 100% Dark Matter (Einstein-de Sitter Model)

In this model, we just use the standard Einstein equations from General Relativity from the previous chapter, with the stress-energy tensor being entirely made up from dark matter. Whilst this sounds like a slightly crazy assumption – we know that all galaxies are at least 90% dark matter, and most normal matter behaves in a very similar way gravitationally to dark matter, so this model is a good approximation. The effects of radiation would have been extremely important in the early universe (particularly when the CMB was formed), but at late times, dark matter's gravitational effect is much more powerful.

Model 2: A universe with 100% Dark Energy (de Sitter Model)

In this model, instead of filling the universe with dark matter, we add what's called a "cosmological constant", 'Λ' , into the Einstein equations, and assume the universe is completely empty! This changes the field equations to the following form:

$$G_{\mu\upsilon} + \Lambda g_{\mu\nu} = 0 \ .$$

Whilst this assumption is again not very realistic (since we know matter actually DOES exist), this model has some very nice mathematical properties that allow us to greatly simplify the horrible complexities of the Einstein equation. It's therefore a really useful testing ground whenever astronomers are testing out other ideas.

The effect of a positive "cosmological constant" is to *accelerate the expansion of spacetime*. Einstein originally suggested the existence of a cosmological constant because he could not believe that the universe actually expanded even though his own theory predicted it. He therefore suggested a *negative value* for this constant – that would cancel out the expansion caused by the matter. Unfortunately for him it turned out that the predictions from that model were total garbage – and when Lemaître and Friedmann used general relativity to predict the expanding universe which was consistent with observations, he gave up on the idea.

The model with a positive cosmological constant was first solved by Wilhelm de-Sitter and so it is called the "de-Sitter spacetime". The model with a negative cosmological constant is called "anti-de-Sitter spacetime". Recently anti-de-Sitter type spacetimes have become fashionable in string theory – although the versions they consider there have more than 4 dimensions!

Model 3: A universe with 70% Dark Energy and 30% Dark Matter (ΛCDM Model)

This model is a "best of both worlds" type approach – where we keep the "cold dark matter" (or CDM) in, but we also allow for the possibility of a cosmological constant, "Λ".

Now the Einstein equations look like:

$$G_{\mu\upsilon} + \Lambda g_{\mu\nu} = \kappa T_{\mu\upsilon} \ ,$$

and the behaviour of the scale factor is usually somewhere in between the Einstein-de Sitter model and de Sitter spacetime, depending on what the proportion of cosmological constant to dark matter that you put into the model is.

Main Activity 1

You will be supplied with the theoretical predictions of the three separate cosmological models for the luminosity-redshift relation and the corresponding time-evolution of the scale factor:

You must:

- Create template graphs using MS Excel showing the theoretical predictions for the distance-redshift relation for the three models.
- Create template graphs using MS Excel showing the theoretical predictions for the evolution of the scale factor in each of the three models.
- Use your graphs to estimate the age of the universe.

Main Activity 2

You will be supplied with an observational data set, detailing the redshifts and distances to a catalogue of Type 1a supernovae. This is the same catalogue used by Perlmutter's Supernova Cosmology Project that ultimately led to the award of the 2011 Nobel Prize (in conjunction with Riess et al from the High-z Supernova Search team).

You must:

- Create a detailed, well labelled graph with error bars using MS Excel, showing the measured data. Make sure you use appropriate units for the distance measurements.
- Compare the predictions of the three models to the real data, and decide for yourself which model is most likely to be correct. Write a short [5-6 sentence] justification for your choice of model. What age of the universe does your chosen model predict?

Dark Energy

Until the 1990s, CDM type models like the Einstein-de Sitter model were the only show in town. Researchers have tried to predict the value of the cosmological constant using quantum theory, but not only have they failed – the calculated value they came up with was too small by a factor of…

$$10^{120}\ .$$

Yes, that's a ten with *one hundred and twenty zeros after it*. This prediction is the only known incorrect prediction of quantum theory – yet not only is it wrong, it's wrong by **one of the largest numbers ever imagined**. This problem is known as the "cosmological constant problem".

No researchers thought the cosmological constant actually existed until the results of the Supernova Cosmology Project and the High-z Supernova Search team were published. Any matter that could play the role of the cosmological constant would seem to have extremely bizarre properties – for example its pressure would **increase** as its volume increased (think about how weird this is – usually if you squish something, i.e. decrease the

volume, you expect the pressure to increase, and if you allow it to expand, the pressure will decrease…). The observation of the accelerated expansion of the universe seems to be telling us of the existence of some hypothetical new unknown particle. In research circles, this new particle/force/phenomenon has become known as "**dark energy**" – although "dark energy" really only tells us that **we don't know anything about it at all**.

In the absence of knowledge, all sorts of dark energy models have sprung up, ranging from modified gravity type models that modify general relativity in such a way that the modifications only become noticeable at extremely large distances, to predicting new types of particle, inspired by string theory.

The next generation of telescopes that are currently being built are all focusing on trying to investigate and measure dark energy. These include the Large Synoptic Survey Telescope (LSST) which is under construction in Chile, the Square Kilometre Array telescopes in South Africa and Australia, and the Euclid space mission.

Figure 5.5 : LSST in Chile, as photographed from neighbouring telescope, Gemini.

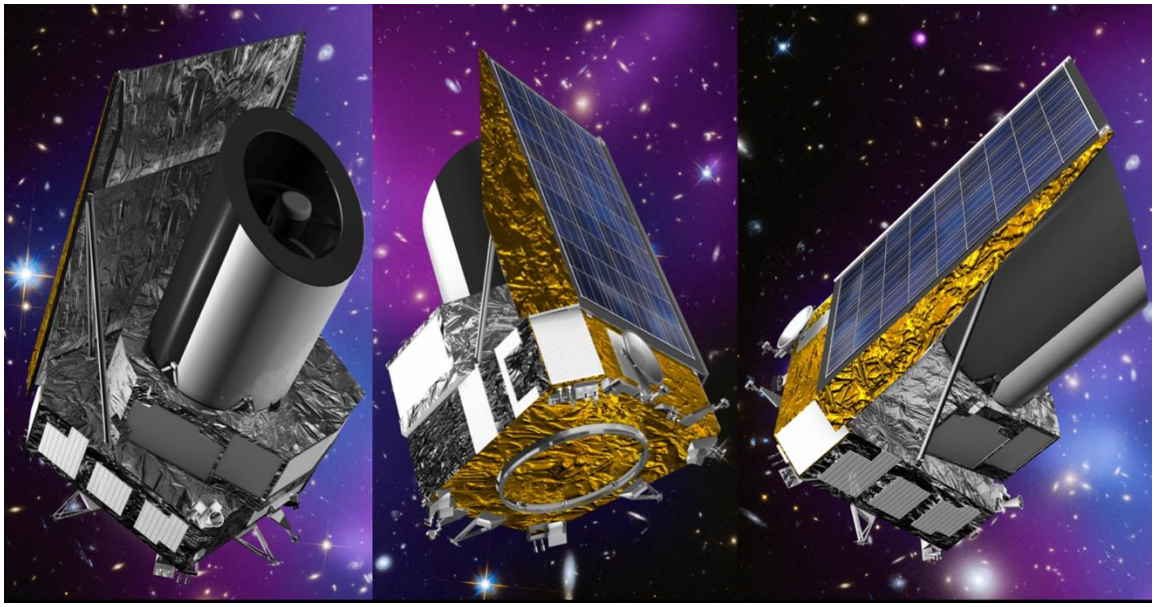


Figure 5.6 : An artist's impression of the Euclid space telescope.

These new observations will aim to map out the precise large scale structure of all the dark matter that permeates the universe, in a massive array of filaments, superclusters and voids.

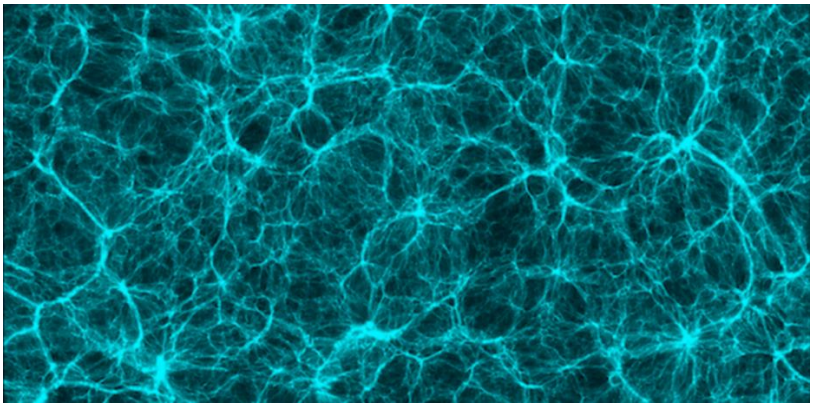


Figure 5.7 : The large scale structure of the dark matter in the Universe. This picture came from the Millennium Simulation – an enormous computer simulation of the growth of structure.

These remarkable structures form a vast *cosmic web* that even seems resemble those formed in…the human brain…

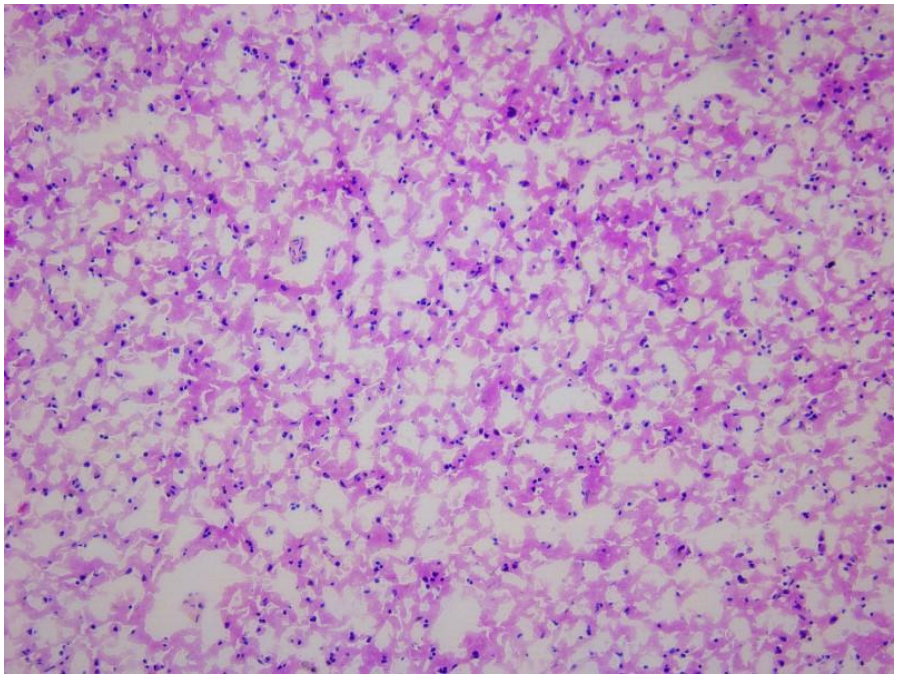


Figure 5.8 : A slide showing some brain tissue under a microscope.

Could our universe somehow be *conscious?*

Different dark energy models tend to affect the growth of this structure in different ways, with each model leaving its own "signature" in the resulting patterns. Cosmologists hope that by understanding how this structure came to be, they might find the fingerprints of new physics that isn't measurable any other way on Earth.

Dark energy really is a modern mystery – whilst we're pretty sure that dark matter exists, and that it is definitely some type of matter with a gravitational pull – dark energy is nothing more than a big question mark that seems to indicate that everything we know about physics is completely wrong.

Could **you** be the one to solve the mystery…

Final assignment

Write a short scientific report (approximately 2000 words) discussing your findings from the work we did in Tutorial 5 in the wider context of physical cosmology.

Your report should be structured as follows:

- **Abstract** (100 words)

  Here you will write a very short (<100 words) summary of your report. Think of it as like an 'advert' for someone who might be interested in your findings!

- **Introduction** (approx. 400 words)

  Here you should discuss the topic of cosmology on the whole. Try to include information from throughout the tutorials and cover a broad range of topics. Try to explain why they are important and exciting!

- **Theory** (approx. 600 words)

  In this section you should give a short description of the scientific theories that underly the work you did. Give a short description of how the models work and where they came from. Include the theoretical template graphs that you created in Tutorial 5 – these are your theoretical predictions!

- **Results** (approx. 300 words)

  In this section you should include the graph of the Union2.1 dataset that you made in Tutorial 5. Be sure to give your graph an appropriate title and well labelled axes so the reader knows what they're looking at! You should explain what the results show.

- **Discussion** (approx. 300 words)

  In this section you should discuss the *meaning* of the results. What are the results telling you about the validity of the different models you used and their predictions? What are the results telling you about the validity of the assumptions that went into those models? What are the implications for our understanding of the universe?

- **Conclusion** (approx. 300 words)

  Here you should summarise everything that you have done. Discuss current how research endeavours to learn more about these mysteries. Feel free to speculate on the nature of dark energy (provided that you argue your case!), and suggest your own ideas for how we could learn more!

Success Criteria:

- Give a brief overview of the subject area of cosmology. Explain what cosmologists do, and why they do it!

- Discuss the implications of the results of the observational measurements of the Supernova Cosmology project on our choice of cosmological model. Explain why you come to the conclusion that you do. Discuss other pieces of cosmological evidence that support your conclusions.

- Explain the impact these results have on our understanding of where the universe came from, and where it is going.

- Discuss the standard way theorists make models that are compatible with the measurements of Riess et al.
- Discuss why these models may be unsatisfactory on a fundamental level.
- Discuss two popular alternative explanations for the results of Riess et al. Explain what your personally preferred explanation is and why.
- Explain the difference between dark matter and dark energy. Discuss evidence for the existence of dark matter.
- Discuss the status ongoing observational efforts to understand dark matter/dark energy.
- Briefly discuss the issue this project has addressed in the broader context of physical cosmology on the whole. How important do you think this issue is relative to other cosmological mysteries?
- Briefly discuss this problem in the context of our fundamental understanding of physics.
- Properly reference your sources of information.
- Use correct spelling and grammar throughout.

Tutorial 6 – Feedback tutorial

What is the Purpose of Tutorial 6?

- To receive feedback on your final assignment
- To respond to the feedback from your Uni Pathways teacher
- To write targets for improvement on your final assignment

Final assignment feedback from your Uni Pathways Teacher

(Remember to look at the mark scheme to help you understand what you have done well so far, and how you can do even better in your final assignment)

Here are three things that my Uni Pathways Teacher thought I did well in my draft assignment
•   •   •

Here are three things that my Uni Pathways Teacher thinks that I could do to get a higher mark in my final assignment
•   •   •

Tasks from my Uni Pathways Teacher to do during the feedback tutorial to help me improve
•   •   •
My response:

Actions I will take to improve my final assignment after this tutorial…
•   •   •

Hand in date for my final assignment:	

Tutorial 7 – Final tutorial

What is the Purpose of Tutorial 7?

- To receive feedback and a grade on your final assignment.
- To reflect on the programme including what you enjoyed and what was challenging.
- To ask any questions you may have about university.

Final assignment feedback from my Uni Pathways Teacher

Final mark:	University style grade:
Feedback: Here are three things that my Uni Pathways teacher thought I did well in my final assignment • • •	

Here are three things that my Uni Pathways teacher thinks I should remember for when I am doing this kind of study in the future
• • •

University

What questions do you still have about University after taking part in Uni Pathways?
•   •   •

Reflecting on Uni Pathways

What did you most enjoy about Uni Pathways?
•   •   •

What did you find challenging about the programme?	How did you overcome these challenges?
• • •	• • •

Appendix 1 – Referencing correctly

When you get to university, you will need to include references in the assignments that you write, so we would like you to start getting into the habit of referencing in your Brilliant Club assignment. This is really important, because it will help you to avoid plagiarism. Plagiarism is when you take someone else's work or ideas and pass them off as your own. Whether plagiarism is deliberate or accidental, the consequences can be severe. In order to avoid losing marks in your final assignment, or even failing, you must be careful to reference your sources correctly.

What is a reference?

A reference is just a note in your assignment which says if you have referred to or been influenced by another source such as book, website or article. For example, if you use the internet to research a particular subject, and you want to include a specific piece of information from this website, you will need to reference it.

Why should I reference?

Referencing is important in your work for the following reasons:

- It gives credit to the authors of any sources you have referred to or been influenced by.
- It supports the arguments you make in your assignments.
- It demonstrates the variety of sources you have used.
- It helps to prevent you losing marks, or failing, due to plagiarism.

When should you use a reference?

You should use a reference when you:

- Quote directly from another source.
- Summarise or rephrase another piece of work.
- Include a specific statistic or fact from a source.

How do I reference?

There are a number of different ways of referencing, and these often vary depending on what subject you are studying. The most important to thing is to be consistent. This means that you need to stick to the same system throughout your whole assignment. Here is a basic system of referencing that you can use, which consists of the following two parts:

1. **A marker in your assignment**: After you have used a reference in your assignment (you have read something and included it in your work as a quote, or re-written it your own words) you should mark this is in your text with a number, e.g. [1]. The next time you use a reference you should use the next number, e.g. [2].
2. **Bibliography**: This is just a list of the references you have used in your assignment. In the bibliography, you list your references by the numbers you have used, and include as much information as you have about the reference. The list below gives what should be included for different sources.

    a. **Websites** – Author (if possible), title of the web page, website address, [date you accessed it, in square brackets].

    E.g. Dan Snow, 'How did so many soldiers survive the trenches?', http://www.bbc.co.uk/guides/z3kgjxs#zg2dtfr [11 July 2014].

    b. **Books** – Author, date published, title of book (in italics), pages where the information came from.

    E.g. S. Dubner and S. Levitt, (2006) *Freakonomics*, 7-9.

    c. **Articles** – Author, 'title of the article' (with quotation marks), *where the article comes from* (newspaper, journal etc.), date of the article.

    E.g. Maev Kennedy, 'The lights to go out across the UK to mark First World War's centenary', Guardian, 10 July 2014.

Notes

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